

PROCEEDINGS

OF THE

PHYSICAL SOCIETY OF LONDON.

From October 1895 to October 1896.

VOL. XIV.

LONDON:
PRINTED BY TAYLOR AND FRANCIS,

RED LION COURT, FLEET STREET.

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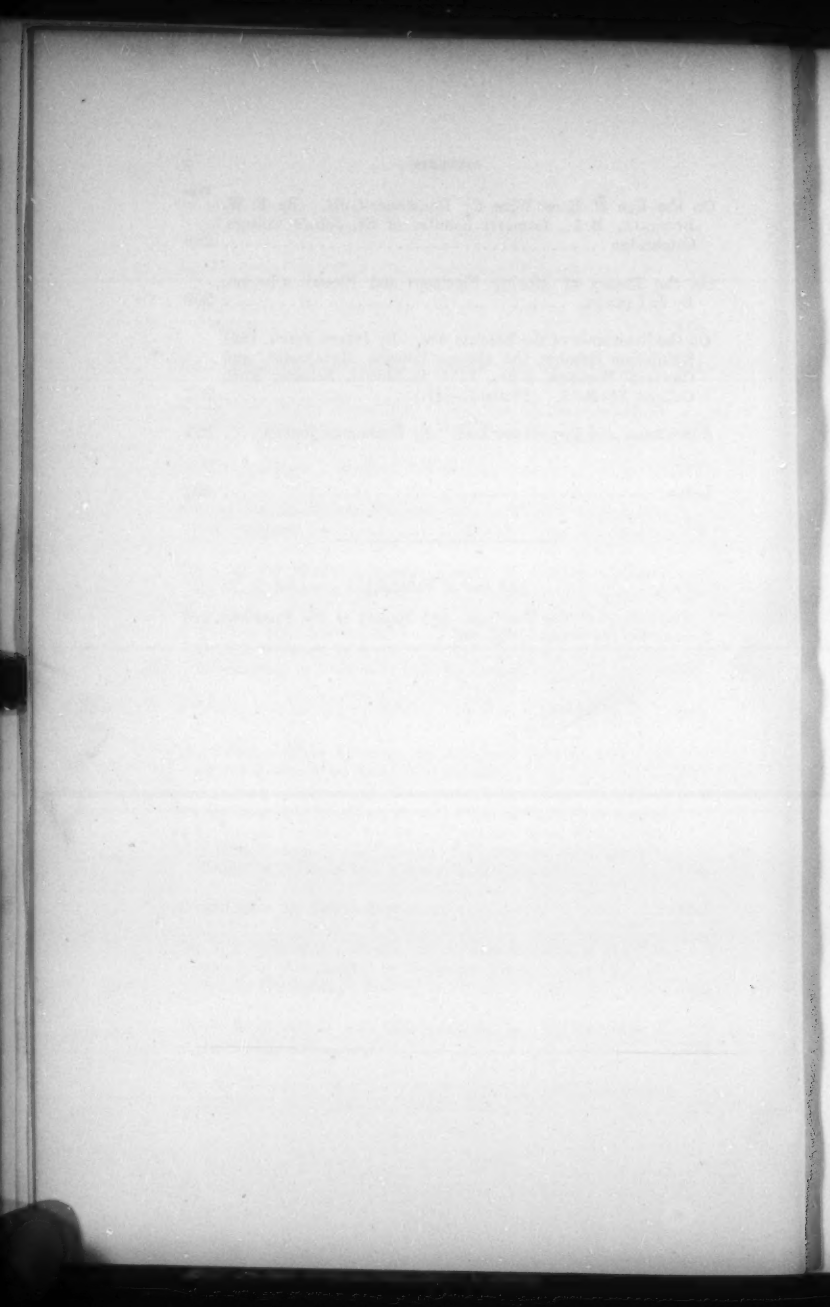
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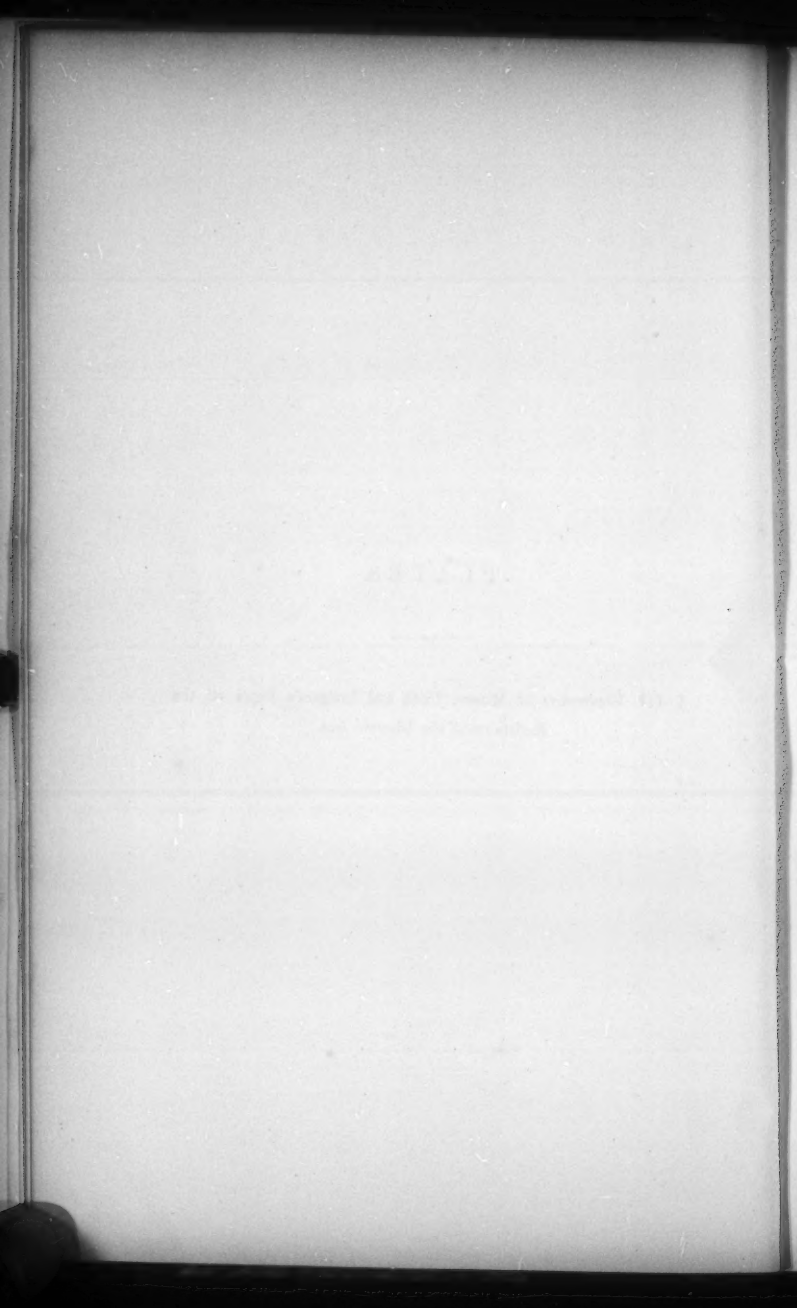
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PHYSICAL PROCEEDINGS.

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PROCEEDINGS
AT THE
MEETINGS OF THE PHYSICAL SOCIETY
OF LONDON.

SESSION 1895-96.

February 8th, 1895.

Captain ABNEY, President, in the Chair.

The following were elected Members of the Society:—A. E. BRISCOE, H. F. W. BURSTALL, E. H. GRIFFITHS, A. A. CAMPBELL SWINTON, and W. E. WILSON.

Mr. W. B. CROFT gave an exhibition of some Simple Physical Apparatus, of Photographs of Optical Phenomena, and of Clock Springs broken during frost.

Mr. S. SKINNER read a paper "On the Tin-Chromic Chloride Cell."

February 22nd, 1895.

Captain ABNEY, President, in the Chair.

W. R. COOPER, J. H. GARDINER, and J. W. RODGER were elected Members of the Society.

A communication was made by Mr. G. H. BRYAN, "On the Mechanical Analogue of Thermal Equilibrium between Bodies in Contact."

March 8th, 1895.

Mr. WALTER BAILY, Vice-President, in the Chair.

C. E. ASHFORD, H. BORNS, S. H. BURBURY, E. CATCHPOOL, C. W. S. CRAWLEY, N. EUMORFOPOULOS, S. G. MOSTYN, P. E. SHAW, H. STANSFIELD, F. T. TROUTON, and J. WATT were elected Members of the Society.

Mr. NABER exhibited "a New Form of Gas Voltameter."

Dr. JOHNSTONE STONEY, F.R.S., exhibited:—

- (1) "The Local Helio-stat;"
- (2) "An Improved Siderostat."

Mr. G. U. YULE read a paper "On a Simple Form of Harmonic Analyser."

Prof. MINCHIN, F.R.S., and Mr. H. N. ALLEN made a communication, "On the Energy Movements in the Medium separating Electrified or Gravitating Particles."

March 22nd, 1895.

(At the Royal College of Science, South Kensington.)

Mr. R. T. GLAZEBROOK in the Chair.

H. E. ARMSTRONG, D. K. MORRIS, W. B. MORTON, and J. J. THOMSON were elected Members of the Society.

Prof. A. W. RUCKER and Mr. E. EDGER read a paper "On the Objective Reality of Combination Tones."

Dr. BURTON made the following communications:—

- (1) "Some Acoustical Experiments;"
- (2) "Objective Demonstration of Combination Tones."

April 26th, 1895.

Mr. WALTER BAILY, Vice-President, in the Chair.

A. CAMPBELL, ELEANOR M. WYNN EDWARDS, A. HAY, G. R. KAYE, S. A. NOTCUTT, W. RHEAM, H. E. SOPER, and E. THOMAS were elected Members of the Society.

Prof. S. P. THOMPSON read a note "On a Neglected Experiment of Ampère."

Mr. W. G. RHODES read a paper on "A Theory of the Synchronous Motor."

Mr. BRYAN communicated a paper "On a Simple Graphic Interpretation of the Determinantal Relation of Dynamics."

May 10th, 1895.

Captain ABNEY, President, in the Chair.

C. A. BUCKMASTER, E. RIGBY, G. L. THOMAS, and S. C. F. WHITE were elected Members of the Society.

Mr. HERROUN read a paper "On the Iodine Voltameter."

Mr. A. SHARP read a paper "On a New Method in Harmonic Analysis."

May 24th, 1895.

Captain ABNEY, President, in the Chair.

Sir FREDERICK BRAMWELL was elected a Member of the Society.

Dr. KUENEN read a paper "On the Condensation and the Critical Phenomena of Mixtures of Ethane and Nitrous Oxide."

Mr. BURSTALL communicated a paper "On the Measurement of Cyclically Varying Temperature."

June 14th, 1895.

Captain ABNEY, President, in the Chair.

S. BEALE and T. H. LITTLEWOOD were elected Members of the Society.

The reading of Mr. BURSTALL's paper, "On the Measurement of a Cyclically Varying Temperature," was completed.

Mr. N. F. DEERE made a communication on "The Thermal Constants of the Elements."

June 28th, 1895.

Dr. GLADSTONE, Vice-President, in the Chair.

CATHERINE E. BERRIDGE, AGATHA LEONARD, DOROTHY MARSHALL, and J. J. STEWART were elected Members of the Society.

Mr. BOWDEN read a note "On an Electromagnetic Effect."

Mr. RHODES read a paper "On the Armature Reaction in a Single-Phase Alternating-Current Machine."

Mr. SHELFORD BIDWELL read a paper "On the Electrical Properties of Selenium."

October 25th, 1895.

Mr. WALTER BAILY, Vice-President, in the Chair.

Sir ANDREW NOBLE and F. W. SHURLOCK were elected Members of the Society.

Prof. J. PERRY, F.R.S., and Mr. H. F. HUNT read a paper "On the Development of Arbitrary Functions."

Mr. LANCHESTER read a paper "On the Radial Cursor: an Improvement of the Slide Rule."

November 8th, 1895.

Prof. A. W. RÜCKER, Vice-President, in the Chair.

Mr. W. H. EVERETT read a paper "On the Magnetic Field of any Cylindrical Coil or Plane Circuit."

Mr. GRIFFITHS and Miss MARSHALL read a paper "On the Latent Heat of Evaporation of Benzene."

Prof. W. RAMSAY and Miss MARSHALL read a paper "On a Method of Comparing the Heats of Evaporation of different Liquids at their Boiling-points."

November 22nd, 1895.

Captain ABNEY, President, in the Chair.

This was a Special Meeting. The following resolution with reference to the Articles of Association was passed:—

"In Article 33 to strike out the words 'by the payment of £10 in one sum,' and in place of this to insert the words 'the

composition fee shall be for every member who shall not have paid ten annual subscriptions, fifteen times the amount of the annual subscription payable by each member; and for any member who shall have already paid ten or more annual subscriptions, ten times the amount of the annual subscription payable by such member."

At the Ordinary Meeting of the same date, G. M. CLARK, W. H. EVERETT, J. FRITH, W. LARDEN, R. A. LEHFELDT, and J. H. VINCENT were elected Members of the Society.

Dr. JOHNSTONE STONEY exhibited some Photographs of Spectra.

Mr. APPELYARD made the following communications:—

- (1) "A note on the Action of Sulphur-Vapour on Copper;"
 - (2) "A Direct-reading Platinum Thermometer;"
 - (3) "An Historical Note on Resistance and its Change with Temperature."
-

December 13th, 1895.

Prof. REINOLD, Vice-President, in the Chair.

The Resolution of the Special General Meeting held on November 22nd, 1895, was confirmed.

H. G. BELL, G. W. COBHAM, E. D. FRIDLANDER, L. G. RYLANDS, and R. C. SLATER were elected Members of the Society.

Dr. JOHN SHIELDS made a communication on "A Mechanical Device for performing the Temperature Corrections of Barometers."

Prof. RÜCKER communicated a paper "On the Existence of Vertical Earth-Air Electric Currents in the United Kingdom."

January 24th, 1896.

Captain ABNEY, President, in the Chair.

H. R. J. BURSTALL, W. CASSIE, and H. S. S. HARVEY were elected Members of the Society.

Mr. CAMPBELL SWINTON exhibited some Photographs taken by Prof. Röntgen's Method.

Mr. E. SCOTT exhibited and explained some Geometrical Instruments invented by himself and Signor MONTICOLA.

Prof. J. D. EVERETT communicated a paper "On Resultant Tones."

Annual General Meeting.

February 14th, 1896.

Captain ABNEY, President, in the Chair.

The following Report of the Council was read by the Chairman :—

Since the last General Meeting of this Society, though no great change has been set on foot, the Members of the Society have had fuller opportunities for forming an opinion of those which were instituted in January 1895 than they possessed last February.

The monthly issue of ' Abstracts ' and of ' Papers ' in one cover has proceeded steadily for fourteen months. During the first twelve months 791 Abstracts have been made and printed, and the whole furnished with a Name Index, a Classified Index, and a General Index, all of the most copious and complete character.

Members are therefore amply able to judge of the advantages of this important step, and the Council believes that any movement in a contrary direction would now be viewed by the Members, individually and collectively, with very great regret.

The Council, therefore, takes this occasion to emphasize the fact that much additional outlay necessarily attends the publication of these Abstracts, and that the scale of payments hitherto in use is altogether inadequate to meet the expense.

The Society, at a Special Meeting held on November 22nd, 1895, has already raised the Composition Fee by 50 per cent. In any case, this was a matter that required adjustment, on account of the low rate of interest of appropriate securities at the present time.

But it is the intention of this Council to strongly recommend to their successors in office the introduction of much more far-reaching measures, the details of which have occupied much attention during the past year, and the chief features of which have already been presented to the Members of the Society in a circular letter. The Council commends this letter to the serious consideration of every Member.

Since the last General Meeting there have taken place fourteen ordinary business meetings, all (with one exception) held in the usual way at the rooms of the Chemical Society, Burlington House.

The Members of the Society now number about 420.

Among the deaths which have taken place in the ranks of the Society are those of Mr. T. H. Huxley and Mr. J. E. Gillett.

The following additions have been made to the Library during the past year:—

Newspapers and Magazines:—

Nature.
The Electrical Review.
The Electrician.
Electrical Plant.
Engineering.
The Philosophical Magazine.
Wiedemann's Annalen.
Beiblätter.
Journal de Physique.

Journals of Societies, British:—

Journal of the Society of Arts.
Proceedings of the Birmingham Philosophical Society.
Transactions and Proceedings of the Cambridge Philosophical Society.
Transactions and Proceedings of the Royal Dublin Society.
Journal of the Institute of Electrical Engineers.
Memoirs and Proceedings of the Manchester Literary and Philosophical Society.
Proceedings of the Institute of Mechanical Engineers.
Quarterly Journal of the Royal Meteorological Society.
Proceedings of the Royal Institution.
Proceedings of the Royal Society.
Proceedings of the Edinburgh Mathematical Society.
Journal of the Camera Club.
Proceedings of the London Mathematical Society.
Monthly Notices of the Royal Astronomical Society.

Journals of Societies, Colonial:—

Journal and Proceedings of the Royal Society of New South Wales.
Geological and Natural History Survey of Canada.
Hong Kong Observatory Observations.

Journals of Societies, Foreign:—

Proceedings of the Academy of Natural Sciences, Philadelphia.
Journal of the Physico-Chemical Society of Russia.
Verhandlungen, Berlin Physical Society.

Mémoires and Séances of the Physical Society of France.
Journal of the College of Science of Japan.
Transactions of the Seismological Society, Japan.
Séances and Bulletin of the International Academy of Sciences
of Cracow.
Report of the Ohio Meteorological Bureau.
Report of the Ohio Weather and Crop Service.
Prace Matematyczno-Fizyczne, Warsaw.

Books, &c. :—

Œuvres de Laplace, Tome xi.
Stanley's Nebular Theory.

THE RIGHT HON. THOMAS HENRY HUXLEY, Professor of Biology and Dean of the Royal College of Science, London. Born at Ealing, Middlesex, May 4th, 1825; died at Eastbourne, Sussex, June 29th, 1895, in his 71st year; buried at the Cemetery of St. Peter's Marylebone, East Finchley.

As a man of science Huxley will best be remembered as the Champion of the doctrine of Evolution, under whose influence that permeated all branches of science not wholly mathematical, and revolutionized thought, and as having introduced rational methods into the teaching of Biology.

Huxley early desired to become an engineer, and the appreciation of the constructive side of nature runs throughout his scientific work. In connection with a course of lectures delivered at the London Institution, and afterwards in the South Kensington Museum, which led up to his Manual of "Physiography," he materially extended the limitations of that term, and laid the foundation of a system of elementary scientific education, which has done as much for the popularization of physical science as of geology and biology. In the year 1856 he accompanied his friend Tyndall on his first visit to the Glaciers of the Alps, the result being the presentation, during the following year, to the 'Philosophical Transactions' of a joint monograph in experimental physics, entitled "On the Structure and Motion of Glaciers."

Huxley was President of the Royal Ethnological, Quaker Microscopical, and Geological Societies. In 1869 he was prompted by the writings of Lord Kelvin (then Sir W. Thomson) to address the latter body on the "Limitations of Geological Time," and he worked out a theory of the Zöic Period of the Earth's History, which, as to periods of years, was by no means in harmony with that of his distinguished antagonist. It will be within the recollection of physicists that during the last few months of Huxley's life, the work of Perry has led to the conclusion that on the physical basis the age of the planet has been underestimated, and that the greater time demanded by the paleontologists (whom Huxley represented) may well have elapsed.

Huxley was a Hunterian Professor of Comparative Anatomy at the Royal College of Surgeons, a Fullerian Professor of Physiology at the Royal Institution, a President of the British Association, a Rede Lecturer at Cambridge, and a Romanes Lecturer at Oxford. In 1870 he was elected a Member of the London School Board, and he served on several Royal Commissions. In 1892 he was made a Privy Councillor, having previously refused Knighthood. He was a Corresponding Member of the Institute of France, and a Ridare of the Pole Star of Sweden, and he was the recipient of Diplomas, Medals, and Hon. Fellowships of numerous Universities, Institutes, and Academies.

The adoption of the Report was proposed and passed unanimously.

The election of Officers and other Members of Council then took place, the new Council being constituted as follows:—

President.—Capt. W. DE W. ARSEY, R.E., C.B., D.C.L., F.R.S.

Vice-Presidents who have filled the Office of President.—Dr. J. H. GLADSTONE, F.R.S.; Prof. G. C. FOSTER, F.R.S.; Prof. W. G. ADAMS, M.A., F.R.S.; Lord KELVIN, D.C.L., LL.D., F.R.S.; Prof. R. B. CLIFTON, M.A., F.R.S.; Prof. A. W. REYNOLDS, M.A., F.R.S.; Prof. W. E. AYTON, F.R.S.; Prof. G. F. FITZGERALD, M.A., F.R.S.; Prof. A. W. BOCKEL, M.A., F.R.S.

Vice-Presidents.—SHELFORD BIDWELL, M.A., LL.B., F.R.S.; Major-General E. R. FESTING, R.E., F.R.S.; Prof. J. PERRY, D.Sc., F.R.S.; G. JOHNSTONE STONEY, M.A., F.R.S.

Secretaries.—T. H. BLAKESLEY, M.A., M.Inst.C.E.; H. M. ELDER, M.A.

Treasurer.—Dr. E. ATKINSON.

Demonstrator and Librarian.—C. VERNON BOYS, F.R.S.

Other Members of Council.—WALTER BAILY, M.A.; C. V. BURTON, D.Sc.; L. FLETCHER, M.A., F.R.S.; R. T. GLAZEBROOK, M.A., F.R.S.; Prof. A. GRAY, M.A.; G. GRIFFITH, M.A.; Prof. G. M. MINCHIN, M.A.; Prof. W. RAMSAY, Ph.D., F.R.S.; Prof. S. P. THOMPSON, D.Sc., F.R.S.; Prof. S. YOUNG, D.Sc., F.R.S.

Votes of thanks were passed to the Chemical Society; to the OFFICERS; and to the AUDITORS.

THE TREASURER IN ACCOUNT WITH THE PHYSICAL SOCIETY, FROM JANUARY 1st, 1894, TO DECEMBER 31st, 1895.

Dr.		Cr.	
	£ s. d.		£ s. d.
Jan. 1, 1895. Balance in the Bank	272 4 10	Jan. 1, 1895. Balance paid to the Treasurer	4 18 6
Entrance-Fees	53 0 0	Taylor and Francis:—	
Life Compositions	255 0 0	Separate Copies	28 1 6
Subscriptions for 1891-94	36 0 0	Miscellaneous	42 10 7
1895	153 0 0	Proceedings	233 16 9
" 1896	4 0 0	Abstracts	214 0 6
	591 0 0		538 9 4
Dividends	17 5 8	Editor	50 0 0
Grant from the British Association	100 0 0	Petty Cash	7 7 6
Sales:—		Abstractors	72 8 10
Taylor and Francis	29 0 8	Periodicals	23 15 1
1 copy Van der Waals	0 4 6	Bridging	2 17 6
Donation, Suspense Account	39 4 11	Cheques and Bank charges	23 13 7
Balance due to the Treasurer	20 0 0	Lavender Fuel	0 9 4
	3 16 2	Attendance	6 0 0
		Reporting	1 17 0
		Tea &c.	17 5 8
		Rent of Name Plate	5 0 0
		Petty Cash:—	2 2 0
		Mr. Blakesley	1 11 2
		Mr. Eider	4 15 6
		Dr. Atkinson	2 12 5
		Balance in the Bank, Dec. 31, 1895	6 19 1
			273 1 9
			<u>£1013 11 7</u>

EDMUND ATKINSON, Treasurer.

Examined and found to agree with the vouchers, books, and accounts.

RICHARD INWARDS, }
CONRAD W. COOKE. } Auditors.

February 1, 1896.

PROPERTY ACCOUNT OF THE PHYSICAL SOCIETY, DECEMBER 31, 1895.

ASSETS.		LIABILITIES.	
£	s. d.	£	s. d.
Subscriptions due, estimated	19 0 0	Subscriptions for 1896 in advance	4 0 0
£400 Furness 4 per cent. Debenture Stock	456 0 0	Donation, Suspense Account	20 0 0
£1000 Midland 4 per cent. Preference Stock	1194 0 0	Balance due to the Treasurer	3 16 2
£200 Metropolitan Board of Works $3\frac{1}{2}$ per cent. Stock	217 0 0	Abstracts, Oct.-Dec.	35 0 0
£270 Lancaster Corporation 3 per cent. Stock	227 6 8	Printers	120 0 0
£254 2s. 9d. New South Wales $3\frac{1}{2}$ per cent. Inscribed Stock	249 7 5	Rent &c.	23 0 0
Balance in the Bank	273 1 9	Balance	2429 19 8
	<u>£2635 15 10</u>		<u>£2635 15 10</u>

EDMUND ATKINSON, *Treasurer.*

Securities examined and found correct. For the New South Wales $3\frac{1}{2}$ per cent. Stock no certificate is given. The Bankers have duly received the last dividend for October 1895.

RICHARD INWARDS, } *Auditors.*
CONRAD W. COOKE. }

February 1, 1896.

PROCEEDINGS
OF
THE PHYSICAL SOCIETY
OF LONDON.

OCTOBER 1895.

I. The Development of Arbitrary Functions.

By Prof. JOHN PERRY *and* Mr. H. F. HUNT.*

STUDENTS at the Finsbury Technical College have for some years carried out the Fourier development idea of the late Professor Clifford by practical Geometrical methods. The curve which graphically represents the function is supposed to be wrapped round a circular cylinder, and the areas of its projections upon two diametral planes enable the coefficients to be found. In 'The Electrician' of June 28th, 1895, one of us gave detailed instructions for carrying out the process. We do not know of any method of development in a Fourier series which can compare with this in either rapidity or accuracy.

In the discussion of Prof. Henrici's paper, April 13th, 1894, one of us described a graphical method of developing any arbitrary function in a series of other normal forms than sines and cosines, such as Bessels or Zonal Spherical Harmonics. It consisted in wrapping the curve which represents the function round a specially shaped cylinder, not circular, and projecting it upon a certain plane. One of us has wasted many months in finding with great exactness a sufficient number of coordinates of the trace of the cylinder suitable

* Read October 25, 1895.

for a Zeroth Bessel development. The labour was unnecessary because the coordinate most troublesome to calculate is not really needed: that is, the actual shape of the cylinder is not needed, and indeed it is better to put aside altogether the idea of a curve being wrapped round a cylinder.

To develop any arbitrary function of x (let us call it y) in normal forms, the real difficulty consists in finding the value of an integral such as $\int_0^a y \cdot Q(x) \cdot dx$, where $Q(x)$ is some tabulated function. If now z is another tabulated function which is the integral of $Q(x)$ the required integral is $\int y \cdot dz$.

Suppose the values of y for 25 equidistant values of x to be known, from $x=0$ to $x=a$ (that is, a is divided into 24 equal parts). Let the corresponding values of z be also tabulated, and let a curve be drawn with the values of y as ordinates and the values of z as abscissæ; the area between this curve and the axis of z gives the value of the integral required.

To illustrate the method we shall show how it is applied in a well-known kind of problem. An arbitrary function $y=f(r)$ of r from $r=0$ to $r=a$ is to be developed in Zeroth Bessels; that is, we must determine the constants A_1, A_2 , &c., in

$$f(r) = A_1 J_0(\mu_1 r) + A_2 J_0(\mu_2 r) + \&c. + A_n J_0(\mu_n r) + \&c., \quad (1)$$

where μ_1, μ_2 , &c., are the successive roots of

$$J_0(\mu a) = 0. \quad (2)$$

Now it is well known that

$$A_s = \frac{2}{a^2 [J_1(\mu_s a)]^2} \int_0^a r f(r) J_0(\mu_s r) \cdot dr; \quad (3)$$

or, letting $\mu_s r = x$, (3) becomes

$$A_s = \frac{2}{a^2 [J_1(\mu_s a)]^2} \frac{1}{\mu_s^2} \int_0^{\mu_s a} x f(x) J_0(x) \cdot dx. \quad (4)$$

Now

$$\int x \cdot J_0(x) \cdot dx = x \cdot J_1(x),$$

which is easily found for any value of x .

To evaluate the integral in (4), take the 25 equidistant values of our function from 0 to a and use them as the ordinates of points in a curve. The values of the corresponding

abscissæ are obtained in the following way. The values of $\mu_1 a$ &c. are the successive roots of $J_0(\mu a) = 0$, and are

$$\mu_1 a = 2.40483$$

$$\mu_2 a = 5.52008$$

$$\mu_3 a = 8.65373$$

$$\mu_4 a = 11.79153$$

and so on.

Take 25 equidistant values of x beginning with 0 and ending with the value $\mu_4 a$: the corresponding values of $x J_1(x)$ are the abscissæ of points in the curve required to find A_4 .

The area of the curve is easily obtained by means of the planimeter. Having found the 25 abscissæ once for all, they may be used for the development of any arbitrary function of r of which 25 equidistant values are given. We now publish four tables giving these abscissæ, which we have carefully calculated for the first four terms of any such development.

In the column headed P we give the number of the point in the curve of which $x J_1(x)$ is the abscissa, there being 25 such points altogether. In Table I. the point 24 corresponds

TABLE I.

P.	$x \cdot J_1(x)$.
0	0.0000
1	.0050
2	.0200
3	.0450
4	.0787
5	.1216
6	.1727
7	.2312
8	.2962
9	.3667
10	.4416
11	.5197
12	.5999
13	.6807
14	.7609
15	.8392
16	.9142
17	.9945
18	1.0490
19	1.1062
20	1.1552
21	1.1949
22	1.2242
23	1.2423
24	1.2485

TABLE II.

P.	$x \cdot J_1(x)$.
0	0.0000
1	+ .0267
2	.1010
3	.2242
4	.3800
5	.5578
6	.7458
7	.9191
8	1.0706
9	1.1825
10	1.2417
11	1.2160
12	1.1141
13	.9418
14	.7052
15	.4132
16	+ .0791
17	- .2801
18	.6455
19	.9963
20	1.3118
21	1.5723
22	1.7605
23	1.8626
24	1.8783
10 A	1.2485

TABLE III.

P.	$x \cdot J_1(x)$.
0	0.0000
1	+ .0640
2	.2434
3	.5036
4	.7921
5	.9485
6	1.2135
7	1.2396
8	1.0986
9	.8366
10	+ .3353
11	- .2107
12	.7807
13	1.2957
14	1.6788
15	1.8667
16	1.8195
17	1.5279
18	1.0163
19	- .3817
20	+ .4154
21	1.1561
22	1.7807
23	2.2002
24	2.3490
6 A	+1.2485
15 A	-1.8783

TABLE IV.

P.	$x \cdot J_1(x)$.
0	0.0000
1	+ .1171
2	.4268
3	.8168
4	1.1373
5	1.2468
6	1.0555
7	+ .5601
8	- .1542
9	.9246
10	1.5554
11	1.8658
12	1.7443
13	1.1825
14	- .2851
15	+ .7485
16	1.6705
17	2.2544
18	2.3065
19	1.8114
20	+ .8497
21	- .3674
22	1.5548
23	2.4214
24	2.7410
4 A	+1.2485
10 A	-1.8783
17 A	+2.3490

to $x=2.40483$. In Table II. the point named 24 corresponds to $x=5.52008$, and so on.

In Table II. an additional abscissa is given corresponding to some unknown point between numbers 10 and 11; this is the extreme abscissa of the curve, and the corresponding ordinate would be a tangent to the curve. In Table III. there are two additional or bounding abscissæ and in Table IV. there are three of them.

As an example we have taken a function which has the value 1 at the centre of a circle and the value 0 at the circumference, and which at any place is a linear function of the radius. The outer radius a of the circle is 1.

In fig. 1 the function is shown graphically by the straight line MDD, and the numbered points in this line correspond to 25 equidistant ordinates. In our actual drawing, of which fig. 1 is a reduction, the scale is such that one unit of value of the function corresponds to 10 inches.

Fig. 1.

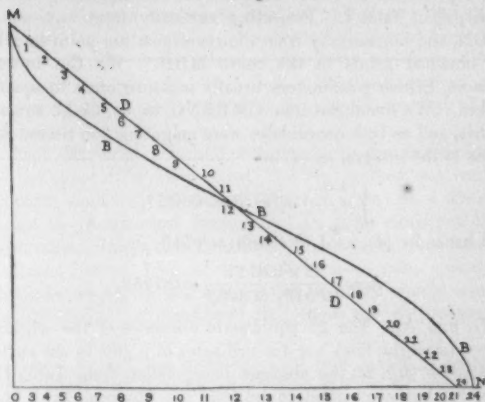
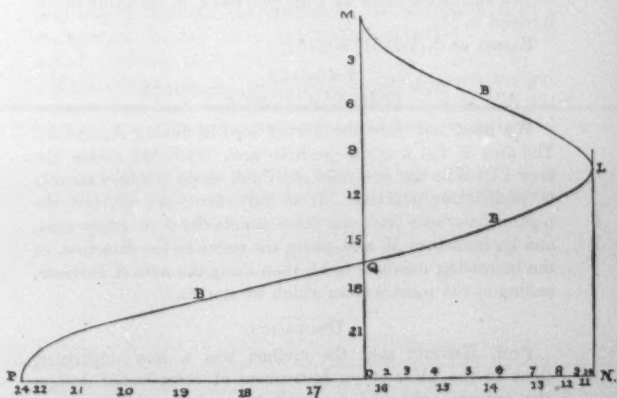


Fig. 2.



The distances from 0 to any number in the axis of abscissæ ON represented in inches ten times the corresponding values of $xJ_1(x)$ of Table I. Projecting vertically from any point in ON, and horizontally from the corresponding point in DD we obtained points in the curve MBB. We use inches because British planimeters usually measure areas in square inches. We found the area OMBBNO to be 61.17 square inches, and as both coordinates were magnified ten times, the value of the integral in (4) or-

$$\int_0^{2.40483} x f(x) J_0(x) . dx = 0.6117,$$

and hence by (4), as $J_1(2.40483) = 0.519$,

$$A_1 = \frac{2 \times 0.6117}{(0.519)^2 (2.405)^2} = 0.7852.$$

To find A_2 . The 25 equidistant ordinates of the original curve (straight line) are the ordinates of points in the curve MBLBBP (fig. 2), the abscissæ being taken from Table II. We found it convenient to multiply the abscissæ by 5 and represent in inches. Our ordinates were multiplied by 10 and represented in inches. The actual area was 2.36 square inches and one-fiftieth of this, or 0.0472, is the value of the integral in (4).

Hence, as $J_1(5.5201) = 0.3403$,

$$A_2 = \frac{2 \times 0.0472}{(0.3403)^2 \times (5.5201)^2} = 0.0268.$$

We need not show the curves used in finding A_3 and A_4 . The area of fig. 2 is the positive area MLNOM minus the area LQPNL, but one need not think about whether an area is positive or negative. It is only necessary to start the planimeter-tracer from the point numbered 0 in every case, and go from 0 to M and along the curve in the direction of the increasing numbers to 24, then along the axes of abscissæ, ending at the point 0 from which we started.

DISCUSSION.

Prof. HENRICI said the method was a new departure, since in the place of an instrument of complicated design, the authors only used a planimeter and pencil and paper,

and obtained the same degree of accuracy. The fact that the series employed to test the method consisted of a finite number of terms seemed to him an objection. Prof. Karl Pearson had, in a recent conversation, informed him of a method for the development of functions which he (Prof. Pearson) had recently discovered. This method was not, however, so simple, at least in most cases, as that of the authors.

Prof. MINCHIN thought it would add to the intelligibility of the paper if it were stated that the method was similar to that employed when expanding in terms of a Fourier series or in spherical harmonics. In these cases you have a function which, when multiplied by other functions of different orders, kills all the terms except one. Graphic methods ought, in his opinion, to be very much oftener employed, and he considered that there was no problem in physical mathematics of which the solution could not be obtained by graphic methods. He would like to know if Prof. PERRY had obtained a graphic method of calculating Bessels.

Mr. TROTTER agreed with Prof. Minchin as to the neglect of graphic methods. He regretted that Prof. Perry did not continue to consider the method as the projection from a cylinder, as he had found the method of wrapping curves round a cylinder most useful.

Prof. PERRY, in his reply, said he had adopted the expansion they had employed under the impression that the test was a particularly severe one. He had not discovered a graphic method of calculating Bessels. The reason they gave up the cylinder was the immense labour involved in calculating the y co-ordinates of the trace, which would afterwards be of no use in the development of the function.

II. *The Radial Cursor: a new addition to the Slide-Rule.*

By F. W. LANCHESTER*.

THE slide-rule in its ordinary form is chiefly useful in working simple problems involving multiplication and division, also involution and evolution where the indices occurring are integers. When an expression involves fractional indices its solution is only possible, on a slide-rule of ordinary construction, by employing the scale of logarithms on the back of the slide in the same manner as a table of logarithms, when the process becomes so clumsy as to practically destroy its utility.

The object of the present improvement is to enable the operator to solve problems of the latter class with the same ease and degree of accuracy as that only previously attainable in connexion with the simpler cases.

In thermodynamic problems such as arise in dealing theoretically with gas- and other heat-engines, the need of some method of computation more rapid than that of ordinary arithmetic involving many references to tables becomes evident, and it was in connexion with work of this description that the author was led to devise the instrument which is the subject of the present paper.

In the case of the isothermal curve of a perfect gas, the ordinary slide-rule is all that can be required, for the curve is represented by the simple expression

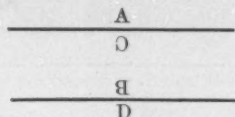
$$pv = \text{constant}.$$

For the sake of brevity let us name the scales in a rule of the ordinary "Gravet" type thus

A
—
B
C
—
D

* Read October 25, 1895.

becoming when the slide is inverted,



Then with the slide in the inverted position the corresponding pressures and volumes of an isothermal can be read off directly on the scales A and G.

In the case of an adiabatic curve, however, we have an expression of the form

$$pv^\gamma = \text{const.},$$

where γ has a value 1.4 more or less according to the physical properties of the gas or mixture of gases we are dealing with. In order to deal with an expression of this form, it is necessary to provide some ready means of dividing the scales A and G proportionately to the value of γ which corresponds to the division and multiplication of the respective logarithms of the quantities dealt with, in the proportion of the indices of p and v , that is $1:\gamma$.

This proportionate division of the scales is effected in the new Cursor by a radial index-arm, indicated in fig. 1 by lines f and g : this is arranged to swing about a stud which is itself carried on a sliding bar running in guides at right angles to the rule. All readings are taken at the points of intersection of the line with the edges of the slide. In the actual instrument the radial arm is made of a transparent slip of celluloid on which is scratched a straight line true to the centre of the stud-hole. The back view shows (fig. 2) the transverse bar graduated to give the indices to which it is desired to work. In fig. 2 this transverse bar is shown set to 1.408, the γ of a perfect gas.

A few words may be added with regard to the method of graduation of the transverse bar. Referring to fig. 3, in which MN is the width of the slide, and O is the centre of

Fig. 1.

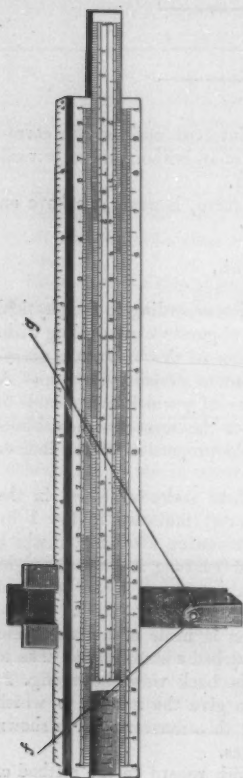
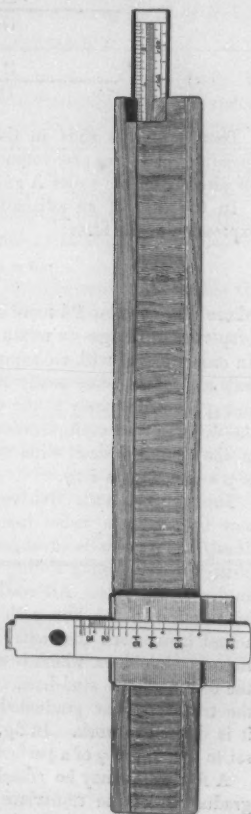


Fig. 2.



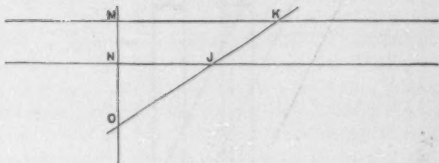
the radial arm stud-hole, for any index γ , we have

$$\gamma = \frac{MK}{NJ} = \frac{MN + NO}{NO} = \frac{MN}{NO} + 1,$$

where OK is any position of the index line, or

$$NO = \frac{MN}{\gamma - 1}.$$

Fig. 3.



The following examples will serve as an illustration of the class of calculation for which the instrument is designed.

1. *To determine the maximum pressure of an adiabatic compression.*

Set the slide on back of cursor to 1.408 (γ). Suppose, for example, that the initial pressure be 15 lbs. per square inch, and that 100 vols. be compressed to 26 vols.

Invert slide. Set index line of cursor to 15 on A, then set 1 (100) on $\mathfrak{A}$ to index line of cursor. Now set index line to 26 on $\mathfrak{A}$ and read on scale A the corresponding pressure = 100 lbs. absolute, 85 above atmosphere (figs. 1 and 2).

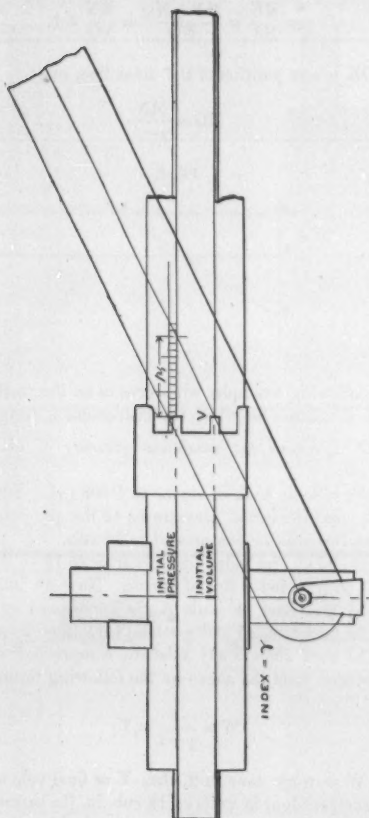
2. The work done in any adiabatic compression can easily be calculated from the above by the following formula:—

$$W = \frac{1}{\gamma - 1} p_1 V;$$

where W = work done in ft.-lbs., V = final vol. of the gas (after compression) in units of 12 cub. in. (to harmonize with ft.-lbs. and pressure in pounds per square inch), and p_1 = difference of pressure between adiabatic and isothermal maximum pressure for the given ratio of compression (fig. 4).

Fig. 4.

$$\text{Work done} = \frac{1}{\gamma - 1} p_1 V.$$



3. The temperature of an adiabatic at any point can be determined from either pressure or volume by using suitable indices for

$$v \cdot T^{\frac{1}{\gamma-1}} = \text{constant};$$

also

$$p \cdot T^{\frac{\gamma}{\gamma-1}} = \text{constant}.$$

In determining temperatures from volumes or *vice versa* it is only necessary to proceed as before (1). In dealing, however, with pressures and temperatures the index is negative (where γ is $>$ unity), the interpretation of this being that an increase of pressure is accompanied by an increase in the temperature. The slide therefore does not require to be reversed as in $p v$ and $v T$ calculations; but with a rule of the Gravet type, owing to the scales C and D being the square roots of A and B, it is necessary to use a fictitious index obtained by dividing $\frac{\gamma}{1-\gamma}$ by the logarithmic ratio of the scales, giving $\frac{\gamma}{2(1-\gamma)}$. It would also be quite possible to

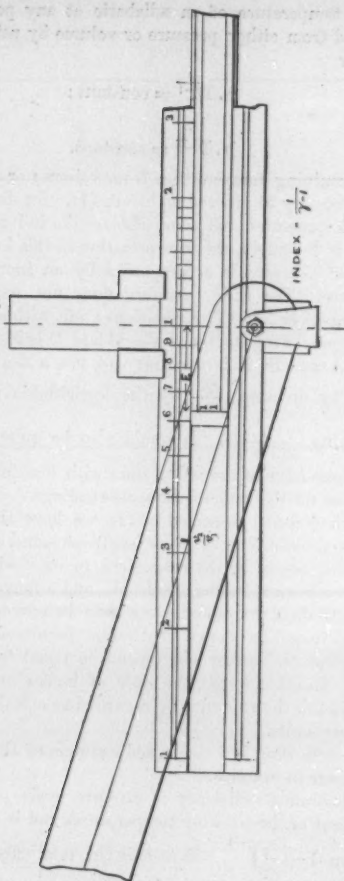
graduate one edge of the index slide with fictitious values to compensate for the ratio of the scale-readings.

4. In the process described in (1) we have the means of plotting a series of adiabatics or isentropics, and each position of the radial cursor on the rule gives in its readings on the index line a corresponding isentropic, and a movement of the cursor to the left corresponds to a gain in entropy and *vice versa*, and it can be shown that equal increments of linear movement of the cursor correspond to equal increments of entropy. In other words, the scale of inches or centimetres on the edge of the rule may be regarded as a scale of entropy in arbitrary units.

5. The following is a very good example of the use of the radial cursor in practice.

The theoretical efficiency of an Otto cycle gas-engine is independent of the working temperatures and is given by the expression $1 - \left(\frac{v_2}{v_1}\right)^{\gamma-1}$, where v_1 is the total effective volume of the cylinder and compression-space, and v_2 = volume of compression-space alone.

Fig. 5.
Efficiency of Otto Cycle Gas Engine $= 1 - \left(\frac{v_2}{v_1} \right)^{\gamma-1}$.



Taking $\gamma=1.408$, $\gamma-1=.408$. This is beyond the range of the instrument, so we will employ $\frac{1}{.408}=2.45$.

Let us suppose that in the case we are dealing with $\frac{v_2}{v_1}$ comes out at .32 (an actual example, see 'The Engineer,' April 6th, 1894).

Set the index slide to 2.45 and place cursor accurately opposite middle 1 on scale A (use one of end lines on slide to centre with), and set index line to .32 ($\frac{v_2}{v_1}$) on scale A (over to the left, fig. 5).

Now using one of the end lines on slide read on scale A opposite to intersection of index line on scale D, *but read backwards from the centre* 1. This will be found to give .37, or the theoretical efficiency of an Otto engine having ratio of expansion .32 is .37.

6. To determine x in such an expression as

$$\gamma^x = 3.$$

Here x will be a fractional index, and we shall first find $\frac{1}{x}$.

Set cursor to 1 in centre of scale A.

Set index line to 7 on scale A, and keeping it there move slide of cursor till index line intersects scale D opposite figure 3 on scale A (use end line of slide).

Now read result on index-scale

$$= 1.76,$$

Or

$$x = \frac{1}{1.76} = .568.$$

DISCUSSION.

Mr. C. V. BOYS said that owing to the kindness of the author he had been able to try the cursor and had found it of great service in dealing with questions of adiabatic expansion. The new addition to the slide-rule suffers under the same disadvantage as the rule itself, namely, that a verbal or written description seems so very much more complex than is the actual operation when using the rule. The author's device might be described as an indiarubber

slide-rule, for it performed the function of a slide-rule, in which the graduations of the slide were made on indiarubber, so that the ratio of the length of the scale on the rule to the length of the scale on the slide might be altered at will, and thus involution and evolution with fractional indices performed.

Prof. S. P. THOMPSON and Mr. TROTTER expressed their admiration for the author's method of "stretching" the scale.

Mr. BURSTALL said he had attempted to apply a similar method to the Fuller rule, but did not succeed, since in this rule there was only one scale. He hoped the author's method could be applied in a form such that a greater accuracy than 1 in 300 could be obtained.

Mr. BOURNE thought the fact that the point of intersection of two lines inclined at an acute angle had to be read was likely to limit the accuracy.

III. *The Latent Heat of Evaporation of Benzene.* By E. H. GRIFFITHS, M.A., F.R.S., *Sidney Sussex College, Cambridge*, and Miss DOROTHY MARSHALL, B.Sc., *University College, London**.

1. THE method of experiment and the nature of the apparatus employed have been fully described in previously published papers. Although it appears somewhat presumptuous to assume, on the part of the reader, a knowledge of such communications, it would on the other hand seem redundant to devote many pages to the mere repetition of what has already appeared in print. We therefore propose to give an explanatory statement of the theory and methods employed, but to omit all detailed description of the apparatus. Full references to former communications will be given when necessary, so that all particulars can be ascertained by any who desire them.

* Read November 8, 1895.

As reference has frequently to be made to the following papers (by E. H. Griffiths) they are, for convenience, denoted as follows :—

Paper J. "The Mechanical Equivalent of Heat," Phil. Trans. vol. clxxxiv. 1893, A, pp. 361-504.

Paper A. "The Influence of Temperature upon the Specific Heat of Aniline," Phil. Mag. Jan. 1895.

Paper W. "The Latent Heat of Evaporation of Water," Phil. Trans. vol. clxxxvi. 1895, A, pp. 261-341.

2. *Description of the Method.*

In order to render our account of these experiments intelligible, we find it necessary to give the following somewhat lengthy extract from Paper W, pp. 270-273 :—

"The method adopted was of such a nature that the results would not be appreciably affected by

- (1) errors in thermometry ;
- (2) changes in the specific heat of water ;
- (3) the capacity for heat of the calorimeter ;
- (4) loss or gain of heat by radiation, &c. ;

and if these points are borne in mind, they may serve to explain some of the contrivances which might otherwise appear uncalled for.

"If the vessel in which the evaporation is taking place is kept at a constant temperature, we are independent of the capacity for heat of it and its contents ; we also dispense with the measurements of changes of temperature. Thus, if matters be so arranged that the loss and gain of heat throughout an experiment are balanced, many fruitful causes of error are avoided. Of course, the actual temperature of the calorimeter during evaporation must be determined, but a small error here is of little consequence. The change in the value of L is small as compared with the changes in θ . In fact, an accuracy of an order of $\frac{1}{10}$ of a degree would be sufficient when determining the actual elevation.

"The heat was supplied to the calorimeter by means of a wire whose ends were kept at a constant potential-difference.

The thermal balance could be maintained in one of two ways:—

- (1) If the heat-supply was too great, the electric current could be temporarily stopped, or, the rate of evaporation of the water increased. (The latter was the method generally adopted.)
- (2) If the cooling was too rapid, the only mode of maintaining the balance was (in the apparatus about to be described) to reduce the rate of evaporation.

“The liquid to be evaporated was contained in a small silver flask, connected with which was a spiral coil of silver tubing 18 feet in length. Both flask and spiral were within the calorimeter, and the water-vapour, after passing through the spiral, emerged from the apparatus at the temperature of the calorimeter. Surrounding the flask, and between it and the spiral, a coil of platinum-silver wire was arranged, and flask, spiral, and coil were entirely immersed in a certain singularly limpid oil consisting of hydrocarbons only.

“The calorimeter (which was filled to the roof with the oil, and the equality of temperature maintained by rapid stirring) was suspended by glass tubes within a steel chamber, whose walls were maintained at a constant temperature. So long, therefore, as the calorimeter and the surrounding walls were at equal temperatures, there was no loss or gain by radiation, &c. If during an experiment the temperature of the surrounding walls changed, the method of experiment involved a corresponding change in the temperature of the calorimeter, and, therefore, some loss or gain of heat would be experienced. The apparatus was so designed that any such change in temperature was extremely small (in no case amounting to $\frac{1}{100}^{\circ}$), yet, in order to estimate the loss or gain, it was necessary to know approximately the capacity for heat of the calorimeter and contents.

“Small differences between the temperature of the calorimeter and the surrounding walls would, during an experiment, be of no consequence provided that the oscillations were of such a nature that the mean temperature of the calorimeter was that of the surrounding space, and it will be found that this condition was fulfilled.

“In addition to the heat supplied by the electric current, there is also a supply due to the work done by the stirrer,

and it was in the estimation of this 'stirring supply' that the greatest difficulties were encountered. Fortunately the heat thus generated was only about $\frac{1}{100}$ of the heat supplied by the current, and thus any small error in that portion of the work becomes of little account.

"Of the accuracy with which the electrical supply could be measured there is no question; and even if the value of the E.M.F. of the Clark cells, or the absolute resistance of the box-coils given by the standardizations performed during the determinations of J , is in any way inaccurate, such errors would now eliminate, since the value of J was determined by means of the same standards as those by which the quantity of heat developed in these experiments was determined. Hence, by assuming the value of J obtained by the use of these standards, we get the comparison in terms of a thermal unit at 15°C ., independently of the numerical value assumed in the reductions.

"One further correction remains to be noticed. The temperature of the calorimeter has been referred to as oscillating about the exterior temperature, and it might happen that at the close of an experiment this difference was not the same as that at the commencement—if any such difference existed. The magnitude of this correction depended, of course, on the ability of the observer to maintain the thermal balance. In these experiments the correction was usually small, and in any case could be determined with great accuracy.

"Having indicated the nature of the observations, we proceed to state the relation between the various sources of loss or gain of heat.

"Let Q_e be the thermal units per second due to the electrical supply;

Q_m be the thermal units per second due to the mechanical supply;

Σq be the total heat-supply during an experiment from any other causes.

"Then, if M be the mass of water evaporated, L the latent heat of evaporation at temperature θ , and if the electrical supply is maintained for a time t_e , and the mechanical for a time t_m ,

$$ML = Q_e t_e + Q_m t_m + \Sigma q. \quad \dots \quad (1)$$

"Now the D.P. at the ends of the coil was always some integral multiple of the D.P. of a Clark cell.

"Let ϵ be the D.P. of a Clark cell, n the number of cells, and R_1 the resistance of the coil at the temperature θ_1 , then

$$Q_z = \frac{\epsilon^2 n^2}{R_1 J} \dots \dots \dots (2)$$

"If the calorimeter at the commencement and end of an experiment was at exactly the same temperature as the surrounding walls, then, if their temperature was unchanged, the term Σq would vanish; but although this term throughout these experiments was of small dimensions, it could not be entirely ignored.

"Let θ_0' and θ_0'' be the temperature of the surrounding walls at the beginning and end of an experiment; suppose the calorimeter temperature (θ_1) to exceed the surrounding temperature by d' at the commencement and d'' at the end of an experiment. Then fall in temperature of calorimeter

$$= (\theta_0' + d') - (\theta_0'' + d'').$$

Hence the heat given out by the calorimeter in consequence of this fall in temperature is

$$C_{\theta_1} \{(\theta_0' + d') - (\theta_0'' + d'')\},$$

where C_{θ_1} is the capacity for heat of calorimeter and contents at the temperature θ_1 .

"If we neglect any small loss by radiation, &c., due to the differences d' and d'' between the temperature of the calorimeter and the surrounding walls, we may conclude that the whole of the heat thus evolved by the calorimeter was expended in the evaporation of water, hence

$$\Sigma q = C_{\theta_1} \{(\theta_0' - \theta_0'') + (d' - d'')\}^* \dots \dots (3)$$

Hence

$$ML = \frac{\epsilon^2 n^2 \times t_z}{R_1 J} + Q_z \times t_z + C_{\theta_1} \{(\theta_0' - \theta_0'') + (d' - d'')\}. \quad (4)''$$

* This apparently clumsy method of representing the quantity of heat evolved or taken up by the calorimeter was adopted because, as the method of experiment involved separate determinations of θ_0' , θ_0'' , d' , and d'' , the actual temperature of the calorimeter at any time could only be obtained in this manner.

3. The following table shows the *comparative* mean values of the terms in equation (4) resulting from our experiments on benzene*.

TABLE I.

No. of Cells.	$Q_n t_n$.	$Q_s t_s$.	Σq .
5	1	·008	± 005
4	1	·011	± 009
3	1	·015	± 001

We found that the thermal balance was most easily maintained when the potential-difference was that of 4 cells, therefore the majority of our experiments were performed with that potential-difference, the experiments with 5 and 3 cells being used as a check upon the results. To secure an accuracy of (say) 1 in 1000 in the total heat-supply, the above table shows that it was necessary to measure

$$Q_n t_n \text{ to (say) 1 part in 2000,}$$

$$Q_s t_s \quad \text{,,} \quad 1 \quad \text{,,} \quad 15,$$

$$\Sigma q \quad \text{,,} \quad 1 \quad \text{,,} \quad 10;$$

but as the sign of the last term was in some experiments positive, in others negative, the above degree of accuracy was not essential, since, when taking the mean of a number of observations, the effect of any error in Σq would be greatly diminished. We believe that the above order of accuracy was exceeded in the measurement of the respective terms. In the case of the experiments with 3 cells at 20° an order of accuracy of 1 in 50 would be required in the value of $Q_n t_n$ to secure the same standard; hence we do not regard our determinations at that temperature as possessing an equal authority with those at higher temperatures.

* (Experiments Preliminary, XIX. a, XXXV., and XXXVI. were not included when calculating this table, as they are in several respects exceptional.)

4. *The Method of maintaining the Space surrounding the Calorimeter at a Constant Temperature θ_0 .*

A full description of the somewhat elaborate apparatus designed for this purpose will be found on pp. 374-378 of Paper J, and some improvements which were subsequently added are described on pp. 274-276 of Paper W. This portion of the apparatus did not, for some unknown reason, appear to be working with the same perfection this year as it did in the summer of 1894. Oscillations in θ_0 (the external temperature), amounting to $\frac{1}{100}^{\circ}\text{C}$., were on several occasions observed, the greatest change during the whole series being 0.145° . This, however, was exceptional and, as an inspection of our final tables will show, the change in the course of an experiment (*i. e.* in about half an hour) was usually only about a few thousandths of a degree at temperatures varying from 20° to 50°C .; the constancy of θ_0 was therefore sufficient for our purpose.

Had it been possible, we should have preferred to determine θ_0 by means of a platinum-thermometer, as we could thus have detected smaller changes; but a third observer would, in that case, have been required, and circumstances did not permit of this addition*.

The mercury-thermometers used for indicating the temperature of the steel chamber in which the calorimeter was suspended were graduated in millimetres, and had been carefully calibrated and compared with two Tonnelot thermometers standardized at the Bureau International†. The stems of these thermometers (where they projected above the tank-lid) were surrounded by glass tubes up which a stream of tank-water was forced by means of a pump driven by a water-motor: thus the stem-temperature was always that (or nearly that) of the tank, and the readings were not affected by changes of temperature in the room. The actual value of θ_0 was of small consequence, an accuracy of $\frac{1}{10}^{\circ}\text{C}$. being

* We believe that such irregularities as present themselves in our final results are partly due to the employment of mercury-thermometers for the determination of the changes in θ_0 .

† See "The Measurement of Temperature," Science Progress, Sept. 1894.

sufficient; but it was necessary to read *changes* of temperature during an experiment to nearly $0^{\circ}001$ C., and this we believe we were able to do. A full description of the various precautions taken will be found in Paper A, pp. 55 and 56, and Paper W, pp. 275, 276.

The observations were taken by means of a reading-telescope fitted with a micrometer-eyepiece, which directly divided 1 millim. on the thermometer-stem into 10 parts: thus $0\cdot1$ of a millimetre could be directly read, and $\cdot01$ could be estimated. There is no doubt that the actual readings could be taken to $\cdot025$ millim., that is about $0^{\circ}001$ C.

Thermometer A had a range of 16° to 26° C., and about 27 millim. were equivalent to 1° C.

Thermometer II. had a range of 28° to 53° C., and about 20 millim. were equivalent to 1° C.

All temperatures, both when obtained from these and from the platinum-thermometers, are throughout this Paper expressed in terms of the nitrogen-scale.

5. *The Calorimeter and its Connexions.*

A full description will be found on pp. 276–281 of Paper W, and drawings and sections on plates 5, 6 of the same paper. The brief account given in the description of the method *supra* is sufficient to indicate the nature of the arrangements to those who are not familiar with that paper.

The *differences* of temperature between the calorimeter and surrounding walls were determined by means of differential platinum-thermometers. A description of the method of standardization and observation of those thermometers is given on pp. 52–56 of Paper A, and some further details on pp. 285–290 of Paper W. Experimental evidence is there adduced in support of the following statement:—"It follows that differences of temperature could be determined to $\cdot0004^{\circ}$, and differences of $\cdot0001^{\circ}$ could be detected."

The following Table shows the difference in temperature corresponding to a difference of 1 mean millim. of the bridge-wire whose opposite ends were connected with the thermometers:—

TABLE II.

Temp.	$\Delta\theta$ for difference of 1 mean millim. bridge-wire.
28	0.009046
30	0.009073
40	0.009101
50	0.009128

By experiments repeated this summer we found that no change had taken place in the value of the mean bridge-wire millimetre. A slight change (particulars of which are given in a subsequent section) had, however, occurred in the position of the null-point, *i. e.* in the reading on the bridge-wire when the temperature of the two thermometers was identical.

6. *Brief Description of the Method of obtaining the Value of the Terms in the Expression*

$$\text{Total heat} = Q_x t_x + Q_n t_n + \Sigma q.$$

$$Q_x t_x = \frac{(ne)^2 \times t_x}{R_1 \times J}.$$

The ends of the platinum-silver coil (immersed in the oil surrounding the evaporating-flask and spiral) were kept at a constant potential-difference by means of the arrangements described in Paper J, pp. 382-388. This potential-difference was always some integral multiple (n) of the potential-difference of a Clark cell (e). During the spring of this year the Clark cells used were again compared with the Cavendish standard (R_1), which has shown no signs of change since its standardization by Lord Rayleigh in 1883 and by Messrs. Glazebrook and Skinner in 1891.

The mean value of the whole set of 30 cells differs from R_1 by 0.00004 volt only; and although individual cells show larger discrepancies than in previous years, their mean potential-difference at 15° C. may be taken as 1.4342 volts (see Paper W, p. 297). A number of these cells were always placed in parallel arc: thus when n is given as 4, we were

really using 12 cells as four files of 3 each. The arrangement for keeping the temperature of these cells at nearly 15°C . has been described in Paper J, p. 385. During the period covered by our experiments, some of the days were extremely hot, and the tap-water became so warm that when turned on by the regulator it was unable to keep down the temperature sufficiently. The extreme range in the temperature of the cell-tank during these experiments was from $14^{\circ}\cdot 8$ to $16^{\circ}\cdot 3\text{C}$. As, however, the movement was always extremely slow, it is probable that the tank-temperature closely corresponded to the effective temperature of the cells, and hence the correction

$$e = 1\cdot 4342\{1 + (15 - \theta) \times 0\cdot 00077\}$$

applied by us gave the value of e with sufficient accuracy.

The time t_{κ} was determined by means of a chronograph controlled by an electric clock, whose gaining rate is now less than 1 in 20,000. Any movement of the keys by which the current was switched on to the calorimeter-coil was automatically recorded on the tape; and thus personal errors were eliminated. The times could have been read to $\frac{1}{100}$ of a second, but it was considered unnecessary to read to nearer than $\frac{1}{10}$, i.e. about 1 in 7000 of t_{κ} .

The value of R_1 is expressed in terms of the "true ohm" as given by the B.A. standards, with which (by kind permission of Mr. Glazebrook) the coils used by us have been directly compared (Paper J, pp. 407-410).

The increase in R_1 due to the rise of temperature caused by the current was determined in the manner described in Paper J, pp. 404-407 (see also Paper W, p. 296).

The value of J assumed by us was $4\cdot 199$ (Paper W, p. 314). We would emphasize the fact to which attention has been previously directed (section 2, p. 3), namely, that even if, in consequence of errors in the standards &c., this value of J is incorrect, it is still the right value to use for the reduction of these observations, for, provided that no change has taken place during the past three years in the standards used (and direct comparisons show no signs of any change which would affect the results), the values of L obtained by us are independent of the numerical values of J and R when expressed in terms of

the same units as those assumed during the determination of the mechanical equivalent.

The value of $Q_s t_s$ is thus expressed in terms of a "thermal unit at 15° C."

7. $Q_s t_s$.

The method adopted for finding Q_s (the thermal units per second due to the work done by stirring) has been fully described in Paper W, pp. 290-293. It was there shown that the value of Q_s varied approximately as r^4 (where r was the rate of revolution of the stirrer), and that this relation was sufficiently close for the reduction of experiments in which the value of r was somewhere between 5 and 6 per second.

The value of Q_s was found to increase rapidly as θ_1 (the temperature of the calorimeter) diminished. Our recent re-determinations of Q_s have explained a discrepancy which caused an uncertainty in the former determination of Q_s at 20° and 50°, and we now find that it was due to an arithmetical error in the reduction of the observations. Fortunately, the resulting correction in no way affects the conclusions arrived at in Paper W; and our present investigation confirms the accuracy of the experimental determinations of Q_s obtained in 1894. As the extreme differences between any of our determinations of Q_s at temperatures above 20° (after the arithmetical corrections at the 20° and 50° points) do not exceed 1 part in 70, even if we include the values found in 1894, and as the mean probable error at any temperature is much below 1 in 100, it is obvious that the values of Q_s have been ascertained with more than sufficient precision.

$$8. \Sigma q = C_{\theta_1} \{ (\theta_0' - \theta_0'') + (d' - d'') \}.$$

We have previously indicated (sect. 4, p. 6) the manner in which θ_0' and θ_0'' (the initial and final temperature of the walls of the surrounding chamber) were determined by the direct observation of mercury-thermometers. The value of d (the difference between θ_1 and θ_0 , i. e. the calorimeter-temperature and that of the surrounding walls) was ascertained by the differential platinum-thermometers previously referred to, and was always small—rarely greater than .01° C. It was usually determined by the reading of the galvanometer-swings

without altering the position of the contact-maker on the bridge-wire.

When evaporating water, the values of d were so small that an approximate value of the swing (in terms of a length of the bridge-wire) was sufficient. During these benzene experiments, however, we found it impossible to maintain the thermal balance with such perfection. The only values of d which were of consequence were d' and d'' (the initial and final values), and, especially in the earlier experiments, the internal temperature (θ_1) rose so rapidly as soon as the last drop of benzene had evaporated, that it was found impossible to switch off the electric current from the calorimeter-coil at the precise moment necessary to reduce d'' to negligible dimensions. As a consequence, the final swings (which were read by means of a micrometer-eyepiece) were often considerable, and therefore it was necessary to determine their value on each occasion; for, although throughout a series of experiments at the same temperature the value varied but little, the changes were sufficient to affect the resulting value of Σq in exceptional cases. In the earlier experiments, especially from Preliminary to No. VII., when the difficulty of the final adjustment had not been fully realized, the values of d'' were unduly large, and their equivalent degree-measurements are therefore somewhat doubtful, owing to insufficient determinations of the value of the galvanometer-swings. We feel sure, however, that from No. VII. onwards any errors due to this cause must be very small—certainly not so great as 1 in 50; for the value of a swing of 100 (in terms of a millim. of the bridge-wire) would, throughout a group of experiments at the same temperature, vary (for example) from 1.31 to 1.34 millim.* This change in value would produce no appreciable effect on Σq when the difference between the initial and final swings did not exceed 50 or 60, as was the case in most of our later, and better, experiments.

The value of C_a (capacity for heat of calorimeter and contents) remains the same as last year with the exception that 0.1 grm. of the oil was removed at the commencement of July when withdrawing the platinum thermometer for purposes

* 1 millim. of the bridge-wire indicated a temperature-difference of about $0^{\circ}.009$ C.

of re-standardization. As the value of C_θ varied with θ_1 from 304 to 323, this loss was negligible.

The quantity Σq represents the heat absorbed by the calorimeter and contents, and it should be remembered that it is by the measurement of a similar quantity that the majority of the determinations of thermal constants have been made by previous observers. We were, however, anxious to diminish the importance of this term as much as possible, for we wished our values of L to be independent of any thermometric errors. As shown by Table II., the average value of Σq was only about $\frac{1}{400}$ of the total heat-supply. In the experiments in which the value of Σq is less than 1 (of which there are many examples), we may say that our final results are independent of such errors, for, as pointed out in Paper W, the value of Q_s is independent of temperature measurements, since it depends on the *ratio* of the rate of rise due to the mechanical supply to the rate of rise due to an electrical together with a mechanical supply (Paper W, p. 331).

The values of C_θ at 20° and 50° used in this Paper differ somewhat from those given in Paper W. This difference is due to the error, previously referred to, in the value of Q_s at those temperatures. Thus

the value of C_θ at 20° is reduced from 307.5 to 305.2,
and " " 50° " " 323.1 to 322.0.

This, however, in no way affects the values of L in Paper W, nor, except in the case of two experiments (viz., Preliminary and XIX. a., *infra*), would it, if left uncorrected, have affected the values of L as given in this communication.

We believe that the values of Σq are correct to better than 1 in 100; and Table I. (p. 21) shows that an accuracy of 1 in 10 would have been sufficient.

9. Measurement of Mass.

The actual measurement of the mass of benzene evaporated presented several difficulties which were not encountered when working with water. A tube resembling a weight-thermometer was filled with benzene, placed in an air-tight case just large enough to contain it, and then weighed against a precisely similar case used as a tare. The weight-thermometer

(termed by us a "dropper") narrowed at its open extremity to a capillary tube, which was doubled back on itself for rather over 1 cm., and again bent near the open end, so that the last 1 or 2 mm. were horizontal. These droppers varied in capacity from about 4 to $6\frac{1}{2}$ c.c., and were filled in the following manner:—The dropper (point uppermost) was lowered by a fine wire to the bottom of a tube about $4\frac{1}{2}$ ft. long, of which the lower 10 inches or so were filled with benzene, while the upper 3 ft. were surrounded by a "condenser-tube" through which tap-water was continually passing. The lower end, containing the benzene, was transferred at regular intervals from a vessel of water at about 86°C. to a vessel of cold water; thus the benzene was alternately boiled and cooled without any escape of vapour into the room. Five or six such transferences were generally required to completely fill the dropper. The containing tube was then placed in a bath at about 65°C. until the temperature of the benzene was steady. The dropper was now removed and allowed to stand in the open air for some time, in order to get rid of any benzene adhering to its surface. Although simple and effective, these operations occupied a considerable time, and, as a rule, the whole of the morning had to be devoted to the filling of the droppers required for the experiments, which were usually performed at night.

Before an experiment the dropper was lowered, by means of a thread passed through a platinum wire sealed into the closed end, into the calorimeter, where it stood in a vertical position. In Paper W (p. 307) it was shown that, when filled with water, the evaporation through the capillary opening between the time of weighing and the commencement of an experiment might be neglected. In the case of benzene, however, it was found that, for several reasons, loss of this kind could not be disregarded:—

- (1) The vapour-pressure of benzene so greatly exceeds that of water at corresponding temperatures that the loss by diffusion through the capillary was appreciable.
- (2) The surface-tension of benzene is so great that the liquid crept up the sides of the capillary to the opening, and the consequent loss by evaporation was increased.
- (3) In order to supply the air necessary for starting the

boiling when the exterior pressure was removed, a capillary tube, closed at one end, had been sealed within each dropper. It was found that this answered very well during some preliminary trials, when the dropper was placed within a glass tube connected with an exhaust pump so that its manner of discharging could be watched; but on a second filling with our purest sample (the first filling having thoroughly cleaned the interior surface of the droppers) no action took place even when the surrounding pressure was reduced to a few millimetres. In order, therefore, that the expulsion of the benzene from the dropper should commence as soon as the external pressure was reduced to the right amount, it was found necessary to leave a very considerable air-bubble within the tube. Precautions had to be taken to prevent the expulsion of the liquid by the alternate contraction and expansion of the air-bubble when the temperature was changed from that of the balance-case to that of the tank.

These difficulties were surmounted in the following manner.

During the process of filling, as above described, the droppers stood in a vertical position, with the doubled over and open end uppermost, and were never inverted after their removal from the filling-tube, at a temperature of about 65°C ., until their insertion into the calorimeter. The coefficient of expansion of benzene is very large, and, on cooling to the room-temperature, the whole of the upper bend together with a couple of centimetres of the neck between it and the body of the dropper was found to be free from benzene. The dropper was then placed in its case and left in a vertical position in the balance-case for, as a rule, some hours, when the small air-space in the enclosing-case no doubt became saturated with the vapour and thus further evaporation ceased. After being weighed, the dropper and case were lowered into a large tube placed within the tank and left until they acquired the tank temperature θ_0 ; they were then rapidly withdrawn and for the first time inverted—the air-bubble rising to the closed end. The case was opened for a moment and the dropper at once lowered into place by means of the

previously attached thread: this operation only occupied a couple of seconds. As the dropper left the case the latter was instantly closed by a second operator to prevent the escape of any benzene vapour left within it. Thus from the time of inversion no change took place in the temperature, and therefore in the volume of the air-bubble; also no appreciable change in the temperature of the calorimeter (θ_1) was caused by the introduction of the dropper and contents. However, the mere act of opening the tube leading down to the calorimeter caused a slight lowering of θ_1 , and in order to re-establish the equality between θ_1 and θ_0 before commencing an experiment, it was necessary to switch on, for a second or two, the current from the exterior coil in the tank to the calorimeter coil. When the observer at the differential-thermometer galvanometer announced that $\theta_0 - \theta_1$ was small and steady, the time for commencing the experiment had arrived.

10. In our later experiments, when we had become more expert at the various operations, the time from the insertion of the dropper to the commencement of an experiment was from 3 to 5 minutes. In our earlier observations at 30° the time was, however, much longer—from about 8 to 15 minutes. During this interval there was generally apparent a slight lowering of θ_1 , which made us fear that the evaporation through the capillary was appreciable, and it was not until our tenth experiment that a means of meeting this difficulty suggested itself. It was evidently necessary to wait until θ_1 became steady before commencing an experiment, and it was difficult to see how to shorten the time required to establish this condition.

From Experiment X. onwards the procedure was as follows. A glass rod whose upper end passed through a cork fitting the opening of the entrance-tube was lowered into the calorimeter. Round the lower three inches of this rod was strapped (by fine platinum wire) a thin roll of cotton-wool of which the upper two inches were saturated with benzene while the lower end was left dry, so that there was no danger of drops of benzene falling from it into the silver evaporating-flask.

The rod was withdrawn occasionally and more benzene added if the upper portion of the roll had become dry. Observation of the galvanometer showed when the cooling effect caused by the evaporation had ceased, and we were thus able to determine when the flask and connecting tubes were saturated. The rod was not finally withdrawn until the dropper was introduced: thus the space being saturated, no further evaporation took place; and we believe that from Experiment X. onwards any error arising from evaporation in the time preceding the experiment may be disregarded.

After we had completed our experiments, we made some observations with the object of ascertaining the probable magnitude of the error in Experiments I. to VII. due to evaporation when the flask had not been previously saturated. A dropper was placed within the evaporating flask for 20 minutes and kept at a temperature of 30°C . It was then withdrawn, and the loss determined. As might have been expected, it varied slightly according to the droppers used, probably owing to the different sizes of the capillary opening.

In 20 minutes, dropper	II. lost 10 milligrams,
" " "	III. " 12 "
" " "	IV. " 13 "

After Experiment VII. we adopted the plan of noting the time from the insertion of the dropper to the commencement of the experiment, but unfortunately we had not previously done so. We consider that the average time in these experiments must have been from 10 to 15 minutes; the loss during this time would therefore appear to have been about 6 milligrams, or rather more than $\frac{M}{1000}$. Thus the values of L resulting from Experiments I. to VII. are probably too low by about 0.12. Fortunately we have six independent experiments at 30°C . (Nos. XXIX. to XXXIV.) in which this cause of error was absent.

We have entered into this matter fully, as it shows the importance of extreme attention to details in work of this kind, and also it was necessary to explain why we practically neglected Experiments I. to VII. when drawing our final conclusions. The same cause of error would slightly affect

Experiments VIII. and IX. at 40° C. At this time, however, we were attempting to minimize the evil by allowing as short a time as possible to elapse between the introduction of the dropper and the commencement of an experiment. We have also a note of the time, which in both cases was less than 4 minutes. The loss during this interval would probably not have affected the resulting values of L by more than .05, and we therefore do not consider it necessary to reject these experiments.

At the close of an experiment, when observation of the galvanometer showed that all evaporation had ceased, the dropper was extracted by means of a bent wire, immediately placed in its case, and weighed again after standing some hours in the balance-case.

Let m_1 and m_2 be the weights of the case and dropper before and after an experiment; the temperature of the balance-case was usually about 20° C., the specific gravity of benzene at that temperature may be taken as 0.85, hence M (true mass corrected to vacuo)

$$= m_1 - m_2 + (m_1 - m_2) \times .0012.$$

11. Before commencing the experiments it was necessary to ascertain if any alteration had taken place in the values of the various constants and variables since their determination in 1894.

Thermometers II. and A were re-standardized and it was found that a "zero-point rise" had taken place, as is customary with mercury thermometers.

Thermometer No. II. had risen 0.4 mm. = 0° 02 C. since its standardization in August 1894.

Thermometer A had risen 1.4 mm. = 0° 06 (nearly) since its standardization in July 1893, which is about the normal rise of thermometers of this description.

The platinum thermometer (AB) was removed from the calorimeter, strapped to its corresponding thermometer (CD), and placed in the tank whose temperature was raised from 18° C. to 40° C. Observations at different temperatures showed a rise of 0.45 mm. in the null-point whose position is now given by the formula $598.8 + .03\theta$ in place of $598.35 +$

·03 θ . If, however, this change, which probably took place in the arms of the bridge rather than in the thermometers *, had not been detected the resulting error would have been negligible, for a difference of ·45 mm. in the setting of the null-point is equivalent (at 40°) to a temperature difference of about ·0041° C., and the total loss or gain by radiation, &c., corresponding to this difference between θ_1 and θ_0 would not exceed 0·2 thermal gram per half-hour, whereas the actual duration of the majority of these experiments was about 18 minutes.

The corrected formula was, however, used throughout these experiments for the adjustment of the contact-maker.

12. Value of R_1 .

The values of R_1 were redetermined at temperatures 30°, 40°, and 50°. The corrected results showed falls of ·0012, ·0012, and ·0016 respectively from the values of 1894. These quantities have, therefore, to be subtracted from the values given in Paper W, table viii., but the correction is only 1 in 10,000 of R_1 .

The following table gives the values of R_1 used during the reductions of the observations on benzene.

(The suffix to R denotes the potential-difference in terms of a Clark cell.)

TABLE III.

Temp.	R_{1c} .	R_{1e} .	R_{1e} .
20	10·327	10·329	10·333
30	10·351	10·353	10·357
40	10·374	10·377	10·381
50	10·399	10·401	10·406

$$\delta R \text{ per } 1^\circ \text{ C.} = \cdot 0024.$$

The re-standardization of the Clark cells has already been referred to (section 6, p. 24).

* [Note by E. H. G., August 12, 1895.—I have found by re-standardizing the bridge-arms, that the above supposition was correct.]

13. Q_s .

No alteration in the values of Q_s at 30° and 40° C. appears to have taken place.

The values of Q_s at 50° and 20° as given in Paper W, pp. 332, 333, appeared to have undergone alteration, and we therefore made a careful redetermination at those temperatures. The method adopted was that described on p. 292, Paper W, viz., the rate of rise in temperature at null-point was determined

(1) when the heat-supply was that due to the stirring only

$$= \left(\frac{d\theta_1}{dt} \right)_s, \text{ and}$$

(2) when the heat-supply was that due to the stirring and a potential-difference of 3 Clark cells

$$= \left(\frac{d\theta_1}{dt} \right)_{3cc}.$$

Hence

$$\frac{\left(\frac{d\theta_1}{dt} \right)_s}{\left(\frac{d\theta_1}{dt} \right)_{3cc} - \left(\frac{d\theta_1}{dt} \right)_s} \dots \dots \dots (A)$$

gives the ratio of the heat supplied by the stirring to the heat-supply when the potential-difference was that of 3 Clark cells; and as the latter can be calculated if R_1 is known, the value of Q_s can be obtained without any assumptions as to the thermometric scale, the capacity for heat of the calorimeter, &c.

The individual experiments were in close agreement with each other and give the following results:—

At 50° , $\left(\frac{d\theta_1}{dt} \right)_{3cc} = \cdot 14533$ expressed in mm. of bridge-wire
where rate of stirring = $5\cdot380$,

$$\left(\frac{d\theta_1}{dt} \right)_s = \cdot 001155, \text{ at rate } 5\cdot380.$$

Hence Q_s (at rate $5\cdot380$) = $\cdot 003404$;

but (see Paper W, p. 330) $\frac{r^4}{Q_s} = \text{a constant} = \frac{837\cdot8}{\cdot 003404}$,
D 2

hence $Q_s = \cdot 003206$ at rate 5·300.

A second determination gave

$$Q_s = \cdot 003202 \text{ at rate } 5\cdot 300.$$

A third gave

$$Q_s = \cdot 003205 \text{ at rate } 5\cdot 300.$$

We therefore assume

$$Q_s = \cdot 00321 + (r_1^4 - 789) \times \cdot 0000041,$$

as sufficiently accurate.

$$\text{At } 20^\circ, \left(\frac{d\theta_1}{dt}\right)_{20} = \cdot 15941 \text{ at rate } 5\cdot 590,$$

$$\left(\frac{d\theta_1}{dt}\right)_s = \cdot 00427 \text{ at same rate.}$$

$$\text{Hence } Q_s \text{ (at rate } 5\cdot 590) = \cdot 01174 ;$$

$$\text{but } \frac{r^4}{Q_s} = \text{a constant} = \frac{976\cdot 6}{\cdot 01174},$$

$$\text{hence } Q_s = \cdot 00949 \text{ at rate } 5\cdot 300.$$

A second determination gave $\cdot 00951$ at rate 5·300,

A third „ „ $\cdot 00991$ „ „ 5·300.

In this last experiment, however, there is internal evidence of some error in the time over the second interval. If we omit this interval and calculate the value of Q_s from the remaining intervals of that experiment, we get $\cdot 00961$ as the value.

It is evident that $\cdot 00950$ is a sufficiently close approximation.

We therefore assume

$$Q_s = \cdot 00950 + (r^4 - 789) \times \cdot 0000120.$$

Our experiments at 30° and 40° show that the values of Q_s at those temperatures as given in Paper W are correct. The errors at 20° and 50° as given in that paper were due to an arithmetical mistake, a difference having been added, instead of subtracted, in each case.

The following Table gives the expressions by which the value of Q_s can be obtained at any of the temperatures or rates given in succeeding Tables:—

TABLE IV.

Temp.	Value of Q_s (in thermal grms.).
50	$\cdot 00321 + (r_1^4 - 789) \times \cdot 0000041$
40	$\cdot 00466 + (r_1^4 - 789) \times \cdot 0000059$
30	$\cdot 00665 + (r_1^4 - 789) \times \cdot 0000084$
20	$\cdot 00950 + (r_1^4 - 789) \times \cdot 0000120$

These changes in Q_s probably indicate the changes in the viscosity of the oil.

14. Alterations in the Apparatus.

No alterations have been made except in exterior portions of the apparatus.

When working with water in 1894 there were, in the tubes leading to the air-pumps, one or two rubber joints which are now replaced by glass ones.

To prevent condensation, the benzene vapour after issuing from the tank passed over a row of small gas-jets and then down into a small Wolff's bottle, connected with the manometer and containing pumice-stone and sulphuric acid. It then passed through another tap* into a large globe (capacity about 35 litres) also containing pumice-stone and sulphuric acid. By means of a water-pump the pressure in this globe was reduced to that of the aqueous vapour at the temperature of the tap-water. Its capacity was so great that the pressure in the Wolff's bottle could, at any time, be brought below that required, at experiments above 20° , by simply opening the tap communicating with the globe. Thus the water-pump had not to be used during an experiment, and, consequently, the two motors worked with greater regularity. The Wolff's bottle was also in direct communication with a Geissler's mercury-pump, by means of which, when working at the lower temperatures, the pressure was greatly reduced near the close of an experiment in order to secure the boiling-off of the last drop of benzene. We found that the mercury-

* The grease on the core of taps traversed by benzene was replaced by phosphoric acid.

pump had to be kept in constant use during the experiments at 20° . With the exception of the above alterations and additions, the apparatus is the same in every respect as that figured in Plates 4, 5, and 6 of Paper W.

15. *On the Purity of the Benzene.*

The benzene, which was a sample of that used by Professor Ramsay and Miss Marshall for their comparative experiments, was supplied by Messrs. Kahlbaum of Berlin and guaranteed free from thiophene. It was redistilled twice from phosphoric anhydride, until it showed a perfectly constant boiling-point.

16. *Description of an Experiment.*

The dropper was placed in position (as described in section 9) and the contact-maker was then set, by means of a magnifying-glass, to the null-point corresponding to the tank temperature θ_0 .

When the observer (Observer II.) at the thermometer-galvanometer (G_2) announced that θ_1 had become steady, three observations of the galvanometer-swing were taken, and the chronograph-key, being pressed at the second of those swings, recorded the time from which t_s was estimated. At the same moment Observer I. recorded the reading of thermometer II., which when reduced to the nitrogen scale gave the value of θ'_0 ,—the initial tank temperature.

As a rule the initial swings were somewhere between 0 and +50 (a swing of +50 would correspond to about $+0.06^{\circ}$ C.), and, for reasons which will appear later, we preferred to have this initial swing (which gave the value of d') positive.

The Wolff's bottle (condenser A) had previously been exhausted down to the pressure of the large globe (condenser B) already referred to, but the tap between the two condensers had been closed. A tap (immersed in the tank-water) between the silver flask and condenser A was then opened, and the air expanding into A caused a fall in the manometer attached to it. The tap connecting condensers A and B was then gradually opened, and the pressure in the flask fell until some benzene was expelled from the dropper. The instant

this occurred the calorimeter temperature θ_1 commenced to fall, and Observer I. was acquainted with this fact by Observer II. who, throughout the whole experiment, was engaged in calling aloud the galvanometer-swings which resulted from the inversion of the battery connexions in the differential-thermometer circuit by means of a swinging key. The storage-cell current, which had been running for some time through a platinum-silver coil immersed in oil in the outer tank, was then switched on to the calorimeter-coil, the action recording itself on the chronograph-tape, and the potential balance then adjusted by means of the apparatus described in Paper J, p. 283. This balance had previously been approximately obtained while the current was running through the tank-coil; thus only a small additional adjustment was required and, as the temperature of the calorimeter-wire remained constant, the electrical balance required little attention, for the potential-difference rarely altered by as much

as $\frac{E}{100,000}$ throughout an experiment. As the benzene vapour passed into condenser A the pressure in the flask increased, and thus the loss of heat by evaporation diminished. By a rapid movement of the tap between condensers A and B, the pressure could again be diminished and the cooling effect increased.

It was found impossible to control the rate of evaporation with the same perfection when using benzene as was the case in the experiments with water. The taps had to be constantly manipulated, and a moment's inattention on the part of Observer I. was immediately followed by a sudden rise or fall in θ_1 . This was more especially the case during the experiments at low temperatures. From about experiment V. or VI. onwards, however, the swings rarely amounted to as much as 200 or so, except during the first minute when the thermal balance was being obtained, at which time a swing of ± 400 or 500 was generally experienced. Throughout the whole of an experiment care was taken that any positive swing should be succeeded by a corresponding negative one. Although the announcement of swings of " + 200 " appeared alarming at the time, the extreme attention

devoted to the keeping down of these oscillations was really unnecessary. A swing of $+200$ indicated that θ_0 was lower than θ_1 (at 40°) by about $0^\circ.02$ C., and the radiation &c. coefficient of the calorimeter being about $.00009$ (in degrees per second per difference of 1° , Paper W, p. 289), this swing, even if maintained throughout the *whole time* that evaporation was proceeding (on an average less than about 7 minutes), would only have resulted in a *total* loss of about $.00009 \times 318 \times .0216 \times 7 \times 60 = .25$ thermal gram *. In no case, however, was a swing of such magnitude allowed to remain unaltered for more than a few seconds.

The chief difficulty was experienced near the close of an experiment. When working with water, there were always some indications that the end was near, for the pressure had to be diminished in order to maintain the thermal balance if only a drop or so remained. In the case of benzene, however, there were rarely any such indications; for the galvanometer-swing might be announced as $+8$ or $+10$, and before the next announcement could be made it would be found that it had shot up to 300 or 400. After the preliminary experiment, it was of course possible to roughly estimate (knowing the weight of the dropper when filled) the probable duration of an experiment and, by adding it to the observed time of establishing the current, to predict approximately the time when evaporation would cease. Owing, however, to differences in the size of the air-bubbles necessarily left in the droppers, to the different rates of stirring, &c., and to errors in calculations made while all the attention of the observers was needed elsewhere, the current was very often switched off too late, thus increasing the value of the correction Σg unnecessarily. If any mistake was made, the final value of θ_1 was in consequence higher than the initial one, for in those cases where the current was switched off too soon, it was always possible to bring θ_1 up to its initial value by re-establishing the current for a second or two; if, however, θ_1 was too high there was no means of diminishing it, all the benzene having been exhausted. Observer I. endeavoured, if possible,

* About $\frac{1}{2000}$ of the total "heat-supply."

to arrange so that the swing at the close of an experiment should be about -150 , for the following reason. Evaporation having ceased and the current being switched off, the tap between the evaporating-flask and condenser A was closed, and a tap (also immersed in the tank-water) was slowly opened so that air (dried by having passed through sulphuric acid and phosphoric anhydride) was gradually admitted into the evaporating-flask through a 30 ft. copper coil immersed in the tank. Thus the air was at the temperature of the calorimeter. The heat liberated by its compression caused, however, a rise in θ_1 equivalent to a swing of about 120; thus no heat was, on the whole, gained by the calorimeter, for a corresponding loss had been experienced during the exhaustion. On this account we preferred to commence with a small positive swing, as it was not then necessary to reduce θ_1 so far at the close of an experiment to allow for this final increase. When Observer II. found that θ_1 had again become steady three final swings were taken, the chronograph-key being pressed at the middle one, thus giving the termination of the time t_s during which the stirring supply had to be estimated. Thermometer II. was read at the same time, and gave θ''_0 , the final tank temperature. Throughout the experiment, every thousand revolutions of the stirrer had been automatically recorded on the chronograph-tape.

The method of removing and weighing the empty dropper has already been indicated.

17. *Remarks on the Experiments.*

We give particulars of all our experiments with two exceptions. In No. XXI. *a.* the chronograph ceased to work during a critical portion of the experiment, and we were thus unable to determine t_s . In XXIV. *a.* Observer I. omitted to close the entrance tap to the evaporating-flask before connecting with the exhaust, thus the attempt to diminish the vacuum was a failure and the experiment was relinquished after a minute or so. In these two cases we at once performed other experiments to replace the failures, but retained the numbers for convenience of reference.

We have, however, rejected, when drawing our final con-

clusions, several of the experiments whose details are given. Such experiments are marked by a † in the Tables. We have, in no case, rejected any experiment *except as a consequence of some note made during that experiment*; that is, before reducing the observations we had already decided as to those which should be regarded as of little value. Thus we have in no way been guided by the results. For example, XXV. differs from the mean of its group by a greater quantity than either of the rejected experiments at that temperature: however, as we have no note against it, we are compelled to give it equal importance with any of the others. A simpler plan would have been to reject entirely the † experiments, but we think that a fairer idea of the general accuracy of the work is given by including all those that we completed.

The reasons for the rejection of the † experiments are given (as they appeared in our original notes) at the end of each group. We have, however, in each case given the mean of all in a footnote, and it will be seen that (except at 30°) our results would not have been appreciably affected by the inclusion of those rejected ones.

As a rule, we found that the thermal balance was most easily maintained when the potential-difference was that of 4 cells, but this supply of heat would have been somewhat too great at 20°, when we worked with 3 cells only. At all temperatures above 20° we performed experiments with 3, 4, and 5 cells, and we regard the agreement amongst the results as very satisfactory, and as establishing the validity of the various corrections. In thermal investigations it is as a rule difficult to alter all the conditions in so complete a manner as that caused by the changes above referred to. For example, if any "priming" had taken place (the usual cause of inaccuracy in determinations of the Latent Heat of Evaporation), its effect must have been greatly increased when the rate of evaporation was nearly trebled, which was the case when the potential-difference was altered from 3 to 5 cells. Again, the importance of the different terms undergoes such changes that any constant error in the determinations of $Q_{s,t}$ and Σq would cause the values of L when $n=4$ to lie between the

values when $n=3$ and $n=5$. An inspection of the Tables will show that no such effect is visible.

We also varied the "stirring supply" very considerably. The Tables show that (at the *same* temperature) the values of Q_s have been changed from .00276 (Expt. XVII.) to .00427 (Expt. XXIV.).

[*Note by E. H. GRIFFITHS.*—My experience with apparatus of a similar nature to that used in these experiments has convinced me of the severity of the test above referred to. During my attempts at a determination of the mechanical equivalent during the years 1887–1891 the results invariably broke down when thus tested. The following quotation, referring to the experiments anterior to 1892, is from p. 364, Paper J :—

"The agreement amongst individual experiments taken under the *same conditions*, was, if anything, at times more marked than in the experiments of 1892 ; nevertheless, when the final reduction took place, fatal discrepancies invariably showed themselves. . . . Experiments conducted with a high electromotive force invariably gave too great a value for the time as compared with that obtained when a lower electromotive force was used."

It appears to me that it is this power of altering all the conditions which renders electrical methods of such great value when applied to thermal determinations.]

18. *Explanation of the Tables.*

Tables lettered A give the experimental *data* ; the deductions from those data are given in Tables lettered B.

(We have arranged the experiments in order of temperature rather than historical order, as consecutive experiments were not always at the same temperatures.)

TABLES A.

- | | |
|------|---|
| Col. | I. <i>a</i> and <i>b</i> give the number and date of the experiment. |
| " | II. <i>a</i> and <i>b</i> give the number of Clark cells in series, and the number of the dropper used. |

- Col. III. gives the mass of benzene evaporated, after correction to *vacuo* (M).
- „ IV. The time during which the current was maintained (t_x).
- „ V. The time from commencement to end of experiment (t_s), *i. e.*, the time during which the “stirring supply” has to be estimated.
- „ VI. The number of revolutions per second (r_1) of the stirrer.
- „ VII. The difference between the initial (θ_0') and the final temperature (θ_0'') of the surrounding walls.
- „ VIII. Let θ_1' (initial calorimeter temperature) exceed θ_0' by d' , and let θ_1'' (final calorimeter temperature) exceed θ_0'' by d'' ; then this column gives the value of $d' - d''$. Hence
- $$\{(\theta_0' - \theta_0'') - (d' - d'')\}$$
- gives the difference between the initial and final temperature of the calorimeter.
- „ IX. The capacity for heat of the calorimeter and its contents at the mean temperature of the tank (C_{θ_1}).
- „ X. The temperature of the Clark cells.
- „ XI. The value of R_{ne} , *i. e.*, the resistance of the calorimeter-coil at temperature θ_1 , when the potential-difference of its end is ne . (From Table III. p. 34.)

19. TABLES B.

(The numbers of the columns are similar to those in the corresponding tables of Paper W, where full particulars of the reductions are given.)

- Col. XV. The value of $Q_x t_x = \frac{(ne)^2 \times t_x}{R_1 \times J}$, where e is the potential-difference of 1 Clark cell, at the temperature given in Col. X. (*supra*). The values of R_1 will be found in Table III. p. 34, and, as previously explained, the value of $J = 4.199$ (see section 6, p. 24).

- Col. XVI. The value of Q_s deduced from Col. VI. by means of Table IV. p. 37.
- „ XVII. The value of Q_{st} , the “stirring supply,” from Cols. V. and XIV.
- „ XVIII. The term $\Sigma q = C_{\theta_1} \{(\theta_0' - \theta_0'') - (d' - d'')\}$ from Cols. VII., VIII., and IX.
- „ XIX. The sum of Cols. XV., XVII., and XVIII., that is, the total thermal grams (Σ) required for the evaporation of M grms. of benzene.
- „ XX. The mean tank temperature (θ_0) of the experiment, expressed in the nitrogen scale.
- „ XXI. The value of $L = \frac{\Sigma}{M}$.

Remarks on Experiments at 50°.

Exp. XIX. a. “Mistake in switching off current, did not do so till θ_1 had risen so far that d'' could not be obtained by means of galvanometer-swing. Had to readjust contact on bridge at close of experiment. Decided to regard experiment as a failure and repeat it.”

The experiment is included in the table to show that even the large resulting value of the term $\Sigma q (-18.64)$ has but a small effect on the value of L , which differs from the mean at this temperature by less than 1 in 400. This value for $L(98.90)$ should, however, certainly not influence our final conclusions.

Exp. XXIII.—“Mistake in connecting with vacuum, reduced pressure far below right amount, hence could not obtain proper thermal balance throughout experiment.”

Conclusion.

When $\theta_0 = 50.014$, $L = 99.14$.

TABLE V. A.
Experiments at 50° C.

I.		II.		III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
a.	b.	a.	b.									
Number.	Date, July	No. of Cells.	No. of Case.	M.	ℓ_r	ℓ_s	r_1	$\theta_1' - \theta_2''$	$d' - d''$	C_{θ_1}	Temp. of Cells.	R_{ref}
XVII.	14	4	II.	5.0339	658.4	1319.2	5.107	0°	-0.010	322	15.30	10.402
XVIII.	14	4	III.	4.7071	622.4	1118.7	5.118	0	-0.054	"	15.30	"
XIX. a.	14	4	IV.	4.9536	670.7	1204.3	5.126	-0.020	-0.060	"	15.30	"
XIX. b.	14	4	III.	4.7241	615.7	1093.2	5.241	+0.025	+0.046	"	15.64	"
XX.	16	4	V.	4.3654	569.1	997.0	5.171	+0.054	-0.030	"	15.24	"
XXI. b.	16	4	II.	4.9893	652.6	1314.0	5.219	+0.005	-0.030	"	15.70	"
XXII.	17	3	V.	4.3550	1010.1	1521.6	5.195	-0.024	-0.014	"	15.58	10.399
XXIII.	17	5	IV.	4.9726	416.2	1024.2	5.411	+0.024	-0.047	"	15.77	10.406
XXIV. b.	18	5	IV.	5.0056	410.6	874.0	5.692	+0.017	-0.057	"	15.50	10.406
XXV.	18	3	V.	4.3899	1011.4	1405.4	5.653	+0.054	-0.068	"	15.50	10.399

TABLE VI. A.
Experiments at 40° C.

I.		II.		III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
a.	b.	a.	b.	M.	t_g	t_g	r_1	$\theta'_0 - \theta''_0$	$d' - d''$	C_{g_1}	Temp. of Cells.	R_{ac}
Number.	Date, July.	No. of Cells.	No. of Case.									
VIII.	8	4	III.	47228	628.2	1092.0	5.274	0	-0.049	318	16.27	10.377
IX.	9	4	IV.	50424	676.8	1155.0	5.220	+0.010	-0.0192	"	15.90	"
X.	9	4	II.	50212	665.4	1169.3	5.171	-0.020	+0.023	"	15.70	"
XI.	9	4	III.	47228	635.5	1074.6	5.176	-0.015	-0.0302	"	15.71	"
XII.	9	4	V.	43846	577.8	1105.3	5.118	-0.020	-0.0016	"	15.72	"
XIII.	10	4	IV.	49094	670.2	1143.7	5.007	-0.045	-0.0085	"	15.45	"
XIV.	10	4	II.	50258	662.0	1194.2	4.859	+0.030	+0.0054	"	15.45	"
XV.	11	4	IV.	50051	668.6	1084.8	5.169	-0.051	-0.0116	"	15.40	"
XVI.	11	4	II.	50286	678.3	1281.4	5.162	+0.051	-0.0430	"	15.50	"
XXVI.	19	3	V.	44191	1035.2	1471.5	5.242	-0.030	+0.0027	"	15.32	10.374
XXVII.	19	5	III.	47469	406.0	779.8	5.237	+0.015	-0.0118	"	15.32	10.381
XXVIII.	19	4	II.	50292	664.6	1066.6	5.238	+0.070	-0.0070	"	15.34	10.377

TABLE VI. B.
Reduction of Experiments at 40° C.

I. a.	XV.	XVI.	XVII.	XVIII.	XIX.	XX.	XXI.
Number.	Q_g	Q_g	Q_g	Σq	Σ	θ_b	L.
VIII.	473.86	-00457	4.99	1.56	477.29	39.965	101.04
IX.	510.68	-00439	5.07	- 5.79	509.96	40.030	101.12
X.	502.22	-00423	4.95	- 3.08	504.90	40.068	100.39
+XI.	479.49	-00424	4.56	- 7.12	476.93	40.078	100.99†
XII.	435.08	-00405	4.48	- 1.14	439.42	40.071	100.22
XIII.	505.96	-00399	4.16	- 7.31	502.81	40.055	100.57
XIV.	499.81	-00350	3.94	+ 2.67	506.42	40.065	100.76
XV.	504.79	-00422	4.58	- 5.31	504.06	40.058	100.70
XVI.	512.05	-00419	5.37	- 12.05	505.37	40.053	100.50
XXVI.	439.76	-00446	6.56	- 0.10	446.22	40.021	100.97
XXVII.	478.73	-00445	3.47	- 3.28	478.92	40.025	100.90
XXVIII.	501.77	-00445	4.75	0	506.52	40.025	100.70
Mean (omitting that marked †)							100.71 ± .057
If we include all experiments, we get							100.74 ± .064

TABLE VII.A.
Experiments at 30° C.

I.		II.		III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
a.	b.	a.	b.	M.	t_a .	t_b .	τ_1 .	$\theta_0' - \theta_0''$.	$d' - d''$.	C_0 .	Temp. of Cells.	R_{me} .
Number.	Date, July	No. of Cells.	No. of Cells.									
† Prelim.	4	3	I.	35975	878.8	1617.0	5691	0	-0640	312	15.93	10350
I.	5	4	III.	49107	661.6	1341.4	5515	-0108	-0178	"	15.44	10353
† II.	6	4	IV.	50174	676.6	1210.9	5517	-0082	-0212	"	15.59	"
III.	7	4	II.	51313	695.0	1102.3	5473	0	-0302	"	15.57	"
IV.	7	4	IV.	50071	674.7	1224.9	5577	0	-0088	"	15.74	"
V.	7	4	V.	45336	697.0	1113.2	5582	-0096	-0121	"	15.70	"
VI.	7	4	III.	48039	636.6	1144.5	5120	+0103	-0063	"	15.77	"
VII.	7	4	II.	52472	692.4	1251.5	5091	+0061	-0088	"	15.89	"
XXIX.	22	3	V.	44083	1045.0	1438.8	5238	-0088	-0085	"	14.85	10351
XXX.	22	3	IV.	49873	431.1	939.5	5252	+0031	-0178	"	14.85	10357
XXXI.	23	5	III.	47495	406.7	788.5	5227	-0077	+0037	"	14.82	10357
XXXII.	23	3	V.	44459	1050.7	1434.8	5156	0	-0010	"	14.90	10351
XXXIII.	23	5	II.	50477	437.8	840.9	5227	-0108	-0068	"	14.78	10357
XXXIV.	23	3	IV.	50226	1185.7	1645.3	5241	0	-0022	"	14.85	10351

TABLE VII. B.

Reduction of Experiments at 30° C.

I. a.	XV.	XVI.	XVII.	XVIII.	XIX.	XX.	XXI.
Number.	Q_{at}	Q_s	Q_{at}	Σq	Σ	θ_p	L.
† Preliminary.							
I.	373.98	00883	14.3	-19.96	368.32	29.820	102.36 †
† II.	500.60	00779	10.45	- 8.92	502.13	29.990	102.25
III.	511.72	00907	7.35	- 9.17	509.88	29.995	101.61 †
IV.	528.68	00756	8.33	- 9.42	525.59	29.962	102.44
V.	510.41	00815	9.98	- 8.68	511.81	30.000	102.24
VI.	459.80	00818	9.11	- 6.77	462.24	29.971	101.67
VII.	451.59	00589	6.64	+ 1.25	489.48	29.969	101.80
	523.62	00567	7.10	+ 4.65	535.37	29.977	102.02
XXIX.	445.19	00935	9.14	- 5.40	448.93	30.104	101.85
XXX.	509.70	00641	6.02	- 4.59	511.13	30.117	102.46
XXXI.	489.95	00629	4.96	- 1.25	484.66	30.034	102.05
XXXII.	447.50	00596	8.55	- 0.31	455.74	30.047	102.50
XXXIII.	517.80	00829	5.29	- 5.49	517.60	30.042	102.55
XXXIV.	504.26	00636	10.46	- 0.69	514.03	30.052	102.34
Mean of Experiments XXIX. to XXXIV.							
† Mean of all (omitting Preliminary and II.)							
						30.066	102.30 ± 0.76
						30.006	102.22

Remarks on Experiments at 40° C.

Exp. XI.—“Reduced pressure far too much at commencement, was unable to obtain thermal balance until near end of experiment.”

Exp. VIII. & IX.—As pointed out (section 10, p. 33), the values of L are here probably in error by about 0.05. This, however, does not seem sufficient reason for rejecting them.

Conclusion.

When $\theta_0 = 40.045$, $L = 100.71$.

Remarks on Experiments at 30° C.

Preliminary Exp.—The oscillations in θ_1 were very violent, as this was our first attempt at balancing when evaporating benzene; also, as we had no idea when to finish, the value of Σq is very great. We have, however, inserted it to show how little such matters affect the result.

Exp. II.—“After inserting the dropper, had to withdraw it again owing to its sticking in the tube; dropper exposed for some time.” There is little doubt that, owing to the cooling when withdrawn, some air was sucked back, and hence there must have been some expulsion on re-introduction. We regret that we continued the experiment: the result is useless.

20. It will be noticed that the irregularities in this Table are greater than in preceding ones. These are due to two causes:—(1) The want of practice in the observers; (2) The value of the galvanometer-swings were not determined with sufficient accuracy until after Experiment VII., and, as the values of Σq are large, any error might affect L , but probably not by more than ± 0.1 .

In Section 10, p. 32, we have given in full our reasons for considering that all these experiments (I. to VII.) are too low by about 0.12. The mean value of L given by them (omitting Preliminary and II.) is 102.14: thus it is probable that, had the flask been previously saturated with vapour, we should have obtained about 102.26, which is in fair agreement with the mean of the last six (102.30), where this cause of error

TABLE VIII. A.
Experiments at 20° C.

I.	II.		III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
	a.	b.									
Number.	Date, July		M.	t_g .	t_s .	r_1 .	$\theta_0' - \theta_0''$.	$d' - d''$.	C_{θ_1} .	Temp. of Cells.	R_{nc} .
+ XXXV.	24		47593	1129.6	1575.5	5.562	+0041	-0104	305	15.30	10.327
XXXVI.	24		50083	1182.1	1833.2	5.586	-0052	-0143	"	15.40	10.327

TABLE VIII. B.
Reduction of Experiments at 20° C.

I. a.	XV.	XVI.	XVII.	XVIII.	XIX.	XX.	XXI.
Number.	$Q_{E't_g}$.	Q_g .	$Q_{E't_g}$.	$2g$.	Σ .	θ_0 .	L.
+ XXXV.	482.02	-01151	18.14	-4.67	495.49	19.047	104.10 †
XXXVI.	504.42	-01172	21.48	-5.95	519.95	19.047	103.82

was non-existent. In any case, taking *all* the values as they stand in Col. XXI. (rejecting the † experiments), we get 102.22. The number 102.30, however, is the most probable value.

Conclusion.

When $\theta_0 = 30.066$, $L = 102.30$.

Remarks on Experiments at 20° C.

Exp. XXXV.—This should be regarded as a preliminary experiment only. We had no idea of the pressure required to produce equilibrium, and had great difficulties in sufficiently reducing the pressure. " θ_1 was extremely irregular; the galvanometer-swing was often off the scale for a considerable period."

Exp. XXXVI.—"This was a very fair experiment; the thermal balance well under control."

We regret that we were unable to perform more experiments at 20° C. It was found necessary, however, to work the mercury-pump almost continuously, and this could not be done without further assistance*. As such assistance was but rarely available, and as our time was drawing to a close, we were compelled to content ourselves with the above experiments.

Conclusion.

When $\theta_0 = 19.947$, $L = 103.82$.

21. *Discussion of the Results.*

The values of L over the range 20° to 50° appear to be practically a linear function of the temperature.

If we assume

$$L = 107.05 - 0.1581\theta, \quad \dots \quad (\alpha)$$

we obtain a very close approximation to the experimental values, as shown by the following Table:—

* We return our sincere thanks to Mr. C. T. Heycock, F.R.S., for his help during these experiments at 20° C.

TABLE IX.

Temperature. Nitrogen-scale.	L (expressed in terms of a thermal unit "at 15° C.").	
	By formula (a).	Experimental results.
50.014	99.14	99.14 ± .034
40.045	100.72	100.71 ± .057
30.068	102.30	102.30 ± .076
19.947	103.90	103.82

It is evident, therefore, that the curvature over the above temperature-range must be very slight.

The value of L cannot continue to be a linear function of θ at very high temperatures, either in the case of benzene or water; for the critical point of benzene would fall at about 677° C., and that of water at about 990° C. It has, however, been shown that in the case of water the curvature from 0° to 100° (that is, over a pressure-range of about 4 mm. to 760 mm.) may be neglected (Paper W, pp. 316-321); and the above Table would indicate that we cannot be far wrong in making a similar assumption with regard to benzene, especially when we consider that both the temperature-range (0° to 80°·2 C.) and the pressure-range (26 mm. to 760 mm.) are in the latter case diminished.

We originally intended to perform a group of experiments at 60°, which is about the highest temperature to which it is advisable to expose the apparatus; but before doing so it would have been necessary to standardize a third mercury-thermometer—a work which would have occupied at least a week of the short time at our disposal.

Assuming formula (a) to hold to 760 mm., and taking the boiling-point at that pressure as 80°·20 C. (Ramsay and Young, Phil. Mag. 1887), we get

When $\theta = 80.2$, $L = 94.37$ ("thermal units at 15° C.").

The results of the experimental work described in this paper may be summarized as follows:—

The Latent Heat of Evaporation of Benzene over the temperature-range 20° to 50° C. (nitrogen-scale) is represented by the equation

$$L = 107.05 - 0.158 \theta,$$

where L is expressed in terms of a "thermal unit at 15° C."

Assuming this expression to hold over the range 50° to 80° , the resulting value of L at $80^{\circ}.2$ C. (760 mm. pressure) is 94.37.

22. Historical.

We have been unable to find records of any determinations in addition to the following:—

REGNAULT. *Mémoires de l'Académie*, 1862, vol. xxvi. p. 761: $L = 109.0$.

R. SCHIFF. *Liebig's Annalen*, 1886, vol. ccxxxiv. p. 338: $L = 93.4$ at $80^{\circ}.35$.

K. WIRTZ. *Wiedemann's Annalen*, 1890, vol. xl. p. 438: $L = 92.9$ at $80^{\circ}.1$.

JAHN. *Zeitschrift für Physikalische Chemie*, 1893, vol. xi. p. 79: $L = 107.6$.

IV. *A Method of Comparing directly the Heats of Evaporation of different Liquids at their Boiling-points.* By Miss DOROTHY MARSHALL, B.Sc., and Prof. W. RAMSAY, Ph.D., F.R.S., University College, London*.

1. THE heat of evaporation of a liquid is usually determined by measuring with the aid of a calorimeter the amount of heat liberated during the condensation of a known mass of the vapour; or conversely, by measuring the heat absorbed during the vaporization of a known mass of the liquid. The former method is the one more frequently adopted, *e. g.* by Berthelot and by Schiff.

Apart from the direct experimental difficulties of the method (and these are by no means inconsiderable), it is open to the objection that it measures, not simply the heat set free during the passage of a given mass of the substance from the gaseous to the liquid condition, but the sum of this quantity of heat with that liberated during the cooling of the liquid from the temperature of its boiling-point to the final temperature of the calorimeter. It is therefore necessary to make a separate determination of the thermal capacity of the liquid between these temperatures.

Whatever the cause may be, the fact remains that the data regarding heats of vaporization are very scanty. The subject has been worked at by few observers, and the results obtained show very considerable discrepancies. Taking, for instance, the case of ethyl alcohol, the heat of vaporization of a gramme is given as 208 calories by Despretz, 227 by Brix, 202.4 by Andrews, and 208.9 by Favre and Silbermann. It is clear that no method can be really satisfactory that leads to such discordant results.

2. Professor Ramsay suggested some time ago that it should be possible to compare directly the heats of evaporation of two liquids, by raising the temperature of each to its boiling-point (surrounding each vessel with a jacket of its own vapour), and then determining the loss of weight sustained by each vessel when a current of electricity was passed

* Read November 8, 1895.

through a carbon filament immersed in the liquid. All other conditions being made the same for the two vessels, the ratio of their losses of weight should give the inverse ratio of the heats of evaporation of the liquids. This method of comparison we have since been engaged in working out; and having once overcome the initial difficulties of apparatus &c., we have been able to obtain some very satisfactory results, and intend to apply the method to a number of other liquids that we have not yet had time to investigate.

In such an arrangement as this there is no direct heat measurement, and therefore a fruitful source of error is eliminated. As the current passes, heat is developed in each carbon filament; and this heat is entirely expended in converting a portion of the liquid into vapour. The thermal capacity of the liquid no longer needs to be known, since the experiment is conducted entirely at the temperature of its boiling-point.

Each vessel is weighed at the temperature of the atmosphere at the beginning and end of the experiment; and the loss of weight of the vessel during the experiment gives the amount of liquid vaporized. If the same current be passed for the same time through two filaments of equal resistance, equal amounts of heat will be developed: hence the ratio of the loss of weight of the first vessel to the loss of weight of the second is the inverse ratio of the heats of vaporization of the liquids employed.

If the filaments have not equal resistances, the amounts of heat developed will be directly as the resistances. If the heat of evaporation of any one liquid be known, the absolute value of the heat of evaporation of any other liquid can be calculated from the ratio directly found.

3. [*Note, July 1895, by D. Marshall.*—At the time when this work was begun, the only liquid for which L was at all certainly known was water; but it was not considered advisable to compare directly with water in all the experiments: (i.) because the heat of evaporation of water is so very much greater than that of any other liquid; (ii.) because it is so exceedingly difficult to get it sufficiently pure; and (iii.)

because the continued use of water seemed to make the apparatus more liable to break.

Ethyl alcohol was in every way more suitable for a standard, and I therefore tried to make a particularly careful comparison between alcohol and water. But both in those first attempts, and again in several other attempts that I made later on, I had the greatest difficulty in getting even moderately concordant results; and I have never, when working with water, been able to approach the degree of accuracy that I know from my other results I am justified in expecting.

Thus I was finally obliged to conclude that water is not a liquid to use in these experiments at all, much less is it fitted to form our ultimate standard. It became necessary therefore to find some other liquid for which L was accurately known; and as we did not feel sufficient confidence in any of the published values with which we were acquainted, Mr. E. H. Griffiths, F.R.S., kindly consented to make a determination of the Heat of Evaporation of Benzene by the same method that he used with such conspicuous success in the case of water.

His experiments (in which I also took part) measured the heat of evaporation over the range 30° to 50° C.; and gave by extrapolation the value at $80^{\circ}\cdot 2$, the boiling-point. This number is therefore the one finally adopted by us in the calculation of our results; and we take this opportunity of recording our sincere thanks to Mr. Griffiths for furnishing us with such a trustworthy standard.

It should be observed that this number finally obtained for the heat of evaporation of benzene at its boiling-point does not in any way affect our work regarded as a method of comparison; but it does enter as the most important feature into the calculations by which we pass from our ratios to the absolute values of the heats of evaporation, and bring our results into line with those of other observers.]

4. *Description of the Apparatus.*

In the first experiments vessels were used closely resembling the ordinary incandescence-lamps, with carbon filaments; but a good deal of trouble was caused by the extreme diffi-

culty of making satisfactory connexions. After the preliminary experiments, these lamps were therefore rejected in favour of others which had a spiral of fine platinum wire in place of the carbon filament: the ends of the spiral were attached to stout platinum terminals sealed into the glass; the terminals were gilt and amalgamated, and thus a safe and good contact was secured. The upper part of each lamp was drawn out into a rather narrow open tube, through which the liquid could be introduced, and which could be securely closed to prevent loss during weighing. The lamp was set up in an ordinary vapour-jacketing arrangement provided with side-bulb and condenser. The jacket was closed at the bottom by an indiarubber cork, through which passed two U-tubes containing mercury; the terminals of the lamp rested on the inner ends, into the outer ends dipped the wires carrying the current. The cork was protected by a layer of mercury so that it could not come into contact with the liquid. Each lamp was jacketed with the vapour of its own liquid, so that the temperature of its contents would be raised to the boiling-point without the possibility of ebullition taking place until the current was started; the liquid then lost by boiling from the lamp was condensed in the jacket, and there was no loss of material.

The two lamps were connected in series.

In the earlier experiments the current used was that supplied at 110 volts to run two incandescence-lamps of 32-candle power (in parallel): it was further cut down to a convenient strength by the introduction of an appropriate resistance. At first this resistance consisted of a solution of copper sulphate—therefore both indefinite and variable: in the later experiments a coil of wire of about 20-ohms resistance was substituted for the copper-sulphate solution.

In performing an experiment, the liquid in the side-bulb of the jacket was first caused to boil; and the current was not started until the whole contents of the lamp were judged to have reached the temperature of the condensing vapour: if this were so, the liquid would pass into tranquil ebullition the moment the circuit was completed. It was generally found advisable to drop into the lamp a little glass capillary

tube to provide a starting-point for boiling, as most of the liquids showed a great tendency to become superheated and bump. A number of experiments were rejected for this reason.

5. *Preliminary Experiments.* February 1894.

In the experiments with carbon lamps, the resistance of each filament was determined by the Wheatstone-bridge method, using a post-office box of coils for bridge, at five different temperatures; and the results were plotted on squared paper. The observations for each lamp lie approximately on a straight line, the resistance falling off uniformly as the temperature rises.

The resistance of each filament at 0° and 180° , the decrease per degree, and the coefficient of change of resistance between 0° and 180° were found from the curves.

TABLE I.

	I.	II.	III.
Resistance at 0°	42.5 ohms.	43.17	42.60
Resistance at 180°	36.69 "	36.58	36.10
Decrease per degree	0.0323	0.0366	0.0361
Coefficient of change of resistance...	0.00076	0.00085	0.00085

These experiments gave no satisfactory results.

6. *First Series of Experiments.* March 1894.

Two lamps were now made with spirals of fine platinum wire in place of carbon filaments. These platinums were each 18 inches long, and coiled into spirals with all possible care to prevent neighbouring turns from touching and short-circuiting. The resistances of the platinums were determined by the Wheatstone-bridge method. The coefficient of increase of resistance of platinum with rise of temperature being (approximately) known, the resistance of each spiral for any given temperature could be calculated. It was assumed that

the temperature of the spiral was the same as that of the liquid boiling round it.

A comparison of chlorobenzene and alcohol may serve for an example :—

Date.	B.	Weight evaporated.		R ₁ .	R ₂ .	Corrected Weights.		Ratio $\frac{L_2}{L_1}$.	L ₁ .
		C ₆ H ₅ Cl.	C ₂ H ₅ OH.			C ₆ H ₅ Cl.	C ₂ H ₅ OH.		
March 15.	746.3	14.5830	4.7975	4.89	4.27	14.5830	5.5252	0.364	76.1
April 20.	763.0	16.6645	5.4195	4.87	4.31	16.6645	6.1236	0.367	

TABLE II.

Substance.	Date.	B.	Ratio.	L.
Alcohol referred to { water*.....}	May 1	767	0.387	208.1
	" 4	755	0.388	
CS ₂	Mar. 11	741.9	0.405	85.0
Acetic Acid	May 10	754.4	0.475	98.6
" "	" 15	756.8	0.472	
" "	" 16	757.7	0.472	
Ether.....	" 17	752.7	0.396	82.5
"	" 30	752.3	0.397	
"	June 4	753.9	0.396	
Ethyl Acetate	" 5	754.4	0.409	85.1
" "	" 6	753.8	0.409	
Methyl Formate	" 7	755.2	0.514	106.7
" "	" 11	754.1	0.511	
Methyl Butyrate	" 12	758.3	0.389	81.0

* The other substances are referred to Alcohol.

7. Second Series of Experiments. June 1894.

Up to this time the resistance of the platinum spiral during the passage of the current had never been directly measured, but was calculated from the cold resistance and the coefficient of increase with rise of temperature, on the assumption that the temperature of the spiral was the same as that of the surrounding liquid. The truth of this assumption was now called in question, and it was found to be unjustifiable. The resistance of the spiral was directly measured, first when surrounded by a liquid at its boiling-point, then when the current was passed so that the liquid boiled.

The resistance was always found to be greater when the current was passing; that is to say, the temperature of the spiral is always higher than that of the liquid boiling round it*. One does not exactly know what conditions govern the difference of temperature between spiral and liquid. The probability is that the ratio of the resistances when two liquids are compared will be altered, as well as the absolute values of the resistances themselves; and this would tend to alter all the results hitherto obtained.

It should also be observed that the coefficient of increase of resistance of platinum with rise of temperature is that given by Callendar (1·338; Phil. Trans. clxxxii. A, p. 136) for a particular sample of wire employed by him; but as this coefficient is found to be different for different samples of wire, the value adopted may very probably not hold for the particular wire here used.

It was therefore necessary to determine directly the resistance of the platinum spirals in each experiment. The resistance of the wire during the passage of the current was now determined by Poggendorff's method:—viz., comparison with the resistance of a standard wire carrying the same current. [For diagram of arrangements *vide infra*, p. 65.]

The standard resistance was a piece of uninsulated german-silver wire, immersed in cold water to keep its temperature constant.

Table III. shows the results obtained.

TABLE III.

Substance.	Date.	Ratio $\frac{L_1}{L_2}$.	L_1 .
Water	July 2.	0·391 }	209·74
"	" 6.	0·390 }	
"	" 11.	0·391 }	
Chlorobenzene	" 18.	0·355 }	75·2
"	" 19.	0·363 }	
"	" 19.	0·358 }	
Ether	" 12.	0·398 }	83·5
"	" 13.	0·398 }	
Ethyl Acetate	" 20.	0·392 }	82·2
"	" 23.	0·392 }	
Acetic Acid	" 24.	0·471 }	100·2
"	" 25.	0·484 }	

* Griffiths, Phil. Trans. clxxxiv. A (1893), pp. 400-407.

8. *Third Series of Experiments.* February 1895.

In the early part of this year the apparatus was further modified with a view to increasing the accuracy of the resistance measurements.

The readings of resistance had been found very troublesome and very uncertain. In many experiments it was almost impossible to obtain anything like a point of balance on the bridge-wire: it was useless to attempt to read to within 1 centim. length of the wire; and during any one experiment the points of balance did not remain steady at all, but seemed to shift indefinitely. This might be the consequence either of variations in the strength of the current in the main circuit, or of change in the resistance of the bridge-wire from unequal heating. [The bridge consisted of a coil of uncovered platinoid wire immersed in cold water and connected in series with a platinum wire stretched against a scale.] Probably both causes were working together.

Two important changes were made:—

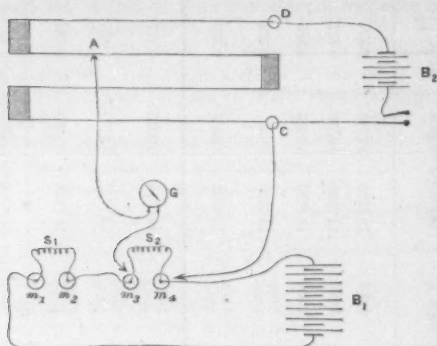
(i.) The current in the primary or lamp-circuit was taken from 8 secondary cells, instead of being taken from the mains through a (probably) variable resistance.

(ii.) A new bridge was made, consisting of four parallel german-silver wires of approximately equal resistance stretched against four metre-scales.

The whole of the resistance in the secondary circuit now falls into the bridge itself; whatever heating-effect there may be is distributed all along the bridge, and its effect is much lessened if not altogether removed. The resistance of the bridge-wire does not need to be known, provided it is tolerably uniform; because we are only concerned with comparative measurements, which are simply proportional to the lengths measured.

For the same reason the standard wire of known resistance is really unnecessary, and was omitted in all the remaining experiments.

It became evident at once—working in these improved conditions—that the whole method had been made very much more sensitive and accurate than before; hence it was of



S_1, S_2 are the platinum spirals of the lamps.

$m_1, \dots, m_4$ are mercury-cups by which connexion is made with S_1 and S_2 .
 B_1, B_2 are the batteries supplying the current in the primary and secondary circuits respectively.

C, D are the terminals of the fourfold potentiometer wire.

A is a sliding contact, and has to be moved until the fall of potential on the bridge between C and A is equal to the fall of potential between m_4 and m_3 , or m_2 and m_1 .

G is a simple reflecting-galvanometer.

the utmost importance to repeat the fundamental comparison between alcohol and water from which all the other results were calculated. These experiments are entered in Table IV.

It was at this point that we gave up the idea of taking water for our standard, and looked round for some other liquid to use in its place. It is quite clear, from the numbers obtained, that there is something in the water itself that interferes with the experiments. Probably there is some electrolysis during the passage of the current along the un-insulated wire; it appears impossible by any ordinary means to get water so pure that its conductivity may be disregarded.

In the later tables the numbers given for the heats of

TABLE IV.

No.	Date.	B.	Weights evaporated.		V ₁ .	V ₂ .	Corrected weights.	Ratio $\frac{L_1}{L_2}$.
			Alcohol.	Water.				
1.....	March 1.	754.1	15.2470	5.4645	274.7	1.057	15.2470	0.379
2.....	" 2.	754.7	14.7365	5.2370	326.2	1.060	14.7365	0.377
3.....	" 5.	757.8	14.1615	5.1310	227.2	1.057	14.1615	0.383
4.....	" 6.	756.0	13.5130	4.7255	{ 208.0 248.5 }	{ 1.063 1.065 }	13.5130	0.372
5.....	" 7.	755.6	15.3565	5.5785	{ 207.4 247.4 }	{ 1.064 1.061 }	15.3565	0.385
6.....	April 9.	759.5	14.0570	5.6460	{ 327.2 275.1 }	{ 1.063 1.064 }	14.0570	0.427
7.....	" 9.	759.5	13.7710	5.5770	{ 327.9 275.7 }	{ 1.066 1.066 }	13.7710	0.432
8.....	" 10.	761.3	13.7425	5.5420	{ 328.3 276.2 }	{ 1.067 1.067 }	13.7425	0.430

Note.—Experiments 1 to 5 were made with the ordinary distilled water of the laboratory; experiments 6 to 8 were made with some water that had been specially purified for use in water-analysis.

evaporation are calculated from the absolute value for benzene, viz. :— $L = 94.4$ at $80^{\circ}2$.

Table V. records the results obtained by us for benzene, toluene, metaxylene, acetic acid, and ten of the lower esters prepared by Prof. Sydney Young. Each liquid is compared with alcohol.

In Table VI. we give for each liquid :—

- (A) the ratio compared with benzene ;
- (B) the value of L in thermal units at 15° ;
- (C) the value of L obtained from the thermodynamic equation (see p. 71, *infra*) ;
- (D) values obtained by other observers.

In Table VII. we give for each,

L , the heat of evaporation ;

t , the temperature of the boiling-point ;

M , the molecular weight ;

$\frac{ML}{T}$, the quotient of the molecular heat of vaporization by the absolute temperature ; according to Trouton's Law this should be a constant.

9. TABLE V.

No.	Date.	B.	Weights evaporated.		Ratio $\frac{V_1}{V_2}$.	Corrected weights.		Ratio $\frac{L_1}{L_2}$.	Mean.
<i>Benzene.</i>			Alcohol						
1 ...	May 21.	752.8	16.976	8.193	0.907	16.976	7.431	0.438	
2 ...	" 22.	755.7	18.662	8.950	0.906	18.662	8.1085	0.434	
3 ...	" 23.	757.9	19.010	9.150	0.901	19.010	8.244	0.434	
4 ...	" 24.	756.7	17.098	8.286	0.905	17.098	7.499	0.438	0.436
<i>Toluene.</i>									
5 ...	May 28.	...	17.359	7.136	0.972	17.359	6.936	0.400	
6 ...	" 29.	763.2	17.046	7.040	0.972	17.046	6.843	0.401	0.401
<i>Metaxylene.</i>									
7 ...	May 30.	755.7	16.309	6.063	1.031	16.309	6.251	0.386	
8 ...	June 6.	767.6	16.755	6.150	1.036	16.755	6.3715	0.380	0.383
<i>Acetic Acid.</i>									
9 ...	June 11.	758.6	19.517	8.956	0.980	19.517	8.777	0.450	
10 ...	" 12.	762.2	17.033	7.754	0.984	17.033	7.630	0.448	
11 ...	" 19.	752.4	19.330	8.613	0.984	19.330	8.475	0.447	0.448

Table V. (*continued*).

No.	Date.	B.	Weights evaporated.		Ratio $\frac{V_1}{V_2}$	Corrected weights.		Ratio $\frac{L_1}{L_2}$	Mean.
<i>Methyl Formate.</i>									
12 ...	March 25.	740.8	15.896	10.1455	0.800	15.896	8.1165	0.511	
13 ...	" 26.	742.3	14.5725	9.289	0.795	14.5725	7.385	0.507	
14 ...	" 27.	736.8	10.1955	6.5265	0.794	10.1955	5.181	0.508	0.509
<i>Ethyl Formate.</i>									
15 ...	April 3.	758.8	17.208	8.883	0.843	17.208	7.4885	0.435	
16 ...	" 3.	758.8	17.8375	9.2205	0.846	17.8375	7.8105	0.437	
17 ...	" 5.	754.3	12.9885	6.712	0.844	12.9885	5.665	0.437	0.436
<i>Methyl Acetate.</i>									
18 ...	March 27.	735.7	17.158	9.0365	0.851	17.158	7.690	0.448	
19 ...	" 28.	731.6	18.1105	9.547	0.853	18.1105	8.1435	0.449	
20 ...	" 28.	731.6	16.220	8.5165	0.853	16.220	7.2645	0.448	0.448
<i>Propyl Formate.</i>									
21 ...	Dec. 20.	...	16.896	7.7695	0.916	16.896	7.117	0.421	
22 ...	Feb. 6.	754.5	17.6385	8.108	0.908	17.6385	7.362	0.417	
23 ...	" 20.	768.1	17.675	8.2215	0.910	17.675	7.4815	0.423	0.420
<i>Ethyl Acetate (referred to Benzene).</i>									
24 ...	Oct. 24.	748.5	16.137	15.451	1.121	16.137	17.321	0.932	
25 ...	" 25.	748.2	15.001	14.275	1.129	15.001	16.110	0.931	
26 ...	" 26.	748.9	15.279	14.449	1.131	15.279	16.342	0.935	0.933
<i>Methyl Propionate.</i>									
27 ...	March 19.	760.5	17.6085	8.012	0.903	17.6085	7.235	0.411	
28 ...	" 22.	758.8	16.204	7.369	0.904	16.204	6.6615	0.411	
29 ...	" 23.	758.0	16.158	7.3375	0.903	16.158	6.626	0.410	0.411
<i>Propyl Acetate.</i>									
30 ...	March 18.	764.9	17.8735	7.206	0.951	17.8735	6.855	0.384	
31 ...	" 28.	731.6	17.508	7.0885	0.950	17.508	6.734	0.385	
32 ...	" 29.	736.8	14.545	5.9035	0.948	14.545	5.5965	0.385	0.385
<i>Ethyl Propionate.</i>									
33 ...	April 4.	762.9	18.5595	7.3745	0.942	18.5595	6.947	0.374	
34 ...	" 4.	762.9	14.111	5.676	0.943	14.111	5.3625	0.380	
35 ...	" 5.	754.3	16.2815	6.5735	0.944	16.2815	6.2055	0.381	0.378
<i>Methyl Butyrate.</i>									
36 ...	March 29.	736.8	14.836	5.738	0.952	14.836	5.4625	0.368	
37 ...	April 2.	755.7	15.207	5.8455	0.950	15.207	5.553	0.365	
38 ...	" 2.	755.7	15.064	5.853	0.951	15.064	5.466	0.372	0.368
<i>Methyl Isobutyrate.</i>									
39 ...	April 8.	759.9	17.0765	6.4145	0.926	17.0765	5.940	0.348	
40 ...	" 8.	759.9	15.7095	5.814	0.926	15.7095	5.384	0.343	0.346

TABLE VI.

	A.	B.	C.	D.
Benzene	1.000	94.4		109.0 Regnault II. 93.4 Schiff X. 92.9 Wirtz XI. 107.6 Jahn XII. 83.6 Schiff X. 78.3 Schiff X.
Toluene	0.920	86.8		
Metaxylene	0.877	82.8		
Alcohol	2.293	216.5		202.4 Andrews I. 227 Brix VI. 208 Despretz VII. 208.9 Favre & Silbermann VIII. 206.4 Schall IX. 205.0 Wirtz XI. 201.4 Longuinine XIII.
Acetic Acid	1.028	97.0	92.7	120.8 Berthelot IV. 84.9 Berthelot & Ogier V. 101.9 Favre & Silbermann VIII.
Methyl Formate ...	1.167	110.1	119.1	117.1 Andrews I. 115.2 Berthelot III.
Ethyl Formate ...	1.000	94.4	93.4	105.3 Andrews I. 100.4 Berthelot III. 92.2 Schiff X. 113.25 Jahn XII.
Methyl Acetate ...	1.028	97.0	96.8	110.2 Andrews I. 94.0 Schiff X. 113.86 Jahn XII.
Propyl Formate ...	0.956	90.2	87.3	85.3 Schiff X.
Ethyl Acetate	0.933	88.1	84.3	92.7 Andrews I. 83.1 Schiff X. 84.3 Wirtz XI. 102.14 Jahn XII.
Methyl Propionate	0.943	89.0	87.7	84.2 Schiff X.
Propyl Acetate ...	0.881	83.2	79.8	77.3 "
Ethyl Propionate..	0.867	81.8	79.2	77.1 "
Methyl Butyrate...	0.844	79.7	72.5	77.3 "
Methyl Isobutyrate	0.794	75.0	76.3	75.5 "

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TABLE VII.

	Ratio to benzene.	L.	t.	M.*	$\frac{ML}{T}$
Benzene	1.000	94.4	80° 2	77.40	20.65
Toluene	0.920	86.8	110.8	91.30	20.61
Metaxylene	0.877	82.8	138.5	105.20	21.03
Water		536.6	100°	17.86	25.64
Alcohol	2.293	216.5	78.2	45.66	28.09
Acetic Acid	1.028	97.0	118.5	59.52	14.72
Methyl Formate	1.167	110.1	31.8	59.52	21.45
Ethyl Formate	1.000	94.4	54.3	73.42	21.13
Methyl Acetate	1.028	97.0	57.1	73.42	21.53
Propyl Formate	0.956	90.2	80.9	87.32	22.38
Ethyl Acetate	0.933	88.1	77.15	87.32	21.93
Methyl Propionate ...	0.943	89.0	79.7	87.32	21.99
Propyl Acetate	0.881	83.2	101.25	101.22	22.45
Ethyl Propionate	0.867	81.8	99.2	101.22	22.22
Methyl Butyrate	0.44	79.7	102.7	101.22	21.43
Methyl Isobutyrate ...	0.794	75.0	92.3	101.22	20.74

* The molecular weights are calculated by taking

$$\begin{cases} H=1. \\ O=15.86 \text{ (Rayleigh \& Scott).} \\ C=11.90 \text{ (Stas:—C=12.005 when O=16).} \end{cases}$$

The absolute zero is taken as $-273^{\circ}7$.

10. The results of these experiments may, we think, be regarded as satisfactory so far as they go.

A good many difficulties have been encountered, but most of them have been overcome; and with a few more improvements in matters of detail we believe that the method will become almost as simple in practice as in theory, and will give results as good as can be required for ordinary purposes.

We do not profess or desire to aim at such accuracy as Mr. Griffiths has reached in his work on water; our method does not and cannot lend itself to such refinements.

We rather hope to obtain for a large number of liquids values that are certainly within one per cent. of the truth, and often much nearer.

Up to the present our experiments have mostly been limited to Dr. Young's esters, which were ready to hand; but we propose to extend them now, as far as possible, to any other liquids that we can obtain pure, and whose boiling-points fall between 30° and 150° .

We desire, in conclusion, to express our thanks to Prof. Carey Foster for the use of his laboratory in the later part of the work, and to Mr. A. W. Porter for his kind assistance and advice in the matter of the resistance measurements.

Appendix.

I. Calculation of L from the thermodynamic equation; worked out for the case of *Methyl Acetate* from experimental data given by Prof. Young (J. C. S. lxiii. p. 1191):—

$$L = (S' - S) \cdot \frac{dp}{dt} \cdot \frac{T}{J}.$$

(i.) To find $\frac{dp}{dt}$, by means of Biot's formula

$$\log p = a + b\alpha^t.$$

t .	P.
50°	588.2 millim.
60°	837.5 „
70°	1167.0 „

hence $\log p = 5.2852200 - 2.515695 \times (.9937262)^{t-50}$
 at 57°·0 . . . $p = 755.0043$ millim.
 at 57°·2 . . . $p = 760.2895$ „

$$\therefore \frac{dp}{dt} = 26.4260 \text{ millim. per } 1^\circ \text{ at } 57^\circ 1.$$

$$\frac{dp}{dt} = 2.64260 \times 13.596 \times 980.94 \text{ dynes per sq. cm. per } 1^\circ.$$

(ii.) S' = vol. of 1 gr. vapour at 57°·1 = 350.6 c.c.

S = vol. of 1 gr. liquid at 57°·1 = 1.1 c.c.

$$\therefore (S' - S) = 349.5 \text{ c.c.}$$

$$T = 273.7 + 57.1 = 330.8.$$

$$*J = 4.199 \times 10^7.$$

Hence $L = 96.81$ thermal units.

* The value of J given here is that determined by Griffiths (Phil. Trans. vol. clxxxiv. 1893 A), and used in working out the experiments on benzene; even if not correct, it is still the right value to use here, because it was determined by means of the same standards as those by which the quantity of heat developed in the benzene experiments was determined; so that any errors would eliminate.

This assumption as to the value of J is the only one that enters into the calculation; all the other data were obtained by direct experiment.

II. *Regarding the Purity of the Liquids employed.*—The benzene was bought from Messrs. Kahlbaum, of Berlin, and guaranteed free from thiophene. It was distilled twice from phosphoric anhydride until its boiling-point remained constant.

The toluene had been used in some of Prof. Ramsay's previous work; it was distilled once and showed a constant boiling-point.

The metaxylene was fractionated three times, and the portion distilling between 138.1 and 138.7 was used; it was probably not quite pure.

The alcohol was obtained from Messrs. Baird and Tatlock, and distilled from quicklime, and finally with a little sodium, until its boiling-point remained constant.

The acetic acid was bought from Messrs. Kahlbaum; it was fractionated by freezing, and the portion melting at 16.2 was separated and used.

The esters were prepared by Prof. S. Young, F.R.S., and all details may be found in his paper (J. C. S. lxiii. p. 1191).

DISCUSSION.

Prof. RAMSAY drew special attention to the table giving the values of the quotient $\frac{ML}{T}$, where M is the molecular weight, T the absolute temperature, and L the latent heat. Very curious differences are noticeable in the case of water, alcohol, and acetic acid.

Prof. CAREY FOSTER expressed his admiration for the method, since it obviated the necessity for knowing the specific heat of the liquid or vapour.

Prof. SILVANUS THOMPSON said the difficulty experienced in the case of water, due to electrolysis, might be obviated by the employment of a spiral of lower resistance and a larger current, so that the difference of potential between the ends of the spiral should be less than 1.7 volts.

The CHAIRMAN said Capt. Abney had asked him to enquire to what extent the temperature of the liquid was affected by radiation.

Mr. J. W. RODGER asked if any direct experiment had

been made to determine if the temperature of the liquid was not above its true boiling-point. In some cases differences of as much as 2° might exist between the temperature of the liquid and that of the vapour given off. The differences in the value of $\frac{ML}{T}$ in the case of water, alcohol, and acetic acid might be due to the fact that the vapours of alcohol and water were simple, while the vapour of acetic acid was complex.

Mr. R. APLEYARD suggested that the differences obtained in the case of water might be due to the presence of dissolved air.

Mr. GRIFFITHS said that the objection to the adoption of Prof. Thompson's suggestion was the fear that, with short wires, an excessive difference in temperature between the wire and the liquid might exist.

Mr. RHODES asked if Mr. Griffiths could trust his determinations of temperature to $\frac{1}{80000}$ th of a degree?

Mr. GRIFFITHS, in reply, said that he thought there was no limit to the accuracy with which a difference of temperature could be measured; the absolute temperature, however, he only relied upon to $\frac{1}{1000}$ th of a degree.

Prof. RAMSAY said the fact of superheating existing would not affect the results, since near the temperatures at which they were working the latent heat did not vary appreciably with the temperature. In reply to Captain Abney, he said some previous experiments by Dr. Young and himself had shown that the vapour-jacket was quite impervious to radiant heat from without.

V. A "Direct-reading" Platinum Thermometer.

By ROLLO APPELYARD*.

MESSRS. CALLENDAR and GRIFFITHS and their colleagues at Cambridge have proved that the platinum thermometer is capable of measuring temperature with an accuracy of *one thousandth* of a Centigrade degree. For general purposes, temperature is never required to be measured with a precision greater than *one tenth* of a degree. It is my object to consider how the reserve of precision may be utilized in facilitating the processes of testing. The results seem to indicate that it is possible, with a platinum thermometer, to determine ordinary temperatures by a single operation, of such a kind that the readings are "direct"—that is to say, no calculation requires to be applied to them.

In the course of some experiments upon the variation of the electrical resistance of dielectrics with temperature, an apparatus was needed which, by a quick and simple operation, would measure the temperature within the substance of the dielectric. As dielectrics, generally, are bad conductors of heat, it is desirable to take simultaneous readings of temperature at many points within their mass. A platinum thermometer may be regarded as having an *extensible bulb*, for the wire may be distributed to various points within the dielectric; the temperature deduced from the platinum thermometer, at any instant, is therefore equivalent to the mean of the simultaneous readings of a large number of distributed mercurial thermometers. Further, the dielectrics, in their protecting covers, are submerged in a tank of water; the stems of ordinary mercurial thermometers placed, vertically, beneath a water-surface would be difficult to observe; the platinum instrument is far more convenient for this work, for it has the advantage of a *portable stem and scale*.

The resistance of a dielectric † is by no means such a

* Read November 22, 1895.

† See "Dielectrics," Proc. Physical Soc. xiii. p. 155, 1895; Phil. Mag. 1894, xxxviii. p. 396.

definite quantity as the resistance of a metallic conductor. The difference in quality between samples of the same name, the age of the substance, the nature of the contact between the dielectric and its electrodes, and the so-called "electrification," are such variable factors that the term "dielectric resistance" has only an empirical significance. To arrange for the measurement of temperature with extreme precision is therefore unnecessary.

Range.

For certain practical reasons, the temperatures mentioned in this paper are referred to the Fahrenheit scale. The required range was 32°-120° F.; and, within the present limits of accuracy, it is safe to assume that the resistance, R_t , of the platinum coils, at any temperature t , within this range, is represented by the expression

$$R_t = R_{32}[1 + \alpha(t - 32)]. \quad (1)$$

The Platinum Coils.

There were six platinum coils, forming six thermometers, each having a resistance of about 7 ohms. Each consisted of about 2 metres of pure platinum wire, of 8 mils diameter, wound in double spirals upon an ebonite rod. Thick copper leads were soldered to the ends of the coils. To determine α and R_{32} the six coils were separately tested in melting ice, and then at various temperatures in heated water, side by side with a mercurial thermometer. These measurements were made with the coils submerged in a large tank, the temperatures being controlled by a gas-regulator. The resistances were measured on an ordinary "dial" bridge having german-silver coils. This part of the work occupied much time: α was derived from the mean of the best of 30 determinations; it was somewhat tedious, but the values R_{32} and α proved, in the result, very accurate. The six coils of the six thermometers were afterwards connected in series, and regarded as one thermometer. The mean values were:—

$$R_{32} = 40.05 \text{ ohms (six coils, in series).}$$

$$\alpha = .002097.$$

The six platinum coils, still connected in series, were then

imbedded within their respective dielectrics. *Hollow metal tubes*, for mercurial thermometers, were inserted at the same time within the dielectrics, for check-readings. The dielectrics were then immersed together into one tank of water.

Measurement of R_t by "Dial" bridge.

The apparatus was now complete, and tentative tests were made for R_t at different temperatures. The results, when compared with the mean of the readings of the mercurial thermometers in the tubes, were rather discouraging. The temperatures could not be depended upon within about 2° Fahr. These errors were due either to the german-silver coils, or the plugs, of the dial-bridge. Slightly better results were obtained by reducing the battery to a single cell. It was impossible to determine the bridge-temperature, at any time, with accuracy; and, moreover, even if that temperature could have been found *, the temperature-coefficient for different samples of german-silver varies so widely with different samples, that no dependence whatever could have been placed upon values "corrected" by text-book coefficients. The "dial" bridge was therefore abandoned, and a slide-bridge of special form was designed for the purpose.

Measurement of R_t by Slide-bridge.

The slide-bridge, fig. 1, consists of a 12-ft. plank, P, with a groove G cut along its whole length. A metre-scale, S, can slide in the groove, flush with the upper surface of the plank. Datum-marks are made at successive metres along the plank; subdivisions being read upon the sliding metre-scale. The contact-piece, L, is attached to the metre-scale at its zero-point. This saves the cost of an elaborate graduation of the plank; and keeps the sliding-contact very accurately in position. When necessary, the metre-scale with its contact-piece is easily transferred to other such slide-bridges. There is no distortion of the slide-wire, and indentation is

* A platinum coil interwound with one of the resistance-bobbins would probably be the best method under these circumstances.

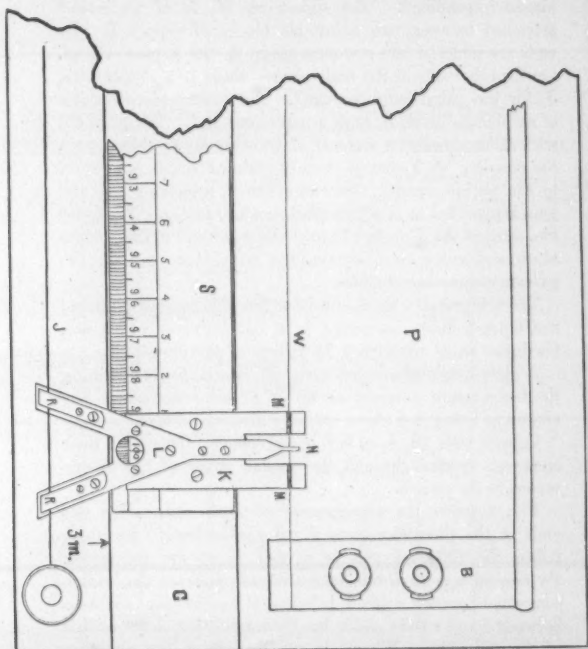
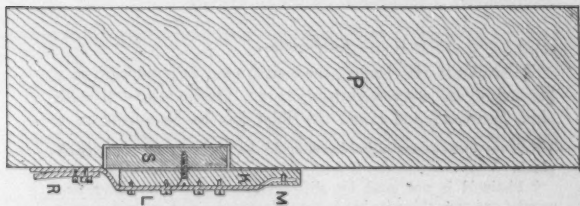


Fig. 1.



almost impossible*. The slide-wire, *W*, is of platinoid † stretched between two adjustable blocks of copper, *B*. Its ends are soldered into saw-cuts made in the copper blocks. On the other side of the scale-groove there is a copper wire, *J*, for the galvanometer-contact. The contact-piece consists of an ebonite block *K*, with a gap at one end. The platinoid wire slides through a saw-cut *M* transversely to this gap in the ebonite. A Y-shaped piece of spring-brass, *L*, is screwed to the ebonite block. One end of the *Y*, projecting over the gap, is provided at *N* with a platinum knife-edge. The other two arms of the *Y* project beyond the other end of the ebonite block, and carry clips, *R*, through which the wire for the galvanometer-contact slides.

The bridge-coils, fig. 2, consist of five silk-covered platinoid resistance-bobbins, immersed in a double vessel of copper, the inner vessel containing $1\frac{1}{2}$ gallons of paraffin-oil.

A glow-lamp, submerged in the oil, is used for maintaining the temperature constant at 80° F.; this temperature was chosen as being just above ordinary air-temperature.

Copper rods (*a*, *b*, *c*, *e*, *f*, *g*, *k*) connect the coils; their ends pass upward, through the wooden cover of the calorimeter, to the switch.

Fig. 2 shows the arrangement of these coils. The two ends of the slide-wire go to *f* and *g* respectively; to which points the 100-ohm coils (*f*, *e*) and (*g*, *a*) are connected. By switching *p* to *l*, the fixed arm (45 ohms) of the bridge, connected between *a* and *b*, is balanced against the 45 ohms between *b* and *e*; the slider has then a position at the middle of the slide-wire. When *p* is switched to *n*, the platinum thermometer is substituted for the (*b*, *e*) coil.

Calibration.

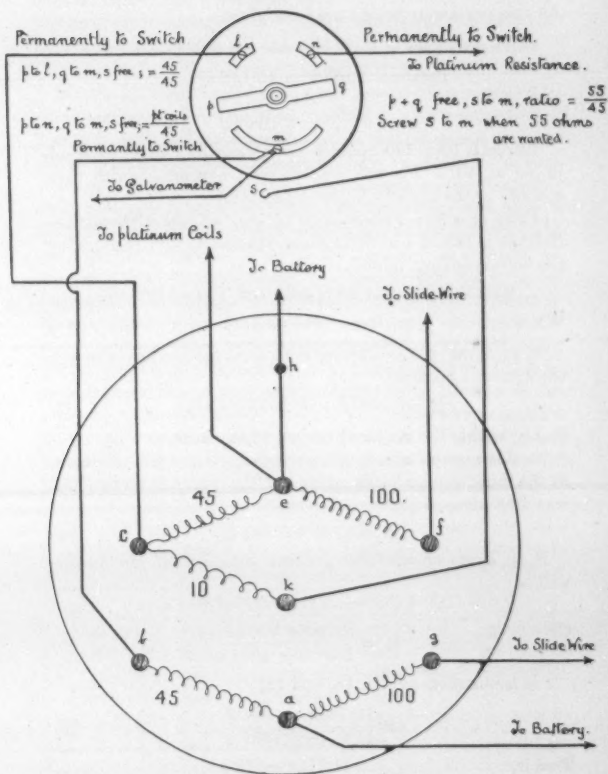
To calibrate the bridge, the switch (fig. 2) is set with *p*, *q* horizontal, and *s* screwed firmly to *m*. This puts $45 + 10 = 55$ ohms between *b* and *e*, and the slider has to be moved to a

* A very convenient optical bench could be made by such a grooved plank.

† Platinoid is not good for slide-bridge wires. After about two months' exposure this wire has become blackened, and I am replacing it by a wire of platinum-silver.

Fig. 2.

SWITCH.



new position along the plank. A simple calculation enables us to find an expression for the temperature, t , of the platinum coils in terms of n , the distance in cms. of the sliding-contact from the extreme left of the plank-scale. The actual figures for this particular bridge are here given.

Corrected values of the bridge-coils, at 80° F.

Between a and $b = 45.138$ ohms.

„ c „ $e = 45.150$ „

„ k „ $c = 10.032$ „

The coils 100, 100 require no correction; they were adjusted at 64°·5 F., and we are only concerned with their equality at the slightly higher temperature.

Value of r , the resistance of 1 cm. of the slide-wire.—Balancing $(45.15 + 10.032)$ ohms against 45.138 and noting the corresponding value* of n , we have

$$45.138[100 + 241.95 r] = 55.182[100 + 58.05 r].$$

Whence $r = .1301$ ohm. (2)

R_t in terms of t .—Putting the values for R_{32} and α in (1), the equation becomes

$$R_t = 40.05[1 + .002097(t - 32)], \quad . . . \quad (3)$$

that is, within the required range of temperature.

To this may be added, without sensible error, the resistance of the thick copper leads going to the platinum coils; this was .025 ohm, so that

$$R_t = 37.39 + 0.084 t. \quad . . . \quad (4)$$

R_t in terms of n .—The general equation of the bridge will be

$$R_t[100 + (300 - n)r] = 45.138(100 + nr).$$

Or

$$R_t = \frac{4513.8 + 5.8745 n}{139.04 - .1301 n}. \quad . . . \quad (5)$$

t in terms of n .—From (3) and (5),

$$t = \frac{10.7406 n - 684.9}{11.677 - 0.0109 n}. \quad . . . \quad (6)$$

That is,

$$n = \frac{11.677 t + 684.9}{10.7406 + 0.01093 t}. \quad . . . \quad (7)$$

* The full length of the slide-wire is $n = 300$ cms.

From (7) the plank may be calibrated "directly," in temperature degrees. It will, however, be more convenient to plot a curve co-ordinating n and t . Such a curve does not greatly differ from a straight line; it is therefore sufficiently accurate to calculate n for every successive five degrees within the range.

Accuracy attained.

Mr. E. H. Griffiths has been good enough to examine my figures, and he agrees that, if the original standardization of the platinum coils is carefully attended to, this apparatus may be relied upon to measure temperature with an accuracy of one tenth of a Fahrenheit degree. Unfortunately, the platinum coils were sealed hard and fast within their respective dielectrics before this slide-bridge method was adopted; so that it has not been possible to check the readings in a liquid, as against a mercurial thermometer. This is very much to be regretted. By taking the mean of the readings of mercurial thermometers in the tank, and the readings of the mercurial thermometers in the tubes within the dielectrics, and comparing this mean with the value of t derived from (6), it is possible to make a very rough comparison. It should be carefully noticed, however, that "tube" temperatures include all the errors which it is the very object of the present method to avoid. The following table must be regarded with that limitation.

TABLE I.

Date.	Tube-temperature.	n .	t , from (6).	Air-temperature.
Aug. 22	76.0 F.	135.8	75.9 F.	81.0 F.
" 23	72.5	132.9	72.6	77
" 28	68.3	128.9	68.1	73
" 29	80.5	139.75	80.4	76
" 30	90.6	147.7	89.6	76
Sept. 2	101.0	156.64	100.1	78
" 3	110.1	164.8	109.9	79

It may be remarked here that, with the same apparatus, by taking the mean of the two values of n , corresponding to

reversal of the battery, a second approximation could have been made. Also the slide-wire is thinner than is required by the temperature range. A thicker wire would have given greater possibilities of accuracy, and it would have been less liable to mechanical injury and temperature fluctuations.

The Action of Sulphur Vapour upon Copper.

While standardizing platinum thermometers in the vapour of sulphur (444° C.) some mica plates, which formed part of the apparatus within the vapour, were bound together with copper wires which passed through holes in the mica. After five hours' exposure to the vapour the apparatus was dismantled, and, as had been expected, the copper wire was completely demetallized. An examination of the resulting wire, a brittle compound of cuprous or cupric sulphide, showed that it was, in some cases, perforated by a small axial hole; the wire had, in fact, become a tube of very small bore.

I am able to show this to you by projecting upon the screen one of the pieces of mica through which the original wire was threaded; the ends of the sulphide wire were broken off flush with each face of the mica, thus forming a section of the sulphide wire.

Prof. Ramsay tells me that, as a rule, it is possible to draw an inner metallic core* from a copper wire which has been treated with sulphur vapour. It therefore seemed probable that my specimen was a piece of sulphide wire out of which a core had fallen. This, however, does not appear to be the explanation, for I have since succeeded in threading an iron wire, of 8 mils diameter, completely through a sulphide wire, which was more than half an inch long and not quite straight. It is impossible that a core of 8 mils could have fallen out of this specimen.

An examination, with the microscope, of the original copper wire does not reveal the slightest perforation; although the end was carefully removed with nitric acid so as to avoid the closing of the hole, if one existed, by any mechanical cutting of the wire.

* I have obtained lately several specimens which show this core.

It was suggested that an occasional hole might occur in the process of drawing the wire. Prof. Boys asked me to try a piece of copper cut from a block. I therefore had a piece, of square section, sawn from a large commutator-bar. After six hours' boiling in sulphur vapour, this copper rod became circular in section; it had a central perforation, as could be proved by passing a wire into it. The colour of the surface was a deep "beetle" blue.

In order to show the amount of expansion which the copper experiences during the change to sulphide, a second piece of copper, of square section, was sawn out of the commutator-bar, half the length being turned down to a circular section. This was put into sulphur vapour for six hours, with the result that the diameter of what had been the square part had increased from 215 to 386 mils; the circular part had increased from 120 to 232 mils; and a third piece, of ordinary drawn copper wire, increased from 31 to 77 mils. During the transformation from copper to sulphide the diameter in all cases is approximately doubled. The length is only slightly increased; all sharp edges disappear; and there is a general tendency towards the circular section. The fracture is crystalline, resembling that of an aerolite.

A piece of Delta metal submitted to the same test came out of the sulphur vapour without appreciable change. It was discoloured but otherwise unaltered, and I propose to adopt this metal in future for the electrodes of platinum thermometers.

Summary.

The foregoing results indicate that platinum-coil thermometry may be reduced to a simple operation, and that the simplified method is sufficiently precise for general work. The exact conditions which determine the numerical limits of accuracy, generally, cannot be very concisely stated. The possible causes of error are well known, and with a little care may be nearly eliminated. The first precaution is to maintain the bridge-coils at some constant temperature just above the maximum air-temperature. For this purpose the oil-bath and glow-lamp will be found satisfactory.

If the temperature of the slide-wire is likely to vary

considerably, r may be redetermined; or a correction-factor may be applied. This will seldom be necessary; no such correction was used in the above tests.

There must be some easy arrangement for calibrating the slide-wire. The auxiliary-coil test is expeditious. The wire now supplied by good makers is sufficiently uniform; and, except where great precision is essential, it will not require to be corrected for differences of diameter; it should be of platinum-silver.

The resistance of the leads can generally be added as in equation (4). In other cases one of the "dummy-lead" methods of Siemens or Callendar must be adopted.

In conclusion I have to thank Mr. E. H. Griffiths, Mr. W. A. Price, and Prof. W. N. Stocker for their help in preparing this paper; and the Silvertown Telegraph Company, who have kindly allowed me to carry out the experiments and publish the results.

DISCUSSION.

Mr. TROTTER said he agreed with the author that the "reserve of precision" at our disposal, on account of the delicacy of some of our modern instruments, ought to be made use of to facilitate the rapid performance of many measurements where the utmost accuracy is not necessary. He had the impression that platinum silver was not now considered the best material for use as the bridge-wire.

Mr. H. F. BURSTALL explained the differences between temperature as measured on the mercury, air, and platinum thermometers. At a temperature of about 40° the platinum thermometer read about $0^{\circ}4$, and the mercury thermometer about $0^{\circ}1$ below the air thermometer. Prof. Callendar had obtained measurements of temperature correct to within $0^{\circ}1$ by using a Weston voltmeter and an ordinary Wheatstone bridge; the variations of resistance, and hence the temperature, being read directly from the deflexions on the voltmeter.

Mr. RHODES thought that, except where extreme accuracy was necessary, the mercury thermometer was very much more convenient than the platinum thermometer.

Mr. BURSTALL said the great convenience of the platinum thermometer lay in the fact that the scale could be read at a distance of many yards from the point where the temperature was being measured, and hence could be used in many places where it would be impossible to read a mercury thermometer.

Mr. BLAKESLEY considered that the author was somewhat bold when he stated that for general purposes it was never necessary to measure temperature to nearer than one-tenth of a degree.

VI. *On the Existence of Vertical Earth-Air Electric Currents in the United Kingdom.* By A. W. RÜCKER, M.A., F.R.S.*

IN a paper by Dr. Adolph Schmidt, read before Section A of the British Association at Oxford (Report Brit. Assoc. 1894, p. 570), the author stated that he had expanded the components of the earth's magnetic force in series, and had deduced expressions, two of which give the magnetic potential on the surface of the earth in so far as it depends on (1) internal, and (2) external forces. "The third series represents that part of the magnetic forces which cannot be expressed in terms of a potential, but must be due to electric currents traversing the earth's surface." The author concludes that such currents amount on the average to about 0.1 ampere per square kilometre.

It appeared therefore desirable that this conclusion, drawn from the magnetic state of the earth as a whole, should be tested by means of those portions which have been most fully studied.

The test to be applied is, whether the line-integral of the magnetic force taken round a re-entrant circuit on the surface of the earth is or is not a vanishing quantity.

The irregular form of the United Kingdom makes the application of this test more difficult than it would otherwise be; but as two detailed Surveys of Great Britain and Ireland have

* Read December 13, 1895.

been carried out by Dr. Thorpe and myself for the epochs 1886 and 1891 respectively, the data at our disposal are so numerous that I thought it worth while to undertake the inquiry.

The actual work of calculation has been carried out almost entirely by two of my students, Messrs. Kay and Whalley. My best thanks are due to them for the care and skill they have displayed.

The facts on which the investigation is based are as follows.

The first survey (1886) included 205 stations, at all of which observations were made by Dr. Thorpe or myself.

The *true*, and therefore irregular, isomagnetic curves were drawn for the epoch January 1, 1886, and the *terrestrial curves*, from which the local disturbances were eliminated, were also calculated for the same date (Phil Trans. vol. clxxxi. A, 1890).

The second survey included observations at 677 stations. These were made by ourselves, or, under our superintendence, by Messrs. Briscoe, Gray, and Watson. The results are about to be published by the Royal Society. The terrestrial isomagnetic curves were drawn for the epoch Jan. 1, 1891. The secular change having been carefully determined by special observations and methods, the values of the elements and the terrestrial curves obtained for the earlier date were reduced to Jan. 1, 1891. Thus the whole of the 882 stations were available for drawing the true isomagnetics for the latter date. The two sets of terrestrial curves obtained from the second survey and from the first survey reduced to the second epoch did not agree exactly, and the lines bisecting the intervals between them were taken as our final result for the terrestrial curves in 1891.

The following sets of curves will be considered in this paper:—

- (1) The terrestrial isomagnetics obtained in the first survey for Jan. 1, 1886. These will be referred to as *the 1886 curves*.
- (2) The same curves reduced by the secular change to Jan. 1, 1891. These will be called *the first survey 1891 curves*.
- (3) The terrestrial curves for 1891 deduced from the second survey. These will be called *the second survey 1891 curves*.

(4) The mean terrestrial curves for Jan. 1, 1891, deduced from (2) and (3). These will be called *the mean 1891 curves*.

(5) Lastly, the true isomagnetic curves deduced from the results at all the 882 stations for the epoch Jan. 1, 1891. These will be called *the true 1891 curves*.

(1) *The 1886 Curves.*

The advantage of using the calculated terrestrial curves is that they can be carried across the sea from England to Ireland, or extended a few miles from the coast by extrapolation. The area included can therefore be made as large as possible. On the other hand, the method of obtaining these curves is such that the errors in their positions will probably be greatest near the boundaries of the land area over which the survey was carried. In order therefore that such errors might affect different calculations as differently as possible, it was determined to take two circuits, which should have their greatest extensions N. and S., and E. and W. respectively. They will be called the α and β circuits.

The α circuit was bounded by long. 2° W., lat. 58° N., long. 7° W. and lat. 52° N.

The β current was bounded by long. 1° W., lat. 55° N., long. 9° W. and lat. 52° N.

In the published account of the 1886 survey (*loc. cit.* p. 322), the values of the declination (δ) and horizontal force (H) are given for all points within the United Kingdom defined by the intersection of whole degrees of latitude and longitude. From these the northerly components of the force ($H \cos \delta$) were calculated for all such points on the lines of latitude, and the westerly components ($H \sin \delta$) for all such points on the lines of longitude which bounded the circuits.

The method of calculating the line-integral of the force may best be shown by an example, for which we may select long. 2° W. between lat. 52° and 58° .

Let C be the number of cm. in a degree of latitude, and N the northerly component of the force. Assume that $N = N_{52} + a'l' + x$, where a is a constant, $l' = l - 52$, and x is a small variable.

Let W be the work done as the unit pole moves due North from lat. 52° to lat. 58° .

Then

$$W = C \int_{52}^{58} N dl = C \{ N_{52} \times 6 + 18a + \int_0^6 x dl' \}.$$

The value of a was found from the values of the northerly components at the points on latitudes 52° and 58° respectively. The integral was calculated by quadrature, graphic methods being employed.

To give an idea of the relative magnitudes of the terms, I append the following data :—

$$\begin{aligned} 6N_{52} + 18a + \int_0^6 x dl' &= 1.00896 - 0.06864 - 0.00152 \\ &= 0.93880. \end{aligned}$$

The constant C was taken = 11119320 cm., so that the work done in this part of the circuit is 1.04387×10^7 ergs.

Treating the other parts of the circuit in the same way, the four quantities, the algebraical sum of which is the work done in completing the circuit, are :—

$$(1.04388 + 0.17082 - 1.00637 - 0.20902) \times 10^7 = -6.9 \times 10^3 \text{ ergs.}$$

Dividing by 4π , we find that the total current within the circuit is -550 C.G.S. units, and, since the area is 2.13×10^8 square kilometres, this amounts to -0.026 ampere per square kilometre. The negative sign indicates that the current flows downwards.

A similar calculation carried out with respect to the β circuit, of which the greatest extension is East and West, and the area is 1.77×10^8 square kilometres, indicates a current of only -0.004 ampere per square kilometre.

(2) *The First Survey 1891 Curves.*

When the 1886 curves are reduced to the Epoch Jan. 1, 1891, by methods which are fully described in the account of the later survey, the results obtained from the α and β circuits are -0.045 and -0.030 ampere per square kilometre respectively. It would at first sight appear as though the fact that these values are larger than those calculated for

Jan. 1, 1886, might be due to errors introduced by the assumed values of the secular change; but, as will immediately be seen, they are not larger than those obtained by another method, which this cause of error does not affect.

(3) *The Second Survey 1891 Curves.*

Treated in exactly the same way as the last, these give values of about the same magnitude but of opposite signs; viz. for the α circuit $+0.016$, and for the β circuit $+0.020$ ampere per square kilometre. Thus two different methods of calculating the same quantity lead to very different results, which point to the conclusion that the apparent effects of the hypothetical currents are due to small errors in the determination of the exact positions of the lines.

In the final calculation of the results of our survey, we have taken the means of the positions of these two sets of lines as the isomagnetic lines for 1891, hence the mean values of the currents deduced from them by the

(4) *Mean 1891 Curves*

are $+0.001$ and -0.005 ampere per square kilometre for the α and β circuits respectively.

(5) *The True 1891 Curves.*

We have further checked the above results by means of the true curves, taking two circuits—one (γ) in England and Scotland, and the other (δ) in Ireland.

The first of these was as follows:—

Long. 1° E. from lat. 51° to lat. 53° .
 Lat. 53° from long. 1° E. to 1° W.
 Long. 1° W. from lat. 53° to 55° .
 Lat. 55° from long. 1° W. to 3° W.
 Long. 3° W. from lat. 55° to 53° .
 Lat. 53° from long. 3° W. to 4° W.
 Long. 4° W. from lat. 53° to 51° .
 Lat. 51° from long. 4° W. to 1° E.

The area is 1.054×10^6 square kilometres. The values of the horizontal force and declination for every $10'$ of latitude or longitude were read off from the maps on which

the values at the different stations were entered, and the true isomagnetics drawn. This operation was performed by Messrs. Kay and Whalley and checked by myself. The northerly or westerly component of the force was then calculated for each of these points, and the average value for each short section was assumed to be equal to the mean of the values at its initial and final points.

No difficulty arose except at a point in Wales, where the curves are closed, and where it was therefore necessary to assume an average value for a section of the line on which a maximum occurred.

The result of the calculation gave a current of -0.008 ampere per square kilometre.

The second circuit was taken in Ireland. It traversed the district of Antrim, in which there are violent local disturbances, and is interesting chiefly as showing to what extent the result may be affected by such causes.

The circuit was as follows :—

Long. $6^{\circ} 30'$ W. from lat. 52° to lat. 55° .

Lat. 55° from long. $6^{\circ} 30'$ to long. 8° .

Long. 8° from lat. 55° to 54° .

Lat. 54° from long. 8° to 9° .

Long. 9° from lat. 54° to lat. 52° .

Lat. 52° from long. 9° to long. $6^{\circ} 30'$ W.

The area is 48×10^3 square kilometres.

The current is -0.046 ampere per square kilometre.

The fact that these different circuits, including areas of very different magnitudes and situated in different parts of the United Kingdom, all give very small values for the hypothetical currents, is strong evidence that the smallness of the calculated numbers is not due to the fact that large positive and negative values mutually cancel each other. It is, for instance, conceivable that the directions of the current-flow might be opposed on what may be called the oceanic and continental sides of the kingdom.

If this were so, it is probable that circuits γ and δ would have given results of opposite signs. By way of testing the matter further, the current was calculated both from the 1886 and the 1891 lines for the relatively small area in the West

of Ireland bounded by latitudes 52° and 53° , and longitudes 9° and 10° .

The results were :—

for 1886,	-0.04	ampere per square kilometre,
„ 1891,	$+0.11$	„ „ „

Hence the difference of direction which characterized the currents deduced from the two surveys when applied to large areas, also distinguishes, and in an exaggerated degree, the results obtained from a small border district. It is therefore evident that either the distribution of the vertical currents has entirely altered in five years, or they are too small to be detected by the method employed.

The former of these hypotheses is negated by the fact that different calculations, based on the first and second survey 1891 curves respectively, lead to discordant results for the same date, and we are therefore compelled to fall back upon the second alternative.

Effect of the Ellipticity of the Earth.

The question may fairly be raised whether, in dealing with such minute quantities, the ellipticity of the earth ought not to be taken into account.

In answer to this, it may be stated that the work done when the unit pole traverses the α circuit was also calculated, using the data as to the form of the earth given by Captain Clarke and quoted by Prof. Everett ('C.G.S. System of Units,' ed. 1891, p. 71). The numerical values thus obtained differed from those given above, but the differences between them were of the same order. Both methods of calculation lead to opposite conclusions as to the directions of the hypothetical currents according as the 1886 or 1891 curves are used; thus proving that the small outstanding uncertainties as to the magnetic state of the United Kingdom are the cause of the discrepancies, which are not reduced by using a closer approximation to the form of the earth.

Conclusion.

I can only conclude from these various figures that the local magnetic surveys furnish no evidence of vertical elec-

trical currents in the United Kingdom. The largest number obtained from the larger circuits is less than half that which Dr. Schmidt assigns as the mean value for the whole earth. Different calculations lead to results of different signs for the same quantity ; and the data which would *a priori* be accepted as the best give the smallest values.

As far as the terrestrial curves are concerned, the final results for the two surveys are embodied in the 1886 and the mean 1891 curves respectively. The local irregularities in the North of Ireland are so great that the calculations based on the true isomagnetics in that country may be neglected as compared with the probably much better results obtained in Great Britain.

Selecting, then, only the most trustworthy values, the results may be summed up as follows in terms of amperes per square kilometre.

	Circuit.		
	α .	β .	γ .
1886.	-0.026	-0.004	
1891.	+0.001	-0.005	
1891.			-0.008

From these figures we may conclude that there is not in the United Kingdom a vertical current amounting on the average to 0.1 ampere per square kilometre. They are not inconsistent with the existence of a current of about a tenth or a twentieth of that amount ; but on account both of the smallness of the results and of the discrepancy between the values obtained for 1891 by two methods, we cannot assert that such a current actually exists. The calculations taken by themselves do not disprove the hypothesis that electric currents traverse the earth's surface, as we cannot argue from the condition of a small portion of the globe to that of the whole. The most that can be said is that no evidence in favour of the existence of vertical currents can be drawn from one district, which has been very minutely surveyed.

P.S.—No reference to Dr. Schmidt's original paper was given in the short notice published in the Report of the British Association. I had therefore supposed that the latter was a preliminary note. Professor Schuster has, however, recently shown to me Dr. Schmidt's complete investigation, and he has kindly calculated the current-density at latitudes 50° and 55° on the prime meridian from formulæ given by Dr. Schmidt. The result is *upward* currents and 0.20 to 0.15 ampere per square kilometre at lat. 50° and 55° respectively. The mean of these two numbers, viz. 0.175, is nearly equal to Schmidt's mean for the whole earth (0.17). It is opposite in direction to and very much greater in magnitude than any vertical current the existence of which is compatible with the results of the Magnetic Survey.

DISCUSSION.

Mr. WATSON said a few words on the difficulty experienced in determining the line-integral in South Wales, due to the presence of closed curves.

VII. *On Resultant Tones.*

By Professor J. D. EVERETT, F.R.S.*

1. THE received theory of the generation of resultant tones in the ear may be summed up with rough accuracy as follows †:—

The drumskin being pulled inwards by the end of the handle of the "hammer," which is attached to its centre, offers unsymmetrical resistance to displacement in the inward and outward directions, so that the equation for the movement of its centre in free vibration would be

$$-\ddot{x} = \omega^2 x + \alpha x^2,$$

or to a closer approximation

$$-\ddot{x} = \omega^2 x + \alpha x^2 + \beta x^3,$$

ω , α , β being constants.

* Read January 24, 1896.

† See *Tonempfindungen*, Appendix xii.; Rayleigh on Sound, art. 68; Bosanquet, Proc. Phys. Soc. vol. iv. p. 240, arts. 57-69.

The value of ω when the second is the unit of time is less than 60, hence the frequency of free vibration, being $\omega/2\pi$, is less than 10.

When two harmonic forces of frequencies m and n act upon the drumskin, they produce, in addition to their own tones, certain "resultant tones," the one of largest amplitude being the "first difference-tone," of frequency $n-m$. The next largest amplitude is about $\left(\frac{n-m}{n+m}\right)^2$ of this, and belongs to the "first summation-tone," of frequency $n+m$. Neither of these tones will be audible unless the excursion x is so large that ax^2 is sensible in comparison with $\omega^2 x$. There will also be difference-tones of frequencies $2m-n$ and $2n-m$, but neither of these will be audible unless βx^3 is sensible compared with $\omega^2 x$.

2. This theory does not appear sufficient to account for the loud resultant tones which are sometimes heard. When a Helmholtz siren is driven rapidly, with the rows of holes 9, 12, 15, and 18 open, the resultant tone of frequency 3 on the same scale is the most prominent tone in the whole volume of sound.

3. The view which I desire to put forward is closely connected with the well-known theorem of Fourier, that every periodic variation can be resolved in one definite way into harmonic constituents, whose periods must be included in the list 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c., where 1 denotes the period of the given variation itself. The corresponding frequencies will be as 1, 2, 3, 4, &c.

In the majority of cases, when this analysis is carried out, the fundamental constituent, represented by 1 in the above lists, is the largest or among the largest; but in the case of a variation compounded of two simple tones with frequencies in the ratio of two integers, neither being a multiple of the other, the fundamental will be absent, and the Fourier series will consist of only two terms, which in the language of acoustics are harmonics of the fundamental.

4. Clearness of thought is facilitated in these matters by supposing a curve to be drawn, in which horizontal distance represents time and vertical distance represents the quantity whose variation is in question. Since the variation is periodic,

the curve will consist of repetitions of one and the same form, in other words it will consist of a number of equal and similar waves, and the wave-length stands for the complete period of the variation.

The point on which I wish to insist is, that if such a curve representing the superposition of two harmonics of the fundamental is in the first instance very accurately drawn, and is then inaccurately copied in such a way that all successive waves are treated alike, the inaccuracy is morally certain to introduce the fundamental.

Let y denote any ordinate, and θ the time (or abscissa) expressed in such a unit that 2π is the numerical value of the wave-length or period; then the amplitude of the fundamental is the square root of $A^2 + B^2$, where

$$A = \frac{1}{\pi} \int_0^{2\pi} y \cos \theta d\theta, \quad B = \frac{1}{\pi} \int_0^{2\pi} y \sin \theta d\theta.$$

In the original curve both A and B vanish. Let y' be the altered value of y in the new and inaccurate curve, and let z denote $y' - y$; then we have, between the above limits,

$$\int y' \cos \theta d\theta = \int y \cos \theta d\theta + \int z \cos \theta d\theta = \int z \cos \theta d\theta,$$

since $\int y \cos \theta d\theta$ is zero.

But z may be regarded as a random magnitude, hence it is infinitely improbable that its different values exactly fulfil the condition $\int z \cos \theta d\theta = 0$. Therefore the new A is finite; and similar reasoning shows that the new B is finite.

If all the ordinates were changed in one uniform ratio, A and B would remain zero, and no new constituent would be introduced; but any other change, unless specially planned to avoid introducing A and B , is practically certain to give $A^2 + B^2$ a finite value.

5. I maintain that such a change is effected in the form of sonorous waves during their transmission from the external air to the sensory fibres by which we distinguish pitch. The waves are transmitted first from the air to the drumskin, then through two successive levers, the hammer and anvil, to the head of the stirrup, while the foot of the stirrup sits upon the membrane of the oval window, and passes on the vibrations

through the membrane to the liquid on the other side in which the sensory fibres are immersed. The levers turn upon ligamentous fulcra, and have rubbing contact with each other. The wave-form cannot run the gauntlet of all these transmissions without being to some extent knocked out of shape. It is much as if a very accurately drawn curve, representing the original wave-form, were copied and recopied, five times in succession, by five different pantagraphs not very firm in their connexions. The final copy so obtained would be sure to exhibit sensible departures from the original.

6. It appears likely that the chief seat of the disturbing actions in the ear is the junction of the hammer and anvil. "When the drumskin with the hammer is driven outwards, the anvil is not obliged to follow it. The interlocking teeth of the surfaces of the joint then separate, and the surfaces glide over each other with very little friction"*. Such action is likely to introduce derangement, increasing generally with the excursions of the drumskin, but not expressible as a definite function of the ordinates of the wave-curve. For a given pressure on the drumskin, the pressure communicated to the liquid in the cochlea will vary according to the relative position and relative motion of the two portions of this joint.

7. The principal resultant tone due to these actions is likely to be that which corresponds to the complete period of the actions, in other words the highest common fundamental of the two primaries, or what old writers called the "grave harmonic." This will not be the same as the "first difference-tone" unless the ratio of the two primaries is of the form $m : m+1$; and I have satisfied myself, both by my own trials and by a study of Kœnig's experimental results, that when the difference-tone and the common fundamental are not identical, the common fundamental is usually the predominant, and often the only audible resultant tone. (See Appendix.)

8. The common fundamental is, however, not the only resultant tone that can be thus accounted for. Similar reasoning to that employed in reference to A and B suffices to explain the introduction of any or all of the harmonics of

* Ellis's 'Helmholtz,' p. 133, 2nd edition.

the fundamental ; but it is to be expected, from the analogy of ordinary experience in harmonic analysis, that the successive constituents will usually be smaller and smaller as we advance in the series. The octave is likely to be the largest of them ; and Kœnig found, in several experiments with primaries in the ratio of 3 : 5, that both the fundamental 1 and its octave 2 were distinctly heard as resultant tones.

9. The following investigation bears on the relation between beats and resultant tones. The expression

$$a \cos m\theta + b \cos n\theta$$

can be reduced to the form

$$A \cos \left(\frac{m+n}{2} \theta - \epsilon \right),$$

where A and ϵ are given by

$$A^2 = a^2 + b^2 + 2ab \cos (n-m)\theta,$$

$$\tan \epsilon = \frac{a-b}{a+b} \tan \frac{n-m}{2} \theta,$$

and the beating together of two tones not differing much in pitch is explained by the fact, definitely expressed in these formulæ, that the whole effect may be regarded as a succession of waves with gradually varying amplitude. The frequency of the beats is the frequency of the maxima of A^2 , and is the difference of m and n .

We have ascribed resultant tones to alterations made in the wave-form by the action of the ear, such alterations being in general largest at those points at which the excursions of the drumskin are largest. These excursions are measured by $\pm A$, and the above investigation shows that their maxima have a frequency corresponding to the difference-tone. This is true whether m and n are commensurable or incommensurable. If they are commensurable, their greatest common measure will be the frequency of the complete cycle of change. This cycle will not be conspicuous in the curve if the ratio of $n-m$ to $\frac{1}{2}(n+m)$ is very small, but will assert itself more and more as this ratio increases ; and these remarks will apply to the comparison of the fundamental with the first difference-tone.

10. If the ear is able so to alter the form of waves impinging upon it as to generate resultant tones, it is natural to seek for some instance of a similar action in external bodies. A violin is very susceptible, like the ear, to vibrations of all frequencies between wide limits, and the sound-post serves, like the ossicles of the ear, to transmit vibrations from one portion to another. It is easy to produce resultant tones by bowing two strings of a violin together. For example, in the ordinary process of tuning, when the fourth and third strings with frequencies as 2 : 3 are combined, the resultant tone 1 is very observable if attention be directed towards it. But more striking effects are obtained when the resultant is at a larger interval from the primaries. The major sixth 3 : 5, the major second 8 : 9, and the minor seventh 5 : 9, are suitable intervals for calling out the fundamental 1, the strings employed being either the first and second or the second and third. The deep resultant tone thus obtained can not only be heard by the ear but felt as a tremor by the hand which holds the instrument. This is clear evidence of its objective existence, and I have succeeded in confirming the fact by means of a Helmholtz resonance-globe, the largest of the ordinary set, responding to C of 128 vibrations. When held with the edge of its mouth resting against the side of the violin, it responds to the combination C of 256 and G of 384 on the 4th and 3rd strings, or to the combination C of 512 and E of 640 on the 2nd and 1st ; or, still better, to the 3rd and 2nd open strings each flattened one note, so as to be C of 256 and G of 384. Here, then, we have distinct evidence that the violin possesses the power which I have ascribed to the ear—the power of manufacturing the fundamental when the two primaries are supplied.

11. Sir John Herschel, in his treatise on Sound (*Enc. Met.* arts. 238, 239), mentions the fact that the common fundamental can be called out by sounding two or more of its harmonics on very accurately tuned strings or pipes, and says that the effect cannot be obtained from a pianoforte tuned in the ordinary way, because the intervals are tempered. I find, however, on trying the experiment with an upright Broadwood of date about 1860, that C of 64 is easily called out by simultaneously striking eight or ten of its harmonics ;

and the effect is greatly enhanced if the key of C 64 is held down. In the latter case its note continues to be heard for a long time after the keys which were struck are released. From these experiments it appears probable that the sounding-board of a piano possesses the same property which we have proved to exist in the violin.

12. I now come to the explanation of the experiments of Professor Rücker and Mr. Edser (Proc. Phys. Soc. vol. xiii. p. 412 ; Phil. Mag. 1895, xxxix. p. 341). They were made with a Helmholtz siren, and in each instance the two primaries were produced in the same box, sometimes the upper and sometimes the lower box. The following explanation is a development of suggestions contained in Appendix xvi. of the *Tonempfindungen*.

The rate of escape of air from the box containing the two rows of holes which are employed may as a first approximation be assumed to be jointly proportional, at each instant, to the aperture for escape and the differential pressure which produces the escape. Again, this differential pressure may be regarded as the algebraic sum of two terms, one of them constant, and representing its average value, while the other represents the difference from the average due to the varying amount of the aperture from instant to instant. As a first approximation, equal increments of aperture must be regarded as producing equal decrements of pressure, so that the variable term will be proportional (with reversed sign) to the excess of the aperture above its mean value. This excess (defect being counted negative) will be a periodic function of the time, and if the ratio of the two primaries in lowest terms be $m : n$, the frequency for the complete period will be represented on the same scale by 1. In other words it will be the period of their common fundamental.

Let the aperture at time t be expressed in a Fourier series, θ being put for $2\pi t/T$, where T is the complete period ; and let the variable part of the expression be denoted by $f(\theta)$, while a_0 denotes the mean aperture, so that the aperture at time t is $a_0 + f(\theta)$. We shall have

$$f(\theta) = A \sin \theta + \dots + a_1 \sin (m\theta + \epsilon_1) + \dots + b_1 \sin (n\theta + \epsilon_2) + \dots$$

The largest amplitudes will be a_1 and b_1 corresponding to the

two primaries ; but A, which corresponds to the fundamental, is likely to be sensible.

The pressure at time t is proportional to

$$C - f(\theta),$$

C being a constant ; and the aperture is

$$a_0 + f(\theta).$$

Hence the rate of escape is proportional to

$$Ca_0 + (C - a_0)f(\theta) - \{f(\theta)\}^2.$$

a_0 is comparable with the maximum value of $f(\theta)$, and C is much greater ; hence a_0 may be neglected in comparison with C.

Developing $\{f(\theta)\}^2$, we shall obtain a term

$$2a_1b_1 \sin(m\theta + \epsilon_1) \sin(n\theta + \epsilon_2) \\ = a_1b_1 [\cos\{(n-m)\theta + \epsilon_2 - \epsilon_1\} - \cos\{(n+m)\theta + \epsilon_2 + \epsilon_1\}],$$

representing a difference-tone and a summation-tone. From $(C - a_0)f(\theta)$ we have the common fundamental

$$(C - a_0)A \sin \theta, \text{ or } CA \sin \theta,$$

and the two primaries

$$Ca_1 \sin(m\theta + \epsilon_1), \quad Cb_1 \sin(n\theta + \epsilon_2).$$

Suppose for simplicity that $a_1 = b_1$, then, taking the amplitude of each of the primaries as 1, the amplitude of the common fundamental will be A/a_1 , and the amplitudes of the summation-tone and difference-tone will each be a_1/C .

When $n - m = 1$, the difference-tone coincides with the fundamental, and their joint amplitude may be taken as the square root of the sum of the squares of a_1/C and A/a_1 .

13. Professor Rücker and Mr. Edser in experiments i. and ii. obtained the difference-tone 64 from five distinct combinations of primaries,

256 & 320, 192 & 256, 320 & 384, 51.2 & 115.2, 96 & 160, their ratios being

$$4:5, \quad 3:4, \quad 5:6, \quad 4:9, \quad 3:5.$$

The second combination appears to have given a stronger effect than either the first or the third ; whence it would appear that low frequencies are favourable to strong effects.

Nevertheless the fourth combination is mentioned as giving a rather feeblér effect than any one of the first three. This confirms our conclusion that the difference-tone is weaker when it is distinct from the common fundamental than when it coincides with it.

Experiment iii. was directed to testing for the presence of the resultant 64 when the primaries were 256 and 576, which are as 4:9. Their common fundamental is 64, and it could not be detected. This may have been because the pitch 576 was too high to give a good effect. Or the failure may be an indication that A/a_1 is decidedly smaller than a_1/C . It would be interesting to repeat the experiment, employing 192 and 320 as the primaries.

14. Near the end of chapter vii. of the *Tonempfindungen* Helmholtz makes prominent mention of the slipping of the hammer on the anvil as an important cause of resultant tones, and appears to regard it as exemplifying his mathematical formula for the restoring force as a function of the displacement. But it is clear that if the hammer, which holds the drumskin, is liable to shift in its supports, the restoring force cannot be a mere function of the displacement, but must also depend on the relative position and relative velocity of the hammer and anvil at the moment considered. I accept all the consequences which Helmholtz deduces in the passage in question from this slipping, including its application to explain first difference-tones; but I regard these consequences as lying outside the range of his general mathematical formulæ as given in Appendix xii.

15. To sum up my objections to the received mathematical theory of resultant tones:—

First. It assumes that the reaction of the drumskin against the air is a definite function of the displacement of the drumskin from a certain fixed position, whereas this reaction depends also on the position and motion of the further end of the hammer at the time.

Secondly. Even if the vibrations of the drumskin were in accordance with the received formulæ, there is plenty of scope for the introduction of additional constituents on the road from the drumskin to the liquid in the cochlea. The auditory ossicles, with their ligamentous supports and attachments,

probably serve to protect the oval window of the cochlea against shocks and jars, and to smooth down asperities in the wave-form, thus mitigating the harshness of sounds and rendering them more musical. The changes thus introduced are very unlikely to fulfil the special conditions required for the vanishing of the common fundamental.

Thirdly. The received theory makes the common fundamental, when not coincident with the first difference-tone, depend on a term involving the cube or some higher power of the displacement. When the primaries are as 3 : 5, the fundamental 1 comes in as $2m-n$, and depends on the cube of the displacement. When they are as 4 : 11, the tone 1, which Kœnig found to be the loudest resultant, is $3m-n$, and depends on the fourth power. When they are in the ratio 4 : 15, as in Kœnig's experiment with the simple tones ut_5 and si_5 , the common fundamental ut_3 , which was the only resultant tone heard, is $4m-n$, and depends on the fifth power of the displacement; the first difference-tone, which depends on the second power and should in theory be the loudest, being inaudible. This is surely a *reductio ad absurdum* of the received theory.

I do not wish to be understood as denying that the theory has any basis of truth. My contention is that the actions to which it is truly applicable play only a subordinate part in the production of resultant tones.

APPENDIX.

Examples selected from Kœnig's *Expériences d'Acoustique*, pp. 103 and 104, illustrating the production of the common fundamental. The "single vibrations" of the original are here reduced to double or complete vibrations :—

ut_5 and si_5 , which are as 8 : 15, gave only ut_3 .

ut_5 and 2816, which are as 4 : 11, gave ut_3 corresponding to 1 louder than any other tone.

ut_5 and si_6 , which are as 4 : 15, gave no audible tone but ut_3 .

ut_5 and 3968, which are as 8 : 31, gave no audible tone but ut_3 .

ut_6 and 3584, which are as 4 : 7, gave ut_4 more distinct than the difference-tone sol_5 .

ut_6 and si_6 , which are as 8 : 15, gave ut_2 distinct, the difference-tone 7 being inaudible.

ut_6 and 3968, which are 16 : 31, gave ut_2 only.

ut_6 and 4032, which are 32 : 63, gave ut_1 only.

DISCUSSION.

Dr. C. V. BURTON, after explaining several portions of Prof. Everett's paper, said that he (Dr. Burton) considered that the author's view in many ways seemed to fit in with the observed facts better than the accepted theory, but still did not appear itself quite free from objection. Prof. Everett supposes that the first term in a Fourier series is always the most important, and although in most cases which occur in practice this may be so, it hardly seems legitimate to take this as a characteristic of a Fourier series.

VIII. *The Magnetic Field of any Cylindrical Coil.*

By W. H. EVERETT, B.A.*

[Abstract.]

APPLYING Ampère's formula for the magnetic force at any point due to an element of current, the force perpendicular to any plane circuit, carrying a current i , is found to be, at any point P,

$$Z' = i \int \frac{r^2 d\theta}{(r^2 + h^2)^{\frac{3}{2}}};$$

h being the distance of P from the plane of the circuit, and r, θ the polar co-ordinates of any point of the circuit referred to the projection of P as origin.

The longitudinal force at any point due to a current in a cylindrical coil, or solenoid, is given by a second integration. It is the sum or difference of two terms, each of the form

$$Z = inh \int \frac{d\theta}{\sqrt{r^2 + h^2}};$$

* Read November 8, 1895.

where h denotes the distance of the point from an end plane of the solenoid, and n the number of turns per unit length of h . The depth of the coil, normal to the cylindrical surface, is assumed to be inconsiderable. The limits of integration are 0 and 2π for any point whose projection, taken parallel to the axis, falls within the solenoid.

Similarly, the transverse force at any point, due to a solenoid, is found to be

$$R = in \Sigma \left(\frac{1}{\sqrt{r^2 + h_1^2}} - \frac{1}{\sqrt{r^2 + h_2^2}} \right) ds,$$

the summation being vectorial.

The latter two formulæ can be readily applied, for approximate calculation, to a cylindrical coil of any cross-section, including coils of circular and rectangular sections. But in the case of rectangular coils the formulæ become integrable.

Let p be the perpendicular distance of any point P from one of the faces of the rectangular coil, and a one of the two parts into which the corresponding side of a cross-section is divided by the perpendicular from P. Then the longitudinal force at P is given by the algebraic sum of sixteen terms, each of the form

$$\begin{aligned} Z &= inh p \int_0^a \frac{dx}{(x^2 + p^2) \sqrt{x^2 + p^2 + h^2}}, \\ &= in \sin^{-1} \frac{ah}{\sqrt{(a^2 + p^2)(h^2 + p^2)}}. \end{aligned}$$

And the transverse force at P is the resultant of eight terms, each of the form

$$\begin{aligned} R &= in \int_0^a \left(\frac{1}{\sqrt{x^2 + p^2 + h_1^2}} - \frac{1}{\sqrt{x^2 + p^2 + h_2^2}} \right) dx, \\ &= \frac{in}{2} \log \left(\frac{p^2 + h_2^2}{p^2 + h_1^2} \cdot \frac{a + \sqrt{a^2 + p^2 + h_1^2}}{a + \sqrt{a^2 + p^2 + h_2^2}} \right). \end{aligned}$$

The first formula in the paper can be used to find an expression for the force due to a circular current, at any point P in the plane bounded by the circle. Draw any chord through P, and call its segments r, r' . Write c for the

distance of P from the centre, b for half the minimum chord through P, and a for the radius. Then for the force at P the formula reduces to

$$\begin{aligned} F &= i \int_0^{2\pi} \frac{d\theta}{r}, \\ &= i \int_0^{\pi} \left(\frac{1}{r} + \frac{1}{r'} \right) d\theta, \\ &= \frac{2i}{b^2} \int_0^{\pi} \sqrt{a^2 - c^2 \sin^2 \theta} \cdot d\theta. \end{aligned}$$

This can be written

$$F = i \frac{s}{b^2},$$

s being the perimeter of the ellipse with a and b as semi-axes, and having some value between $2\pi a$ and $4a$, according to the position of the point considered.

IX. *On the Determination of High Temperatures with the Meldometer.* By WILLIAM RAMSAY, Ph.D., F.R.S., Professor of Chemistry, University College, London, and N. EUMORFOPOULOS, B.Sc., Demonstrator of Physics, University College, London*.

THE meldometer, an instrument devised by Dr. Joly, has been sufficiently described by him (Proc. Roy. Irish Acad. 3rd series, ii. p. 38, or Chem. News, vol. lxx.), and we need therefore only give a very brief account of it here. The essential part of the instrument is a length (about 10 centim.) of thin uniform platinum ribbon, about 1 millim. wide. This is heated by a current of adjustable strength, and the increase in length of the ribbon is measured by a delicate micrometer-screw, the ribbon being kept gently stretched by a small spring. The temperature of the ribbon is, of course, lower where the two forceps hold it; but if it is suitably cut at each end nearly to a point, a length of, say, 6 centim. in the

* Read February 14, 1896.

middle may be made of a very uniform temperature, as can be proved experimentally by taking the reading of the melting-point of the same substance at different points along the ribbon.

An infinitesimal quantity (scarcely visible with the naked eye) of the substance to be melted is placed on the ribbon and viewed with a low-power microscope. The small quantity of the substance required enables it to be purified very completely. The current is then put on and increased rapidly until the substance melts, and thus an approximate reading is obtained. This is repeated more cautiously, to obtain an exact reading. Several readings can be taken by remelting the same, and also by using fresh substance, the latter method being usually adopted.

To translate the readings into temperatures, it is necessary to standardize the instrument by taking the readings with substances of known melting-points. Of these, unfortunately, there are none known with certainty beyond about 350°C . One reading is obtained by taking the temperature of the air, another is the melting-point of potassium nitrate (339°), and for a third one the melting-point of potassium sulphate (1052°) was adopted, for reasons that will appear. The melting-point of silver is irregular, apparently because of absorption of oxygen, and consequent spitting. Gold can be used, and also palladium. As, however, the expansion of the ribbon is almost a linear function of its temperature, and as the observations hitherto taken did not extend beyond about 1050° , it was considered unnecessary to take readings with palladium, the general character of the expansion of the ribbon being already known from Dr. Joly's observations. The question next arises, what is the melting-point of gold? There have been two or three determinations of value, which unfortunately differ from one another.

M. Violle (*C. R.* 1879) determined it by a calorimetric method, and obtains as a result 1045° (on the air-thermometer).

Messrs. Holborn and Wien (*Wied. Ann.* xlvii. and lvi., 1892 and 1895), who give 1072° , compared a thermo-element with an air-thermometer, and then used the former for determining the melting-points of silver, gold, and copper. This

was done by inserting in a porcelain crucible the thermo-element, and also two platinum wires connected by a wire of the substance whose melting-point was to be taken. The platinum wires formed part of a circuit containing a battery and a galvanometer. When the wire melted, the circuit was broken, and the temperature read at the same moment with the thermo-element. The melting of the substance must in general lag a little behind the thermo-element; and as no mention is made of the rate at which the temperature was raised, it is difficult to know how far the results can be trusted. They obtain, however, very concordant readings.

Besides these, there are two determinations with the platinum pyrometer: one by Professor Callendar (Phil. Mag. Feb. 1892), and the other by Messrs. Heycock and Neville (J. C. S. Trans. 1895, p. 160). By a somewhat violent extrapolation* from a formula which, as far as we know, has been compared with the air-thermometer only to about 625°, they obtain:—

	Callendar.	Heycock and Neville.
Freezing- or melting-point of silver ...	982°	961°
gold	1091°	1062°†

* In Holborn and Wien's last paper (*loc. cit.*) the resistance of pure platinum is determined at different temperatures with their thermoelement, and their results cannot be expressed quite satisfactorily ("nur ungenügend") by means of Callendar's formula. The resistance of two of their pure platinum wires began to differ beyond about 900°, while agreeing below this temperature. They therefore consider this property unsuited to extrapolation.

† In a recent paper (J. C. S. Trans. 1895, p. 1025) Heycock and Neville state that "Callendar did not rigidly follow the method of calibration, which was afterwards developed by Griffiths and himself, and which we have always adhered to. That method requires that the resistance of the pyrometer should be determined at three standard temperatures," viz., at 0° , 100° , and the boiling-point of sulphur, $444^{\circ}\text{C}.$ And lower down they say, "If Callendar had standardized his thermometer on the boiling-point of sulphur . . .," hence they infer that Callendar did not use the boiling-point of sulphur for this determination. But the determination of this boiling-point is given in Phil. Trans. 1891, *i. c.*, it was published before the paper referred to, and the latter is also later than another paper (Phil. Mag. July 1891, p. 109), in which the boiling-point of sulphur is directly referred to. We do not quite see how to reconcile the various statements.

Callendar then, taking the melting-point of silver as 945° , makes the melting-point of gold 1037° . Now Violle's determination for silver is 954° , which, using Callendar's formula, would give for gold 1049° . Holborn and Wien's value for silver is 968° . In view of the variance between the numbers, it was determined to take Violle's value, though the correct value may be a few degrees higher (compare Le Chatelier, *C. R.* cxxi. p. 323, 1895). In any case, allowance can easily be made, when further researches have determined the true melting-point.

The gold used by us was a very pure specimen obtained from Messrs. Johnson and Matthey. The salts used, with the exception of some of the iodides, were pure specimens, prepared specially by ourselves. In a few cases the salts so prepared were recrystallized, and melting-points were taken both of the recrystallized salt and also that obtained from the mother liquor. No difference could be detected, and hence no further mention is made of these determinations.

Gold on melting alloys with the platinum, and hence must destroy to a certain extent the uniformity of the ribbon. A remedy for this is to dust the ribbon lightly with finely powdered talc. This, however, is not very satisfactory, and interferes somewhat with the observations; but with care, fairly good results can be obtained, as the following numbers show:—

	With talc.	Without talc.
NaCl.	799°	792°
K_2CO_3	883	880
KCl	752	762
$Ba(NO_3)_2$	583	575

It was then found that the melting-point of potassium sulphate is very little different from that of gold, viz., 7 degrees higher or 1052° ; and this salt was afterwards used instead of the gold; thus there is no need to use talc.

Some melting-points are sharply marked, others are not. In these cases the lowest point was taken at which spreading over the ribbon could be detected.

For purposes of comparison determinations by other observers are given; some determined the melting-points (*e.g.*,

Carnelley, and Meyer, Riddle and Lamb), others the freezing-points (Carnelley, Le Chatelier, Heycock and Neville, and McCrae). The data are taken from the following references:—

Carnelley (calorimetric method): *J. C. S. Trans.* 1876, p. 489; 1877, p. 365; 1878, p. 273.

Le Chatelier (thermo-electric method, assuming melting-point of gold as 1045°): *Bull. Soc. Chim.* t. xlvii. p. 301; *C. R.* t. cxviii. pp. 350, 711, and 802.

V. Meyer, Riddle and Lamb: *Ber.* xxvii. (1894) p. 3129.

—In this method the salt has been previously fused in a platinum tube with a wire down its centre, and to this wire is attached a weight passing over a pulley. When the salt melts, it is pulled out by the weight, and the temperature is determined at the same moment by an air-thermometer.

J. McCrae (*Wied. Ann.* lv. p. 95), relying on Holborn and Wien's results, standardized his thermo-element with boiling diphenylamine (304°) and boiling sulphur ($444^{\circ}5$). For his thermo-element he used platinum against an alloy of platinum rhodium, and also against an alloy of platinum and iridium. The numbers in brackets given below refer to the latter, and the others to the former. It will be noticed that they do not agree perfectly. This may, of course, be due to his metals not being of the same purity as those of Holborn and Wien. The latter also from their observations find that the iridium alloy is not as well suited as the rhodium alloy.

Salts of Lithium.

The lithium carbonate bought as pure gave a conspicuous sodium coloration to the Bunsen flame. It was purified by successive precipitation and washing, until it gave a brilliant carmine coloration to a Bunsen flame, free from yellow fringe, showing an absence of sodium. The melting-points are not very well marked as a rule.

	Ramsay & Eumorfopoulos.	Carnelley.	Le Chatelier.
Li^+SO^4	853	818	830
Li^+CO_3	618	695	710
LiCl	491	598	
LiBr	442	547	
LiI	below 330	446	

It will be noticed that there is here no agreement between the results of different observers. Our results are as a rule about 100 degrees lower than Carnelley's, except that of the sulphate. Our resistances did not allow us to take the melting-point of lithium iodide, as we had not arranged for temperatures below 330° , but its melting-point is below that of potassium nitrate (339°). Lithium iodide is very hygroscopic, so that on placing it on the ribbon it quickly liquefies; then on putting on the current, it becomes solid and then liquid again at the temperature given above, *i. e.*, it has melted, and remains so indefinitely.

Salts of Sodium.

The salts of sodium and potassium were prepared from the bicarbonate, precipitated by carbonic acid from a solution of the pure carbonate. The melting-points are well marked, though all are not equally so, *e. g.*, the iodides.

	Ramsay & Eumorfopoulos.	Le Cha- Carnelley.	Meyer, Riddle & Lamb.	Heycock & Neville.	McCrae.
Na^2SO^4 ...	884	861	860	883	883
Na^2CO^3 ...	851	814	820	849	852
NaCl	792	772	778	815	813
NaBr	733	708	...	758	...
NaI	603	628	...	661	...

The agreement between Messrs. Heycock and Neville's results and our own is only apparent, as they find 1062° as the melting-point of gold, while we assume 1045° . The fact that they are determining freezing- and not melting-points may introduce some difference.

Salts of Potassium.

	Ramsay & Eumorfopoulos.	Le Cha- Carnelley.	Meyer, Riddle & Lamb.	Heycock & Neville.	McCrae.
K^2SO^4 ...	1052	...	1045	1078	1066.5
K^2CO^3 ...	880	834	860	879	...
KCl	762	734	740	800	...
KBr ...	733	699	...	722	...
KI	614	634	640	685	...

M. Le Chatelier gives an earlier value 1015° for potassium sulphate, and also 885° for potassium carbonate. Our value for potassium sulphate is 7 degrees above that of gold.

Messrs. Heycock and Neville's is 4.5 above their value for gold, but they mention that there may be an error of 2 degrees, due to the alkalinity of their potassium sulphate.

Salts of Calcium, Strontium, and Barium.

The salts of calcium, strontium, and barium were prepared from their carbonates precipitated from the purified nitrates. Their melting-points are not well marked, especially those of calcium; and here again the iodides are less well marked than the chlorides.

	Ramsay & Eumorfopoulos.	Carnelley.	Le Cha- telier.	Meyer, Riddle & Lamb.	McCrae.
$\text{Ca}(\text{NO}_3)_2$	499	561			
CaCl_2	710	719	755	806	802
CaBr_2	485	676			
CaI_2	575 (?)	631			
$\text{Sr}(\text{NO}_3)_2$	570	645			
SrCl_2	796	825	840	832	854
SrBr_2	498	630			
SrI_2	402	507			
$\text{Ba}(\text{NO}_3)_2$...	575	593	592		
BaCl_2	844	860	847	922	916 (941)
BaBr_2	728	812			
BaI_2	539				

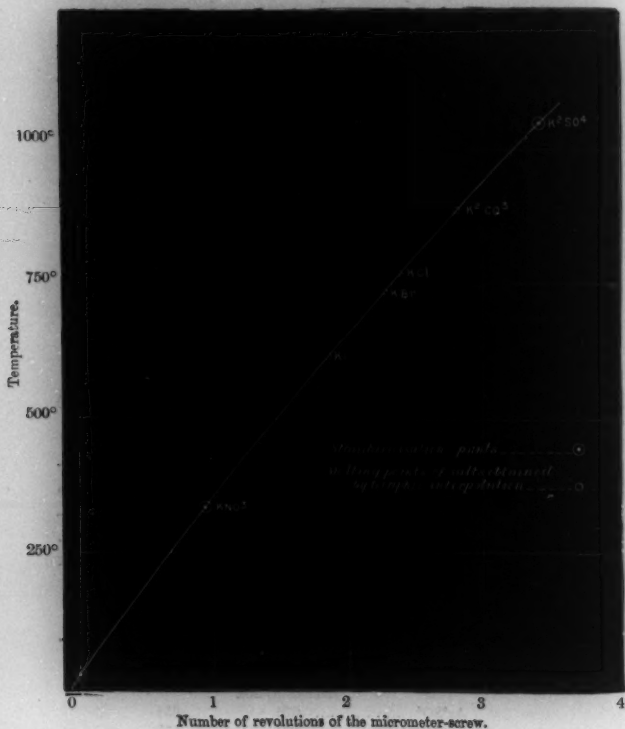
Calcium chloride is very difficult to observe, as it slowly softens. We found it practically impossible to take the melting-point of calcium iodide, as it is exceedingly hygroscopic, and, on heating, it is almost immediately oxidized. We do not think it can be above the value given, though it may be below it.

Salts of Silver and Lead.

These were prepared by precipitation. Their melting-points are well marked.

	Ramsay & Eumorfopoulos.	Carnelley.
Ag_2SO_4	676	654
AgCl	460	451
AgBr	426	427
AgI	556	527
PbSO_4	937	
PbCl_2	447	498
PbBr_2	363	499
PbI_2	373	383

A curve is appended with the melting-points of the salts of potassium marked on it. In drawing this curve there must



of necessity be a small uncertainty, as there is no datum-point between 339° and 1052° ; but this cannot amount to more than a very few degrees.

In conclusion, we may point out that plotting our meldometer-readings against other observers' melting-points does not give a smooth curve.

University College, London.

DISCUSSION.

Prof. RAMSAY added that the chief advantage of the meldometer was that only a very minute fragment of the substance was required for the measurement, so that extreme purity of the sample could be secured. There was the disadvantage, however, that many substances undergo some change when heated in air. In reply to a question from Mr. Blakesley, Prof. Ramsay said that the property of the platinum which was used to measure the temperature was its expansion.

Mr. CAMPBELL asked whether the zero of the instrument was found to be constant. In Cardew voltmeters it often took several hours for the needle to come back to zero after heating.

Mr. EUMORFOPOULOS, in reply, said that the zero was constant to within a quarter of a degree.

Prof. RAMSAY also exhibited a small direct-vision spectroscope, in which the eye-piece is moved in a plane perpendicular to the axis of the instrument by means of a micrometer screw. This form of spectroscope is found to be of great utility in verifying the position of lines in the spectrum.

X. *On some Geometrical Instruments invented by*
Mr. E. SCOTT and Signor MONTICOLO.

[Abstract.]

MR. E. SCOTT, at the meeting on January 24, 1896, showed some Geometrical Instruments invented by himself and Signor Monticolo.

The instrument designed by Signor Monticolo is intended for drawing arcs of circles of such large radius that compasses cannot be employed. It can be used to trace arcs of circles of which the radii vary from 50 cm. to infinity.

The second instrument exhibited was a modified form of hatchet planimeter, which Mr. Scott has devised with a view of avoiding some of the defects of the ordinary form of instrument. Thus, to avoid the cutting of the paper, which occurs when the knife-edge is sharp, and the side-slip which occurs when the knife-edge is blunt, the author uses a wheel with a sharp edge. To avoid the inclination of the instrument to one side, which may easily occur with the ordinary form, a flat celluloid plate with a dot at the centre is used as the tracing point; this plate being kept pressed flat on the surface of the paper. A small wheel with a recording disk is attached, and may be used to measure the distance between the first and last position of the knife-edge.

Mr. Scott also described a form of planimeter which he had invented, and in which the disk and cylinder movement is used to perform the integration

DISCUSSION.

Mr. C. V. BOYS said that an instrument designed by Mr. Clarkson had been exhibited before the Royal Society which was capable of drawing arcs of circles of large radius. This instrument only drew an approximation to a circle, but the approximation was so close that it nowhere was more than the thickness of a thin ink line away from the truth. It would be interesting to hear from the author whether Signor Monticolo's instrument drew a rigorously exact circle or not. The upright position of the hatchet

planimeter might be secured by using two wheels in place of one. The planimeter described was really a modified form of one he (Mr. Boys) had described before the Society in 1881.

Mr. BLAKESLEY gave a short geometrical proof showing that the curve traced by Signor Monticolo's instrument was rigorously an arc of a circle. Mr. Blakesley also drew attention to the fact that the instrument in its present form cannot be used to trace the arc on both sides of the starting point.

Dr. C. V. BURTON described an idea for an instrument for drawing circular arcs which had occurred to him, depending on the use of two wheels of different radii connected by an axle conveying a tracing point.

XI. *An Analytical Study of the Alternating Current Arc.*
By J. A. FLEMING, M.A., D.Sc., F.R.S., Professor of
Electrical Engineering in University College, London, and
J. E. PETAVEL*.

ALTHOUGH the physical phenomena of the Electric Arc have received of late years very considerable attention, there are many points in connexion with the study of the alternating current arc which seem to us to have been insufficiently explored. The investigation which is here described is a contribution to the further examination of the mode in which the variation of the luminous and electric effects takes place in the alternating current arc.

The first portion of the research is concerned with an analytical study of the variation of the light thrown out from different radiating regions in the arc, and the delineation of the periodic value of this illuminating power by graphical methods. The object of the first set of experiments was to record and represent by an appropriate graphical method the periodic value of the light thrown out from the carbons and from the true electric arc region when the arc is supplied with electric

* Read February 28, 1896.

power of known constant amount and varying magnitude, and at the same time to record the periodic variation of current through the arc, and potential-difference of the carbons. The instrumental appliances involved in the first place the employment of means for keeping constant and measuring the mean value of the power supplied to the arc. This was accomplished by the use of a suitable non-inductive resistance and a bifilar reflecting wattmeter, the series coil of which was in the circuit of the arc, and the shunt coil of which was connected across the carbons of the arc. The following is a description of this wattmeter:—

The series coil was wound with 10 turns of thick copper wire (No. 10 S.W.G.) and had a resistance of $\cdot 01$ ohm. The shunt coil was wound with 23 turns of thin wire, and had a resistance of 3.76 ohms, and was placed in series with a non-inductive platinoid resistance of 1200 ohms. The shunt-coil was suspended by two fine silver wires, which also served to conduct the current in and out of the coil. The controlling and deflecting forces so balanced each other at small displacements as to make the angular displacement of the movable coil very nearly proportional to the power passing through the wattmeter. The movable coil was provided with a mirror by means of which the image of a wire illuminated by an auxiliary arc lamp was reflected on to a fixed scale. This scale was carefully graduated, so as to read directly in watts. It was found that the earth's magnetic field caused a small deflexion of the movable coil when using continuous currents and when the shunt current was passing through it, when at the same time no current was flowing in the series coil. This terrestrial field was neutralized by magnets, or else the scale was shifted so that the part of the deflexion of the shunt coil due to the terrestrial field was eliminated. The best way would have been to have turned the wattmeter round through a certain angle, but as it had to be screwed up against the wall in a fixed position for steadiness, it was found that the above method was the simplest plan for obviating this source of error. The vibrations of the movable coil were damped by means of a mica vane dipping into a dash-pot filled with oil. When all the adjustments were made, it was found that this wattmeter produced a deflexion of the spot of

light on the scale almost exactly proportional to the power passing through the instrument. This wattmeter was then connected up to the arc lamp, so that the whole current actuating the arc lamp passed through the series coil, the terminals of the shunt coil being connected to the carbons of the arc.

A series of preliminary experiments were then made for the purpose of enabling us to eliminate from the wattmeter-readings, firstly, the power taken up in the wattmeter itself; secondly, the power taken up in the shunt coil of the arc-lamp and other shunt resistances; and, thirdly, the power taken up in the carbons themselves, right up to the point where the arc is being formed. The observations, when reduced by applying these corrections, gave us at once the true mean power being taken up in the arc. The wattmeter was constructed with all the precautions necessary in making a wattmeter for measuring alternating-current power, so that it served to measure the power taken up either in alternating or continuous current arcs. The current supplied to the arc passed through a series of non-inductive resistances, consisting of carbon plates, in such a manner that the power given to the arc could be regulated with the greatest exactness. In addition to the wattmeter, an ammeter was placed in series with the arc lamp so as to measure the current passing through the arc, and a voltmeter connected to the carbons so as to measure the potential-difference of the carbons, these instruments being suitable both for continuous and alternating currents. In all the experiments the mean power was kept constant in the arc, and this was done by adjusting the current so that the wattmeter took a certain deflexion corresponding to the power desired, and the wattmeter was kept at a constant deflexion by regulating the carbon resistances in series with the arc. In addition to the instruments described above, a lens was fixed so as to enable the length of the arc to be measured in the usual manner.

In the course of the experiments three arc lamps were employed—a hand-regulated arc lamp in which the distance of the carbons was adjustable by a screw with great accuracy; a continuous-current arc lamp (the Waterhouse Arc Lamp); and an alternating-current arc lamp (the Helios Arc Lamp);

these last two being selected as excellent arc lamps of their respective types, our object being to select an arc lamp, the mechanism of which enabled it to be worked with electric powers varying over wide limits, and yet to yield a perfectly steady arc. In addition to these instruments there was set up the apparatus for delineating the curves of current of electromotive force, which have been described by one of us in the 'Electrician,' vol. xxxiv. p. 460, and which consists of a synchronizing alternating-current motor, having its fields separately excited and its armature circuit traversed by a shunt current from the circuit operating the arc lamp. This alternating-current motor was set up on the photometer bench, and it was used to drive an aluminium disk pierced by four openings, in such a manner that the disk revolved synchronously with the alternating current operating the arc. In addition to this duty the alternating-current motor carried a contact-maker on its shaft for the purpose of enabling the curves of current of electromotive force to be delineated. A brief description of the motor is as follows :—

It consists of two sets of field magnets *M, M* (see fig. 1), having eight poles on each side which are secured to two cast-iron disks. Between these field magnets revolves a small armature *A*, the iron core of which is formed of a very thin strip of transformer iron wound up into a ring, the armature coils being wound on this ring. The armature coils are joined up in series with one another, so as to give a series of alternating magnetic poles round the ring when a current flows through the armature circuit. The diameter of this armature is about six inches. The field magnets have eight poles, and the armature eight coils. The field magnetic coils are bobbins about 2 inches long and $1\frac{1}{2}$ inch in diameter, and when joined up in series in the proper manner the field magnets take a current of about eight amperes to give them a proper amount of saturation. The armature is carried upon a hard wooden boss fixed to a steel shaft; and the steel shaft is carried through small ball bearings made like bicycle bearings. In order to prevent any side shake of the armature, there are at opposite ends of the base cast-iron pillars with a gun-metal screw through each, against which the rounded end of the shaft bears; the position of the shaft can thus be

adjusted with great nicety, and runs with great freedom from friction. The ends of the armature circuit are brought to two small insulated collars fixed on the shaft, against which

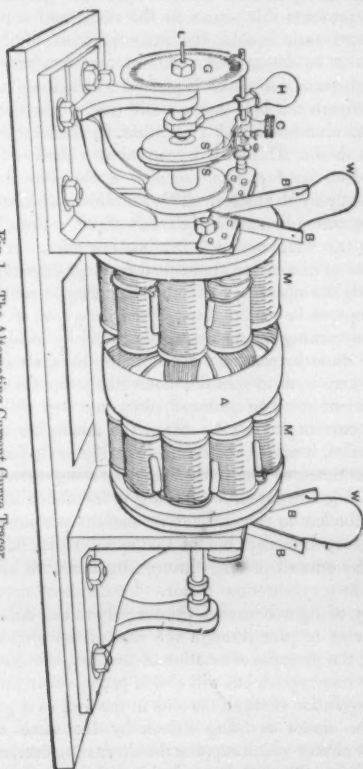


Fig. 1.—The Alternating Current Curve Tracer.
[From the 'Electrician,' vol. xxiv, p. 460, 1896.]

press two light brass brushes marked B, B kept against the shaft by means of an expanding steel wire W. The armature

shaft carries on one side an ebonite disk with a steel slip let into it. Two insulated springs S, S are carried on a rocking arm H; this rocking arm can be traversed through half a circumference, and is centred upon the gun-metal screw which prevents side shake in the shaft, and a pointer and graduated scale enables the exact position of the contact-springs to be determined. One of the stop-screws keeping the shaft from side shake is pierced with a longitudinal hole, and through this hole passes a stiff steel wire; this serves to drive an aluminium disk 27 centims. in diameter and 4 millimetres thick. This disk is carried on a shaft which runs in a cast-iron bearing, and the disk is therefore driven synchronously by the motor. This aluminium disk has four slits in it separated by angular intervals of 90° ; the slits are 0.5 centimetre wide and 4.5 centimetres long. If the field-magnets of the motor are excited by a continuous current of about eight amperes, and if an alternating current of about two amperes is passed through the armature of the motor, then, on turning the motor rapidly round by hand, which can best be done by passing a strap round the shaft and pulling at the strap so as to spin the motor like a top, the motor will, if sufficient speed be gathered, drop into step with the alternating current driving it. Since the motor has eight magnetic poles, it makes one complete revolution in four complete periodic times, so that if the motor is being driven from an alternating-current circuit having a frequency of 100, then the motor has to run at 1500 revolutions per minute before it will drop into step, but at that speed it will fall into step with the current passing through its armature, and will be driven as a synchronous motor. Under these circumstances, if a ray of light is passed transversely to the disk in such a manner as to pass through the slits of the aluminium disk during the progress of rotation of the disk, the beam of light will be interrupted, but will obtain passage four times during each revolution through the slits in the disk as it goes round.

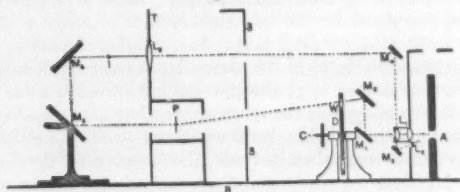
If the motor is being driven by the same alternating current circuit which supplies the alternating current to an arc lamp, it is evident that, on looking through the slits in the revolving disk at the alternating current arc, it will be seen in

one constant condition during its periodic variation, such instant being determined by the position of the slits with reference to the phase of the current. Without entering into a longer description, it will be evident that this synchronizing motor driving a disk and a contact-breaker enabled two things to be done—first, to delineate all the current and electromotive-force curves of the arc taken in the usual way; and, secondly, a ray to be taken from the arc selected at one particular instant during the complete period through which the variation of illumination passes. These arrangements were completed by the construction of a photometer of a particular kind. Owing to the slow variation of position of the electric discharge in the alternating current arc, it would have been useless to photometer the instantaneous value of the light coming from the alternating current arc against any fixed standard of light; but it was found possible to make a very exact comparison between the intensity of the light coming from any part of the arc, and selected at any one constant instant during the complete phase, with the mean value of the light coming from that same part of the arc during the complete period; in other words, it was found possible to photometer the arc against itself, and so eliminate to a large extent the difficulties arising from slow periodic variations of the light sent out from the arc in any one direction. It is well known that the light of an alternating current arc, taken in any one direction, undergoes a slow periodic variation quite independently of the variation of current during the phase, neither is it dependent upon any variation of the mean square value of the current, because it takes place even when that current is perfectly constant. It appears to be due to slow changes of position of the points on the carbons between which the discharge takes place. The discharge is as it were seeking out new points between which to take place, and it continually changes these positions as the arc burns. The photometric arrangements finally adopted were as follows, and are shown in outline in fig. 2.

A represents an alternating current arc. This arc was enclosed in a metal lantern in which were three openings. The light from this arc passed through a lens L_1 in a hori-

horizontal direction and fell upon a mirror M_1 placed at an angle of 45° , capable of being rotated at this constant inclination round a horizontal line co-linear with the axis of the revolving disk, and the motor placed in front of the lens L_1 . The ray was then reflected upwards into another mirror M_2 , and by this mirror reflected at an angle very nearly equal to 45° in such a manner as to pass through the slits in the revolving disk D, when any one of the slits was in a position to allow

Fig. 2.



Arrangement of the Photometer and Revolving Disk.

the ray to pass. The two mirrors M_1 and M_2 were rigidly connected to a rocking-arm so centered that the line $M_1 M_2$ could be rotated round into any required position, always moving parallel to a radial line of the disk D. The disk D was the disk carried on the shaft of the synchronizing motor above described; the motor, together with its associated disk, was placed on the photometer bench in the required position opposite to the arc-light lantern. In fig. 2 the motor itself is not shown, but its position is indicated by the letter C. The angular position which the rocking-arm carrying the two mirrors M_1 and M_2 occupied with respect to the vertical line passing through the centre of the revolving disk could be observed on a graduated scale. It will be clear, then, that if the disk driven by the motor was in synchronism with the alternating current producing the alternating arc at A, an observer, looking through holes in the rapidly revolving disk, would see by reflexion in the mirrors M_1 and M_2 the alternating current arc at A; but he would see it, not as it is seen when looked at directly, but in some constant condition taken

at one definite instant during the phase, which instant would depend upon the position of the line $M_1 M_2$ with regard to the vertical line through the centre of the disk. Thus by rocking over the arm $M_1 M_2$ in various positions the observer could see, through the window W as the disk revolved, the arc at A , either in the condition when the electric discharge is taking place, or when the true arc is extinguished, according to the position in which the arm $M_1 M_2$ was set. If, instead of observing with the eye, a disk of paper with a photometric grease spot upon it was placed at P , the lens L could be so adjusted as to throw an enlarged and well-defined image of the arc upon the disk at P ; and by rocking over the arm carrying the mirror into successive positions, the observer would see the image of the arc pass slowly through all those successive phases which in the arc itself actually take place during one periodic time. As the image of the arc is much larger than the photometer disk, it was possible, by slightly shifting one of the mirrors, to bring any desired part of the image of the true arc or of the craters of either of the carbons to cover the grease spot. In addition to this interrupted ray, another ray was gathered from the same part of the arc by a lens L_2 , placed on the same level with the lens L_1 but slightly to one side of it. This lens gathered a beam which was reflected by a mirror M_3 placed at an angle of 45° , and which reflected the ray upwards to another mirror M_4 . In fig. 2, for the sake of clearness the lens L_2 is shown beneath the lens L_1 , but it must be understood that in the real apparatus the lenses L_1 and L_2 were on the same level and placed side by side. The ray reflected from the mirrors M_3 and M_4 was set horizontally, so as to be received on a lens L_3 , and by this lens L_3 was gathered to a focus at a point I . The lens L_2 was so adjusted as to form a large image of the arc on the screen which carried the lens L_3 , and by slightly moving the mirror M_4 any part of this image could be made to cover the lens L_3 . It will be seen, therefore, that the light gathered together at a focus at the point I could be made to be light coming from any assigned area in the arc or from the craters. A movable stand carried two other mirrors M_5 and M_6 fixed at angles of 45° in such a way as to reflect the ray coming from

focus at l , and reverse its direction so as to bring it round and make it fall on the left-hand side of the photometer-disk P . It will thus be seen that, by moving the mirrors M_5 and M_6 , the light falling on the left-hand side of the photometer-disk P could be made to have any desired intensity within certain limits, and could be gathered from any desired part of the arc or craters; and, moreover, this illumination was the mean illumination, or proportional to the mean illuminative power of any part of the arc selected for examination. It will thus be clear that the arrangement enabled us to project on to the right-hand side of the photometer-disk P the rapidly intermittent ray taken from any part of the arc, and always gathered at one constant phase condition during the complete period; whilst on the left-hand side of the photometer disk we could project a ray gathered from the same part of the arc, but not interrupted. We could therefore compare the mean value of the light proceeding from any part of the arc with the instantaneous value of the light taken from the same part of the arc and selected at any assigned instant during the period. Thus the arc itself became its own standard, and difficulties due to slow fluctuation of the mean light of the arc disappeared. At the same time the contact-maker on the motor enabled us to delineate in the usual manner the curves of current and potential difference of the arc, and thus to record the variation in the arc of the arc current, the carbon potential difference, the power expended in the arc, the resistance of the arc, and the luminous intensity of any part of the arc. A long series of experiments was then made with alternating current arcs of different lengths and powers, the periodic electric quantities being delineated and the light being taken, either from the centre of the true arc halfway between the carbons or from one of the craters of the carbon terminals,—generally the bottom carbon. The process of taking measurements was as follows:—

After setting the lenses and the mirrors so that the lens L_1 and the mirrors M_1 and M_2 gave a sharp image of the arc on the right-hand side of the photometer disk with the selected area of the image covering the grease spot, the mirrors M_5 and M_6 were moved backwards and forwards until the balance

was obtained between the illumination falling on the right- and on the left-hand side of the photometer disk. The right-hand side of the photometer disk being illuminated by an intermittent stream of light always selected in the same phase, when considered as belonging to a periodically varying illuminating beam, whilst the light falling on the left-hand side of the photometer disk is a uniformly illuminating beam of the same quality and colour coming from the same part of the arc but not interrupted, and representing therefore the mean value of the light emitted from that selected area of the arc. Observers were deputed to measure all the various quantities by the different instruments, and a power of constant definite amount was supplied to the electric arc from an alternating-current machine driven by a continuous-current motor. By means of the carbon resistance and the reflecting wattmeter this power was kept constant at a selected value for a sufficient time to enable all the various periodic quantities to be observed at sufficiently frequent intervals during the phase. In all cases the arc was allowed to burn quietly for half an hour to get the carbons into a constant position before any observations were taken. It is hardly necessary to go into the details of delineating the current and electromotive-force curves, as the process of doing this is now well understood. The Kelvin vertical multicellular voltmeter, having a half microfarad condenser placed across its terminals, was employed for the measurement of the potential difference of the carbons in the following manner:—

The voltmeter, with its associated condenser, was connected, through the contact-maker driven by the shaft of the alternating motor, to the carbons of the arc; the contact-maker thus closed the circuit at a certain instant during the phase, and the voltmeter gave the instantaneous value of the potential-difference of the carbons. In series with the arc was placed a non-inductive resistance of suitable magnitude. A switch was arranged so that the voltmeter with the condenser in parallel with it, both being in series with the revolving contact-maker, could be put across either the terminals of this non-inductive resistance, or else between the carbons of the arc. By rocking over the arm carrying the spring-

brushes of the contact-breaker, the voltmeter circuit was closed at a particular instant during the phase, and the voltmeter reading gave therefore, when corrected, the instantaneous value either of the potential-difference of the carbons or the instantaneous value of the current through the arc. As the Kelvin multicellular voltmeter used by us only begins to read at 60 volts, in order to get readings for lower values than 60 volts, it is necessary to add a known electromotive force to the voltmeter circuit in order to block up the needle of the voltmeter to a false zero. This was done by connecting a known number of small Lathanode secondary batteries, the potential of which was determined by the same voltmeter, in series with the voltmeter. In this way the series of observations were successively taken of the following quantities:—First, the instantaneous value of the potential difference of the carbons taken at equidistant intervals throughout a complete period; secondly, the instantaneous values of the current through the arc taken throughout the complete period; and, thirdly, the instantaneous values of the luminous intensity of a certain selected portion of the arc taken at intervals throughout the complete period and expressed in terms of the true mean luminous intensity of the same portion of the arc at that time. These quantities having been obtained, it was then possible to plot them down in a series of curves, and to deduce therefrom curves representing the periodic variation of power through the arc, and the periodic variation of the resistance of the arc. Five sets of experiments were made, taken for different frequencies and different lengths of arc, each set comprising an observation taken with the light proceeding from the centre of the true arc, and also an observation taken with the light proceeding from the centre of the crater of the lower carbon. The frequencies employed were 83, 50, and 26. In all cases the current was kept at 14 amperes (mean square value); the results of these observations are embodied in the following 10 tables arranged in 5 pairs. Table I. A, for instance, gives the results of the observations taken with an arc having a frequency of 83·3, the light being taken from the centre of the true arc. Table I. B gives similar results of observations

for the same arc, the light being taken from the centre of the lower crater. By holding a magnet at the back of the arc, noticing which way the instantaneous image of the true arc was projected by the magnet, and noting the pole of the magnet presented to the arc, it was possible to determine when the lower carbon was positive and when it was negative; and the diagrams corresponding to the above 10 tables are marked so as to show the half of the wave when the lower carbon is positive and when it is negative, in all those diagrams which refer to the light coming from the crater. The results of the observations given in the 10 tables are delineated graphically in diagram in figs. 3 to 12; and on referring to these diagrams a periodic line will be seen in each, delineating the variation of the potential difference of the carbons, and another periodic line indicating the variation of the current through the arc, whilst a third line indicates the periodic variation of the luminous intensity of the selected portion of the arc. In the diagrams figs. 13 to 17 are given curves representing the periodic variation of the light from the crater in the various cases, and extending over several periods. It will be noticed that those diagrams representing the periodic variation of the light from the arc show that this light undergoes a regular fluctuation between a maximum and a minimum, the maxima having equal values. The light in the centre of the true arc never falls quite to zero. This seems to be due to a little luminosity which hangs in the interspace between the carbons, but at the present moment it is difficult to say whether this persistence is due to a very small admixture of stray light (although every effort was made to keep this out), or to a persistence of the illuminating-power of the incandescent vapour in which the arc has been formed. On examining the true arc during its complete periodic variation, it is found that the blue or purple strip of light forming the true arc undergoes a periodic variation in intensity. As far as the eye can judge, the blue or purple light completely vanishes at a certain instant during the phase; but there is, outside the true arc, a dim halo of golden light which is persistent; and it is therefore probably on account of this persistent aureole of faint light

round the true arc that the ordinates of the curve representing the periodic variation of the luminous intensity of the arc never become zero, but always indicate the outstanding constant amount of light. On the other hand, in the diagrams which represent the periodic variation of light coming from the centre of the crater of the lower carbon, we find the luminous intensity of the crater varies between a minimum value and two maximum values of different magnitude. During the time when the crater is positive it reaches a higher maximum intensity of illuminating power than during the time when it is negative, and, moreover, the curve representing the periodic variation of light rises more steeply than it comes down, which indicates a slow cooling of the carbons after they have been heated; in other words, they heat more quickly than they cool. This is particularly noticeable in that part of curve corresponding to the crater being negative; and it is only what would be expected, because after the carbon has reached its negative maximum and is beginning to cool, the opposite carbon is cooling from a condition in which it has been positive, and as it has been heated to a higher temperature than the negative carbon, it must assist by its radiation in keeping up the temperature and retard the cooling of the negative carbon. These diagrams will also show many other interesting facts: they show, for instance, that in the case of the long arc the self-induction of the arc is more marked than when the short arc is employed, and in that case there is a very distinct lag of current behind the potential difference.

It may also be noted in comparing the diagrams V. B and IV. B in figs. 12 and 10, which represent the curves for arcs of the same frequency but of lengths 1.2 centim. and .32 centim. respectively, that in the case of the long arc there is no lag of light behind power as far as regards the points of maximum when the carbon is positive, but in the case of the short arc there is a sensible lag. This might be expected to be the case, because for a long arc the influence of the opposite carbon in keeping up the temperature of its neighbour as this last is cooling is less felt than in the case of a short arc.

Broadly speaking, the facts may be summed up as follows :— The purple light of the true arc undergoes a periodic variation, and, as far as the eye can judge, is completely extinguished during a certain interval during the phase ; it has equal maxima values during the period, at instants slightly lagging behind the instants of maximum power-expenditure in the arc. On the other hand, the illuminating-power of the carbon crater varies between a minimum value and two maxima of unequal values ; the greatest maximum occurring when the carbon is positive and at an instant slightly lagging behind the instant of maximum power-expenditure in the arc. A series of curves are also given (see figs. 18, 19, and 20) in which the periodic variation of the current, potential difference, power, and apparent resistance of the arc for various powers and frequencies are represented ; and it will be seen from these curves that the resistance of the arc, including in this any counter electromotive force which may exist, varies periodically, the resistance being a minimum when the current is a maximum, and *vice versa*.

TABLE I. A.

*Observations on the Periodic Variation of the Intensity of the
Light of an Alternating Current Arc.*

Light from centre of Arc. Short Arc.

Frequency = 83·3 ~
 Length of arc = 0·55 cm.
 Potential-difference (P.D.) of carbons } = 39 volts.
 (mean square value) }
 Current (mean square value) = 14 amperes.
 Power expended in arc = 546 watts.
 Pressure at alternator terminals . . . = 104·5 volts.

Angle of Phase.	Current through Arc (instantaneous value).	P.D. of Carbons (instantaneous value).	Angle of Phase.	Intensity of Light from Arc.
0			26	38
20			38	23
40	+ 5·6	+20·2	58	16
60	+ 0·7	+ 0·2	76	6
80	- 3·7	-24·2	80	16
100	- 9·7	-34·5	102	24
120	-16·2	-38·0	132	47
140	-20·3	-43·0	146	60
160	-21·0	-44·8	156	56
180	-18·4	-43·5	168	68
200	-13·0	-32·5	176	57
220	- 6·2	-19·2	196	48
240	- 0·4	+ 1·9	202	31
260	+ 4·2	+25·5	242	11
280	+10·2	+38·1	252	19
300	+16·3	+40·8	274	12
320	+20·0	+46·5	304	50
340	+20·4	+48·3	320	58
360	+18·4	+50·5	326	55
			340	65
			346	63
			356	68

TABLE I. B.

*Observations on the Periodic Variation of the Intensity of the
Light of an Alternating Current Arc.*

Light from Crater. Short Arc.

Frequency = 83.3 ~.
 Length of arc = 0.42 cm.
 Potential-difference (P.D.) of carbons }
 (mean square value) } = 36 volts.
 Current (mean square value) = 14 amperes.
 Power expended in arc = 504 watts.
 Pressure at alternator terminals . . . = 98 volts.

Angle of Phase.	Current through Arc (instantaneous value).	P.D. of Carbons (instantaneous value).	Angle of Phase.	Intensity of Light from Crater of lower Carbon.
0	+16.5	+42	12	60
20	+10.2	+25.3	33	34.5
40	+ 3.1	+10.2	53	28
60	- 1.1	-14.2	74	31
80	- 5.4	-27.7	95	24
100	-13.4	-32.4	116	44
120	-18.7	-38.4	137	71
140	-20.4	-48.9	157	101
160	-19.8	-52.3	178	148
180	-15	-48.4	200	91
200	- 8.8	-31	205	93
220	- 3.3	-11	226	61
240	+ 1.5	+11.4	247	42
260	+ 6.3	+26.4	268	31.5
280	+13.0	+34.9	289	41
300	+18.7	+39.4	309	56
320	+21.5	+44.4	330	65
340	+21.2	+45.8	351	55
360	+16.5		119	163
160	-19.8	-49		
140	-20.3	-48		

TABLE II. A.

*Observations on the Periodic Variation of the Intensity of the
Light of an Alternating Current Arc.*

Light from centre of Arc. Medium Arc.

Frequency = 50 ~.

Length of arc = 0.63 cm.

Potential-difference (P.D.) of carbons } = 39 volts.
(mean square value)

Current (mean square value) = 14 amperes.

Power expended in arc = 546 watts.

Pressure at alternator terminals = 99 volts.

Angle of Phase.	Current through Arc (instantaneous value.)	P.D. of Carbons (instantaneous value).	Angle of Phase.	Intensity of Light from centre of Arc.	
0	+19.8	+44	10	44	
20	+14.1	+38.5	32	13	Arc out.
40	+ 6.3	+27	55	11	" "
60	+ 2.7	+10.5	77	18	
80	- 2.2	-16	100	17	
100	- 6.3	-36	122	45	
120	-13.5	-37.5	144	74	
140	-18.3	-47	165	68	
160	-19.8	-54	187	52.5	
180	-19.5	-54	208	32	
200	-14.8	-50	230	8.5	
220	- 7.9	-34	252	9	
240	- 1.9	-11.2	273	11	Arc out.
260	+ 1.9	+13	294	26	" "
280	+ 7.2	+31	315	(71)?	
300	+12.6	+36	336	68	
320	+19.5	+40	356	42.5	
340	+21.7	+47.3	348	68	
360	+19.8	+46.3	327	45	

N.B.—Owing to an accidental shift of the brushes, in this table the angle corresponding to the instantaneous light is not the same as that corresponding to the current. The position of the curve of instantaneous light with regard to the E.M.F., current, and power curves cannot therefore in this case be determined.

TABLE II. B.

*Observations on the Periodic Variation of the Intensity of the
Light of an Alternating Current Arc.*

Light from Crater. Medium Arc.

Frequency = 50 ~.
Length of arc = 0.63 cm.
Potential-difference of carbons (mean
square value) = 38 volts.
Current (mean square value) = 14 amperes.
Power expended in arc = 532 watts.
Pressure at alternator terminals . . . = 96 volts.

Angle of Phase.	Current through Arc (instantaneous value).	P.D. of Carbons (instantaneous value).	Angle of Phase.	Intensity of Light from Crater of lower Carbon.
			0	56
			20	43
30	+ 6.3	+17	40	45
50	- 0.5	- 2.5	60	32
70	- 3.9	-22	80	39
			100	52
			120	137
			140	277
			160	277
			180	227
190	-11.3		200	165
210	- 5.5	-22	220	63
230	+ 0.1	- 1	240	64
250	+ 4.2	+16	260	61
270	+11	+28.4	280	78
			300	85
			320	72
			340	87
			360	(37)?
			340	69
			320	75

TABLE III. A.

*Observations on the Periodic Variation of the Intensity of the
Light of an Alternating Current Arc.*

Light from centre of Arc. Medium Arc.

Frequency	= 26 ~.
Length of arc	= 0.63 cm.
Potential-difference (P.D.) of carbons (mean square value)	} = 37 volts.
Current (mean square value)	= 14 amperes.
Power expended in arc	= 518 watts.
Pressure at alternator terminals	= 62 volts.

Angle of Phase.	Current through Arc (instanta- neous value).	P.D. of Carbons (instanta- neous value).	Angle of Phase.	Intensity of Light from Centre of Arc.	
0	+19	+48	7	45	
20	+12.5	+39.5	28	26	
40	+ 5.8	+21.5	49	6	Arc out.
60	+ 0.5	+ 2.5	71	12	" "
80	- 2.2	-22.5	92	7	" "
100	- 9.7	-24	114	14	Arc just starting.
120	-16.8	-34.5	136	67	
140	-19.5	-45.5	158	71	
160	-21.8	-47.5	180	66	
180	-19.3	-50	202	24	
200	-11.5	-42	224	19.5	Arc out.
220	- 4.8	-22	246	4	" "
240	- 0.5	- 2	268	8	" "
260	+ 2.0	+17	290	22	
280	+10.8	+26	312	40	
300	+15.8	+36	333	57	
320	+20.7	+47.5	354	75	
340	+21.3	+50	15	58	
360	+19	+48			
340	+21	+51	345	62	
320	+20	+48	323	70	

TABLE III. B.

*Observations on the Periodic Variation of the Intensity of the
Light of an Alternating Current Arc.*

Light from Crater. Medium Arc.

Frequency = 26 ~.
 Length of arc = 0.63 cm.
 Potential-difference (P.D.) of carbons } = 39 volts.
 (mean square value) }
 Current (mean square value) . . . = 14 amperes.
 Power expended in arc = 546 watts.
 Pressure at alternator terminals . . = 62 volts.

Angle of Phase.	Current through Arc (instantaneous value).	P.D. of Carbons (instantaneous value.)	Intensity of Light from Crater of lower Carbon.
0	+18	+52	64
20	+14.3	+55	77
40	+ 6.7	+24.5	46
60	+ 1	+ 4.5	42
80	- 1.2	-21	43
100	- 5	-27	52
120	-17.5	-37	64
140	-18.2	-47	145
160	-20.6	-52	285
180	-20	-54	310
200	-11	-47	260
220	- 5	-25.5	200
240	- 1	- 5.5	119
260	+ 2	+15	87
280	+ 8.5	+30	50
300	+14	+40	104
320	+19	+45	110
340	+22	+53	130
360	+21		120

TABLE IV. A.

*Observations on the Periodic Variation of the Intensity of the
Light of an Alternating Current Arc.*

Light from centre of Arc. Short Arc.

Frequency = 50 ~
 Length of arc = 0.32 cm.
 Potential-difference (P.D.) of carbons } = 36 volts.
 (mean square value) }
 Current (mean square value) . . . = 14 amperes
 Power expended in arc = 504 watts.
 Pressure at alternator terminals . . = 85 volts.

Angle of Phase.	Current through Arc (instantaneous value).	P.D. of Carbons (instantaneous value).	Angle of Phase.	Intensity of Light from Centre of Arc.	
0	+18.4	+41.4	12	28	
20	+13.8	+38.4	34	15	
40	+ 5.7	+21	56	24	Arc going out.
60	+ 0.6	- 8.6	78	13	Arc out.
80	- 6	-22	100	14	Arc out.
100	-13.1	-25	122	18	
120	-19	-43	144	28	
140	-19	-46.5	166	20	
160	-18.3	-49	188	26	
180	-14.4	-46	210	17	
200	-10.2	-44.6	232	15	Arc going out.
220	- 5.0	-20	253	6	Arc out.
240	+ 1.1	+ 6.8	274	4	Arc out.
260	+ 8.6	+22.6	296	16	
280	+14.5	+29	317	26	
300	+18.9	+37.6	338	23	
320	+22.1	+40.6	360	30	
340	+20.3	+44	350	29	
360	+18.1	+42.6	328	23	
340	+21.5	+45.4			
320	+21.1	+40.4			

TABLE IV. B.

*Observations on the Periodic Variation of the Intensity of the
Light of an Alternating Current Arc.*

Light from Crater. Short Arc.

Frequency = 50 ~
 Length of arc = 0.32 cm.
 Potential-difference of carbons (mean
 square value) } = 38 volts.
 Current (mean square value) . . . = 14 amperes.
 Power expended in arc = 532 watts.
 Pressure at alternator terminals . . = 93 volts.

Angle of Phase.	Current through Arc (instantaneous value).	Angle of Phase.	Intensity of Light from Crater of lower Carbon.
0			
20	+13.1	20	61
40	+ 7.2	41	56
60	+ 0.8	63	45
80	- 5.0	84	58
100		105	51
120		126	101
140		148	220
160		170	227
180		191	216
200		212	111
220	- 6.3	232	64
240	- 0.4	253	73
260	+ 4.6	273	66
280	+12.6	294	63
300	+17.6	315	77
320	+19.0	335	60
340		359	63
360		338	78
		317	88
		36	79

TABLE V. A.

*Observations on the Periodic Variation of the Intensity of the
Light of an Alternating Current Arc.*

Light from centre of Arc. Long Arc.

Frequency = 50 ~.
 Length of arc = 1.2 cm.
 Potential-difference (P.D.) of carbons }
 (mean square value) } = 53 volts
 Current (mean square value) . . . = 14 amperes.
 Power expended in arc = 742 watts.
 Pressure at alternator terminals . . . = 88 volts.

Angle of Phase.	Current through Arc (instantaneous value.)	P.D. of Carbons (instantaneous value.)	Angle of Phase.	Intensity of Light from Centre of Arc.	
0	+20.4	+58.6	10	40	
20	+15.9	+48.6	31	44	
40	+ 8.4	+26.4	52	9	
60	+ 0.7	- 7.6	76	12	Arc out.
80	- 3.6	-46	98	19	" "
100	- 8.1	-53.6	120	24	
120	-15.1	-63.6	142	72	
140	-18.6	-67.6	164	50	
160	-19.6	-68	187	63	
180	-17.6	-67	209	44	
200	-12.6	-53.6	232	12	Arc out.
220	- 5.6	-21	253	7	" "
240	- 0.6	+24	275	7	" "
260	+ 4.4	+47	297	31	
280	+ 8.4	+53	320	62	
300	+17.4	+55	343	87	
320	+21.4	+59.6	348	63	
340	+23.4	+59.6	326	78	
360	+20.4	+55.6			
340	+22.4	+59.6			
320	+21.4	+58.6			

TABLE V. B.

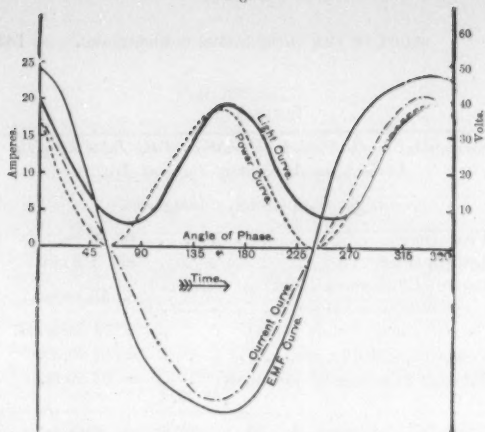
*Observations on the Periodic Variation of the Intensity of the
Light of an Alternating Current Arc.*

Light from Crater. Long Arc.

Frequency = 50 ~.
 Length of arc = 1.2 cm.
 Potential-difference (P.E.) of carbons } = 53 volts.
 (mean square value)
 Current (mean square value) = 14 amperes.
 Power expended in arc = 742 watts.
 Pressure at alternator terminals . . . = 93 volts.

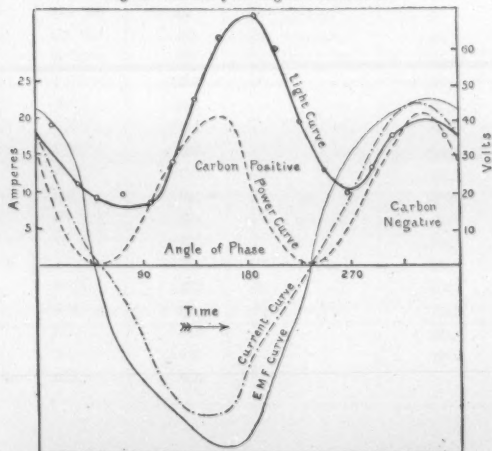
Angle of Phase.	Current through Arc (instantaneous value).	Angle of Phase.	Intensity of Light from Crater of lower Carbon.
0		359	92
20	+15.9	18	92
40	+11.7	36	98
60	+ 1.6	53	84
80	- 4.4	70	70
100		87	76
120		105	74
140		123	207
160		142	175
180		160	280
200		179	265
220	- 6.5	199	193
240	0.0	218	76
260	+ 4.4	238	84
280	+10.4	257	71
300		277	66
320		296	101
340		315	98
360		334	107
		340	120

Fig. 3.



The ordinates of the firm line marked "Light Curve" represent the periodic variation of illuminating-power of the centre of the *true arc* of an alternating-current arc. The other curves marked E.M.F., Current, and Power represent the variation of potential difference of the carbons, current, and power in the arc. Carbons, 15 mm. cored; $\sim = 83.3$; amperes = 14; volts = 39; length of arc, 0.55 cm.; volts at alternator terminals = 104. (See Table I. A.)

Fig. 4.—Intensity of Light from Crater.



Frequency = 83.3 $\sim$. Length of arc = 0.42 cm. (See Table I. B.)

Fig. 5.—Intensity of Light from Arc.

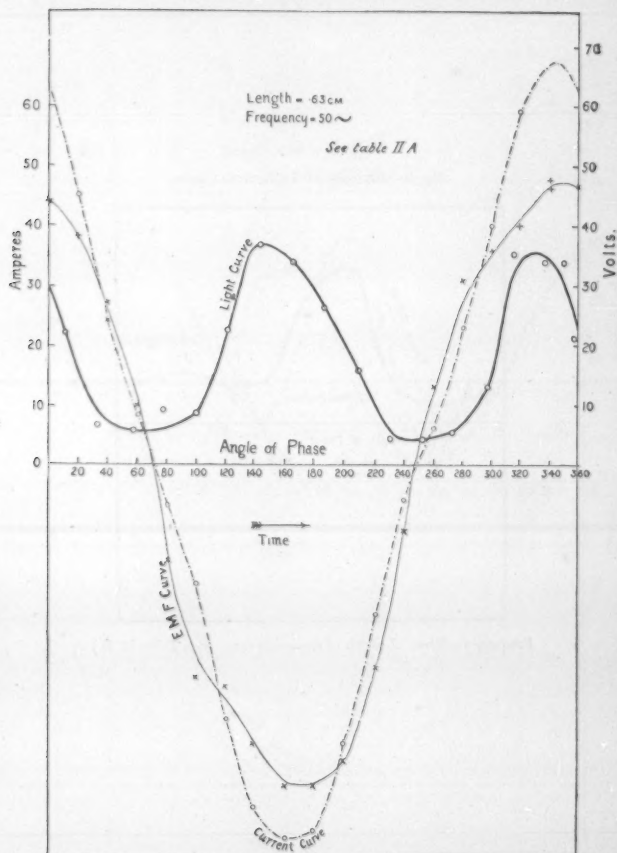
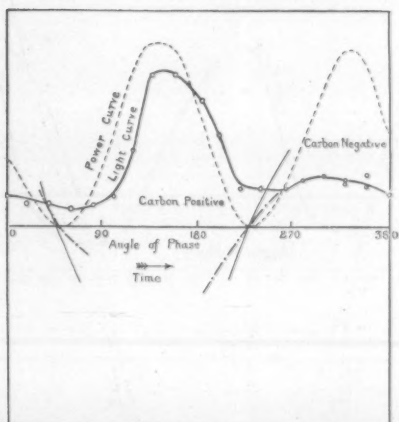


Fig. 6.—Intensity of Light from Crater.



Frequency = 50 ~. Length of arc = 0.63 cm. (See Table II. B.)

Fig. 7.—Intensity of Light from Centre of Arc.

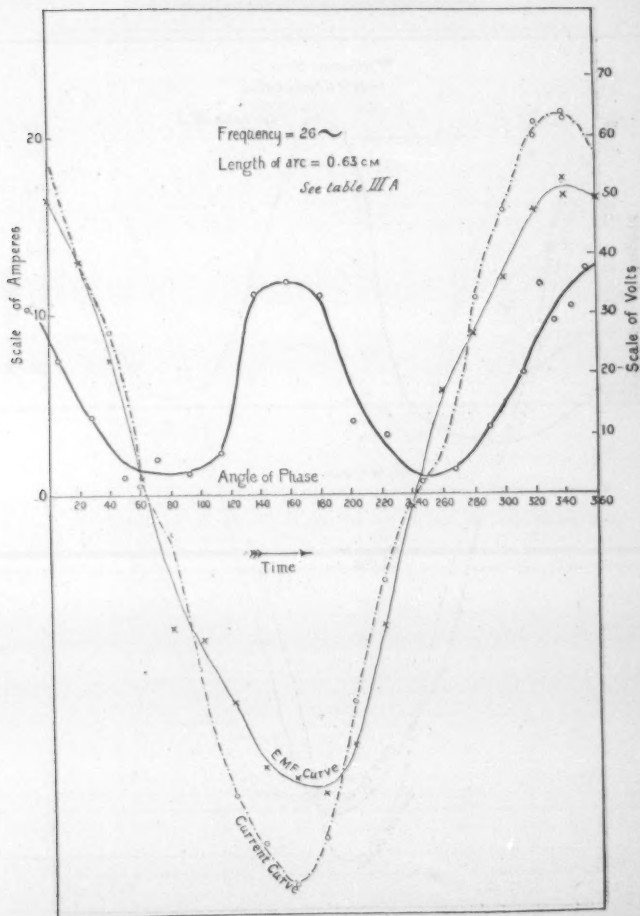


Fig. 8.—Intensity of Light from Lower Carbon.

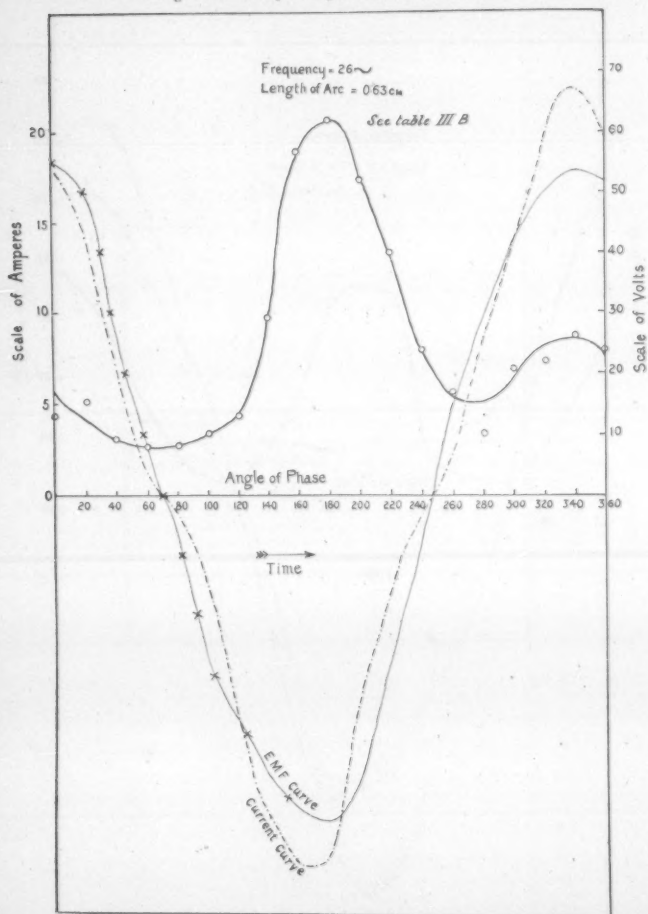


Fig. 9.—Intensity of Light from Centre of Arc.

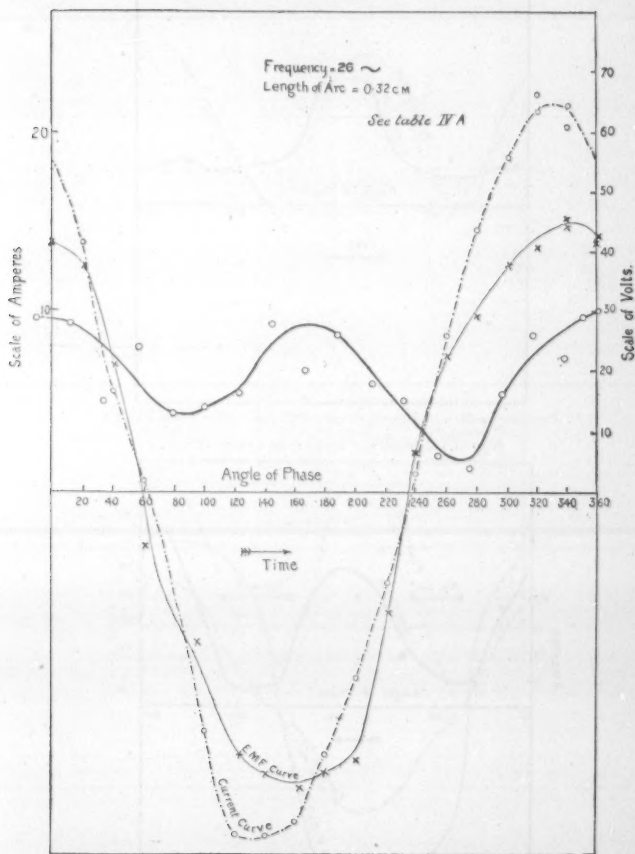
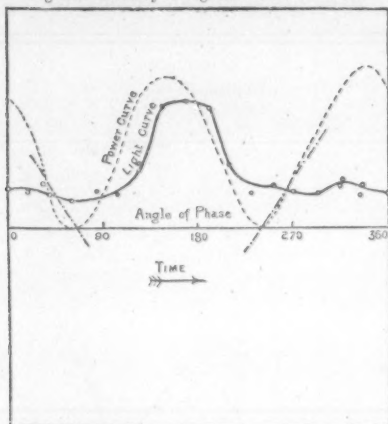
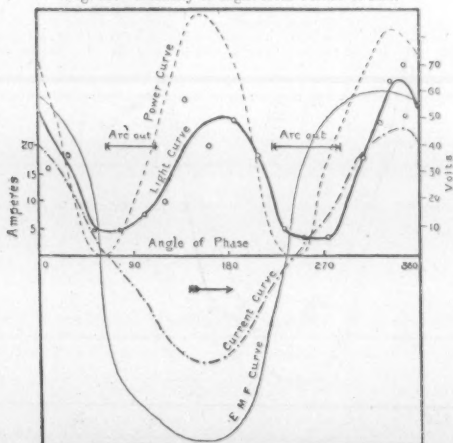


Fig. 10.—Intensity of Light from Centre of Arc.



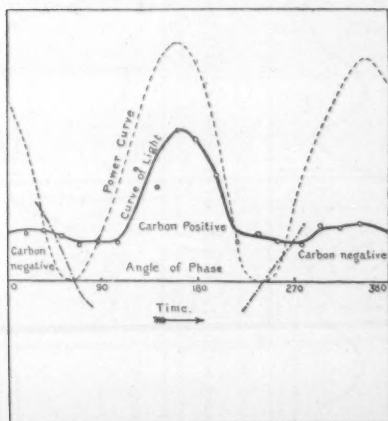
Frequency=50 ~. Length of arc=0.32 cm. (See Table IV. B.)

Fig. 11.—Intensity of Light from Centre of Arc.



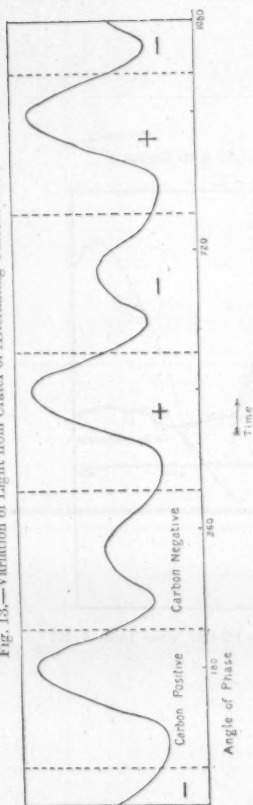
Frequency=50 ~ Length of arc=1.2 cm. (See Table V. A.)

Fig. 12.—Intensity of Light from Crater.



Frequency = 50 $\sim$. Length of arc = 1.2 cm. (See Table V.B.)

Fig. 13.—Variation of Light from Crater of Alternating Current Arc.



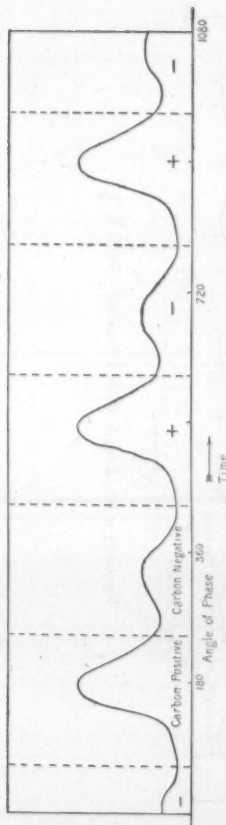
Frequency = 83.3. Length of Arc = 0.42 centim. (Table I. B.)

Fig. 14.—Variation of Light from Crater of Alternating Current Arc.



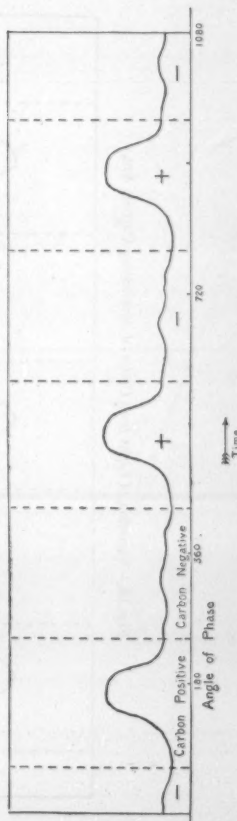
Frequency = 50. Length of Arc 0.63 centim. (Table II. B.)

Fig. 15.—Variation of Light from Crater of Alternating Current Arc.



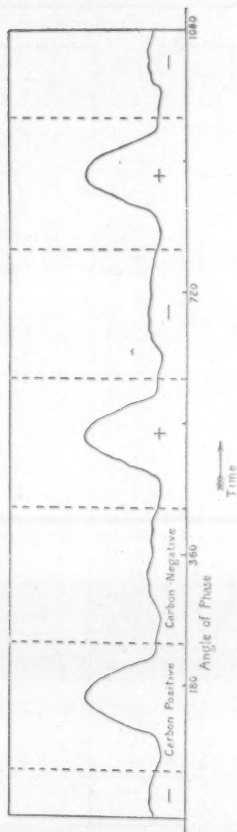
Frequency = 26. Length of Arc = 0.63. (Table III. B.)

Fig. 16.—Variation of Light from Crater of Alternating Current Arc.



Frequency = 50. Length of Arc = 0.32 centim. (Table IV. B.)

Fig. 17.—Variation of Light from Crater of Alternating Current Arc.



Frequency = 50. Length of Arc 1.2 centim. (Table V. B.)

Fig. 18.—Curves of Current, Potential Difference, Power, and Resistance of 360 Watt Alternating Current Arc.

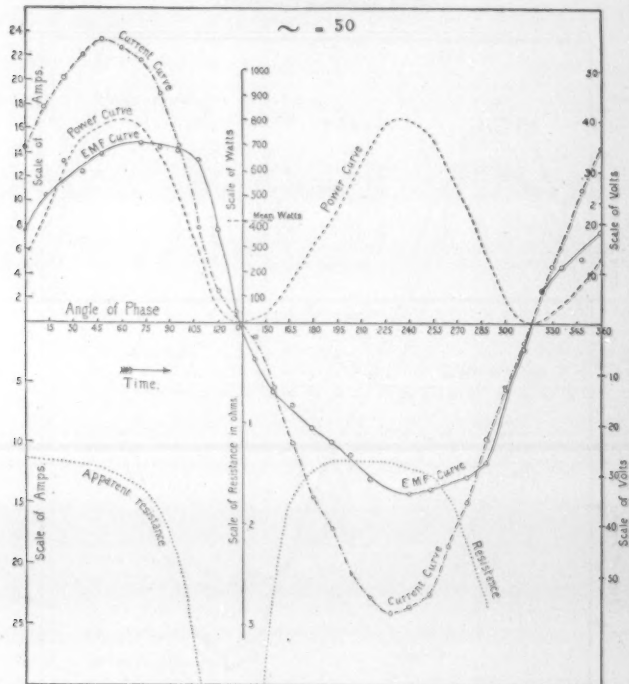


Fig. 19.—Curves of Current, Potential Difference, and Power of 500 Watt Alternating Current Arc.

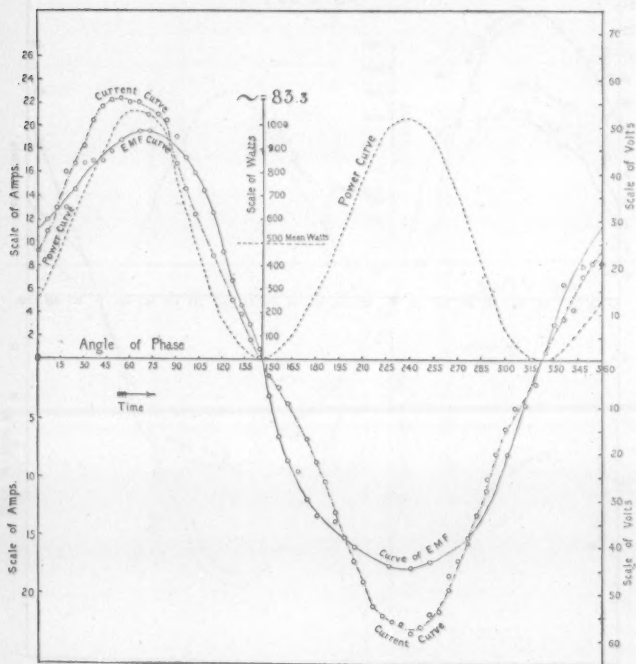
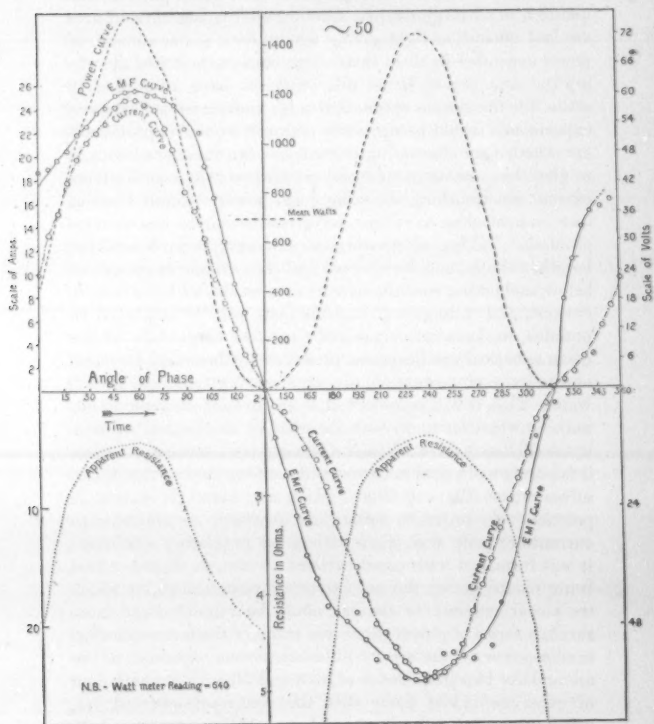


Fig. 20.—Curves of Current, Potential Difference, Power, and Resistance of 600 Watt Alternating Current Arc.



The second portion of the experimental work here recorded consisted in an investigation of the efficiency of the alternating-current arc as a light-giving agent, when compared with the continuous-current arc taking up the same mean power. The question in all its generality, whether alternating-current arcs are less efficient as light-giving agents for a given amount of power expended in them than continuous-current arcs absorbing the same power, is not one which we have attempted to settle, for the simple reason that a far more extended series of experiments would be necessary before it would be possible to say whether an alternating-current arc can or cannot be made to give the same mean spherical candle-power as a continuous-current arc absorbing the same mean power. Limitations of time compelled us to reduce our investigation to one definite problem. Taking alternating-current arcs, formed with the length, voltage, and the current and the carbons as described below, and taking continuous-current arcs also with the length, current, and voltage and carbons most usually employed in practice, we have investigated the relative magnitude of the mean spherical candle-power produced by these arcs for equal expenditure of powers in the arcs varying from 200 to 600 watts. This, it will be observed, is an investigation which has nothing whatever to do with the relative mechanical or electrical efficiency or excellence of the arc-lamp mechanisms, but it is purely a physical measurement made of the two arcs themselves under the conditions which are found to obtain in practice. In order to settle this question, an alternating-current arc lamp was taken with good regulating qualities; it was furnished with cored carbons 15 mm. in diameter, and being placed under the conditions above described, in which the power supplied to the arc could be regulated and measured, a series of observations was made of the mean spherical candle-power of the arc for different powers expended in the arc and for two frequencies of 83.3 and 50~. A similar set of experiments was made with the continuous-current arc, using in one case the same carbons—15 mm. carbons, both cored—and in the other case the positive carbon 15 mm. cored and a negative carbon 9 mm. solid. These sizes were chosen because the 15 mm. cored and 9 mm. solid sizes are those that are frequently employed in continuous-current arc lighting.

The arrangements for obtaining the mean spherical candle-power consisted of a mirror—the coefficient of reflexion of which had been determined—which was employed to reflect the light from the arc proceeding in different directions to the horizon into the photometer, the ray always falling upon the mirror at an incidence of 45° . The standard of comparison was an incandescent lamp, which was worked at rather a high temperature in order to diminish the difficulty of making the photometric comparison by diminishing as much as possible the colour-difference in the lights compared. This incandescent lamp was kept constantly standardized against another standard incandescent lamp worked at a rather lower temperature. The means of comparison was a Sugg's Star Disk Photometer. It was found that by focussing the eye to a point rather nearer the eye than the images of the Star Disk as seen in the two mirrors, the difficulty of discriminating between a small difference in the brightness of the two images in spite of a small colour-difference was to a great extent diminished. We abandoned as perfectly useless any comparison of the two lights in terms of red and green candle-power. By the employment of the reflecting wattmeter and carbon resistance as above described, it was perfectly possible to keep the power expended in the arc constant to a certain number of watts throughout long periods. In the case of the alternating-current arc lamp experiments, the alternating-current arc lamp was turned round its vertical axis so as to take a series of observations quickly in different directions, but at the same angle to the horizon, and the mean of these observations was taken as the effective illuminating-power in any angular direction; at the same time, the current through the arc and the potential-difference of the carbons were observed. These observations having been taken, involving many hundreds of photometric measurements, the results were set out in a series of photometric diagrams, as shown in figs. 21 to 24, which delineate the respective form of the photometric curves for the two arcs and for different wattages expended in the arc. These photometric diagrams were then integrated, and the mean spherical candle-power calculated in the usual way by means of a Rousseau's diagram, and finally the results embodied in one complete table and diagram, as given below (pp. 160, 161).

Fig. 21.—Photometric Curves of Continuous Current Arc.
Both carbons 15 mm. cored.

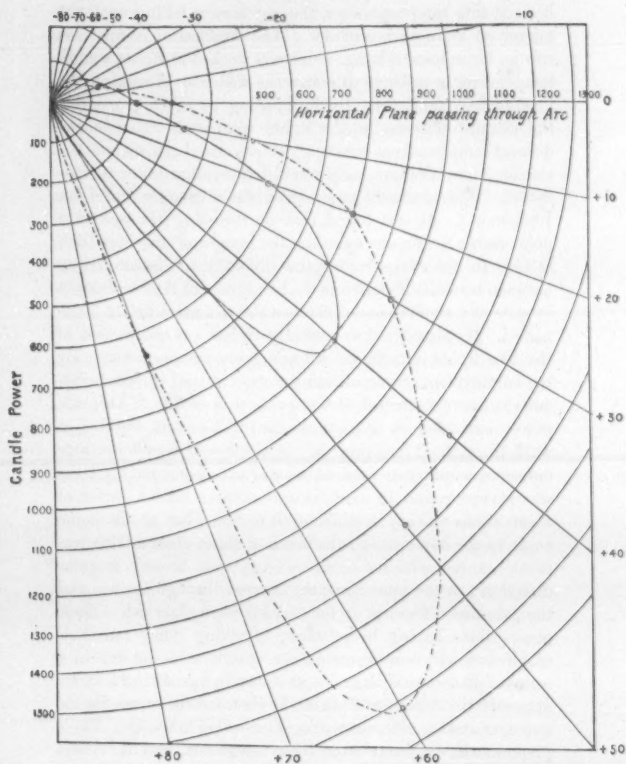


Fig. 22.—Photometric Curves of Continuous Current Arc.

Negative carbon 9 mm. uncored.

Positive carbon 15 mm. cored.

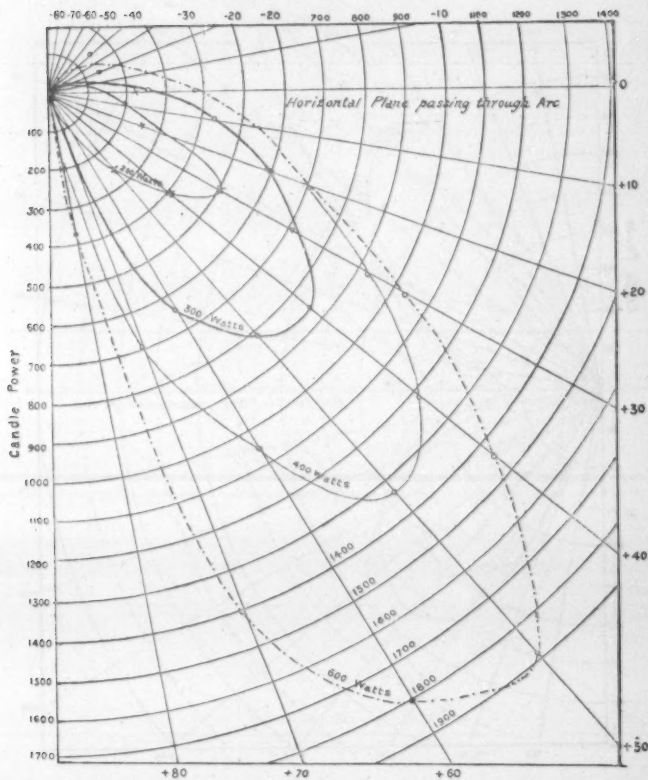


Fig. 23.--Photometric Curves of Alternating Arc Lamp ~ 83.3.

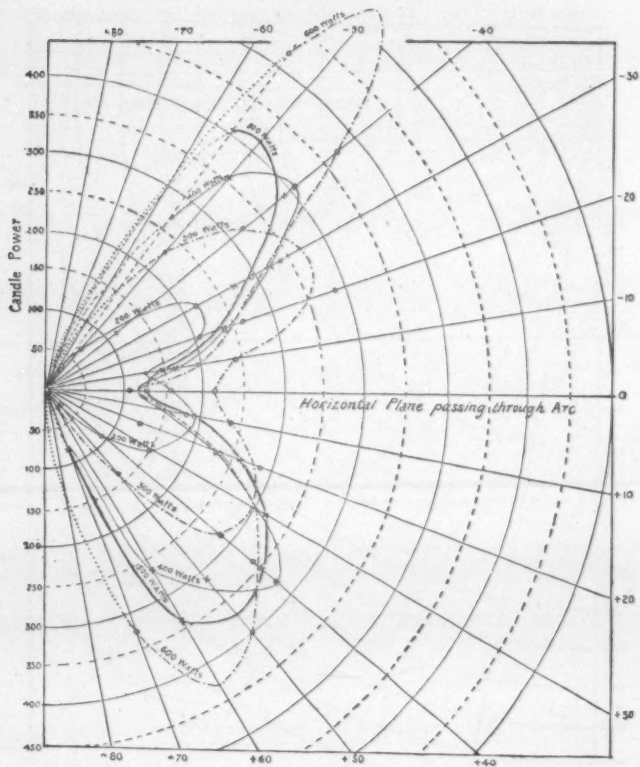
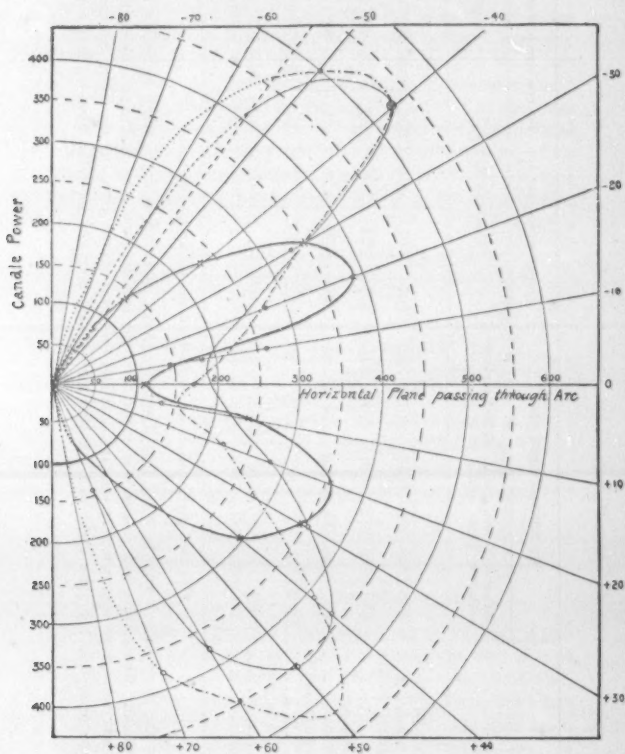


Fig. 24.—Photometric Curves of Alternating Arc Lamp ~ 50.



	Continuous-Current Arc.						Alternating-Current Arc. Both carbons 15 mm. cored.							
	+Carbon 15 mm. cored. -Carbon 9 mm. solid.			Both carbons 15 mm. cored.			Frequency 83.3 ~.							
														Frequency 50~.
Power expended in the arc in watts	582	380	292	*215	607	313	601	501	404	305	*233	596	459	308
Mean spherical candle- power	675	455	372	181	582	344	307	274	256	250	144	326	322	254
Potential difference of carbons in volts ...	56	44	36	26	60	40	37.5	34	28	21	15.3	39	31	22
Current through arc in amperes ...	10.6	8.7	8.1	8.2	10.2	8	16.2	15.1	15.1	15.5	15.4	16	15.1	14.6
Length of arc in centi- metres	0.715	1.0	0.07	0	0.807	0.061	0.628	..	0.125	0.016	0.01	0.7	..	..
Power factor	1.0	0.25	1.0	1.0	1.0	1.0	..	..	0.96	0.96	0.99	0.97	0.99	0.97

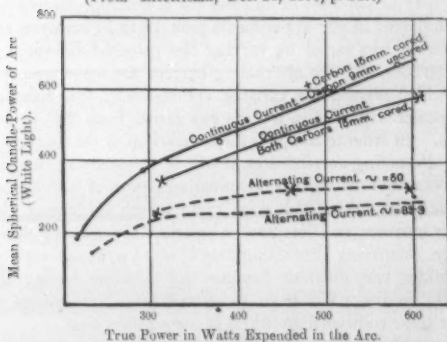
* In these observations the arc was hissing during the whole of the time.

In reading the above table it must be noted that it was the power, measured in watts, taken up in the arc which was kept constant. The current and difference of potential of the carbons were controlled so as to secure this result. The current and volts were approximately constant, and had the mean values given in the table, but the product of these mean values does not quite agree with the true mean power measured in watts taken up in the arc.

The results in the Table are graphically embodied in the diagram in fig. 25, from which it will be seen that, taking the alternating-current arc as employed, the total mean spherical candle-power is always considerably less than that of a continuous-current arc, taking the same mean power. Lowering the frequency seems to increase the efficiency of the alternating-current arc, as one might naturally assume it would do, and it is obvious that increasing the diameter of the lower carbon of the continuous-current arc would diminish its total mean spherical candle-power at any given wattage. This table and diagram therefore, we think, settle the question that for a given expenditure of power in the arc a greater

Fig. 25.

(From 'Electrician,' Dec. 20, 1895, p. 247.)



amount of mean spherical illumination can be obtained, if that power is supplied in a continuous-current form than if it is supplied in an alternating form. It has been suggested that, with proper carbons and under proper conditions, the alternating and continuous-current arc should give the same mean spherical candle-power for the same expenditure of power, as has been shown to hold true in the case of the incandescent lamp by the investigations of Profs. Ayrton and Perry. We think, however, that no *à priori* reasoning can apply in the case of the arc. It is perfectly clear that, owing to the

interval of cooling that elapses, the mean temperature of the two carbons in the case of the alternating-current arc must be less than the temperature of the positive crater of the continuous-current arc, and that therefore the result we have obtained is only what might be expected. If the question is asked, how do we account for the difference in efficiency? the answer must be that the energy absorbed in the case of the alternating-current arc is radiated at a lower temperature, and the two arcs were therefore exactly in the same condition as regards comparison that two incandescent lamps would be, both taking the same total power but worked at different temperatures, and therefore different watts per candle, and therefore giving different mean spherical candle-powers with the same total power expended in them.

It may be observed in all cases the alternating-current arcs we employed in our experiments took 16 to 17 amperes, and the power was varied by varying the potential-difference of the carbons, and the alternating-current arc lamp used was one which effected this variation automatically, even although the power expended in the arc was varied from 200 to 600 watts. In order to complete the comparison of the continuous and alternating-current arcs, it will be necessary to compare the behaviour as regards illuminating-power of alternating-current arcs, taking the same mean power but formed with larger currents and less carbon potential-differences; that is to say, comparing alternating arcs of equal power-absorption, but taking very different currents and therefore having different lengths. We hope to extend this investigation to cover these additional questions at some future time.

The above observations have necessitated an enormous number of photometric and electrical measurements, and we have in the above work been very efficiently aided by Messrs. L. Birks, W. H. Grimsdale, A. M. Hanbury, E. N. Griffiths, and others, to whom our thanks are due.

DISCUSSION.

Prof. AYRTON said the behaviour of the alternate-current arc was of great interest, for the power supplied could not be measured by simply multiplying the current by the electromotive force, since the current lags behind the volts.

The resistance, *i. e.* the ratio of the current to the E.M.F., also lags, but the authors do not appear to have made any attempt to measure the *true* resistance. The authors were to be congratulated on the guarded tone they had adopted as to the bearing of these experiments on the question of the relative efficiency of the alternating and continuous current arcs. In a previous communication, one of the authors had stated that the alternating-current arc must necessarily be a less efficient light-producing agent than the continuous-current arc. Although the last set of curves given in the paper might appear to support this supposition, he (Prof. Ayrton) felt that the difference obtained was probably due to the fact that the alternating-current arc was not being worked under proper conditions. The quality of the carbons and the length of the arc have a most important influence on the efficiency of an arc. At present, our knowledge is not sufficient to allow of our stating definitely whether or not an alternating-current arc can be made as efficient as a direct-current arc, but there is no doubt that it will be possible to get much better results than are at present attainable.

Prof. S. P. THOMPSON said that when the fact of the existence of the difference in phase between the current and volts in an alternating arc was first published, he made some experiments which showed that there was a lag, and not a lead, *i. e.*, that the arc acted as if it possessed self-induction. The resistance also lagged, and he thought this lag might be due to a thermal lag. The temperature of the arc will lag behind current, both when it is increasing and when it is decreasing; and if the resistance of the arc depends on the temperature of the vapour in the arc, then the resistance would also lag behind the current. It was not possible from *a priori* reasoning to say whether or no an alternating-current arc could ever be obtained of an efficiency equal to that of the direct-current arc. With suitable carbons, length of arc, current, and volts, it seemed to him that it might be possible to obtain an equal efficiency. The light-giving process in an arc is not merely an irreversible degradation of electric energy into heat; for the difference of potential between the carbons may be written $V = a + Cbl$; where a may be regarded

as a back electromotive force, and bl as a true resistance. The first term of this expression does not vary with the length of the arc (l), but the second term does. Multiplying through by the current (C), the equation

$$\text{Watts expended} = Ca + C^2bl$$

is obtained. It is the first of the terms on the right-hand side, which is a reversible effect, and corresponds to the power expended in driving the current against a back electromotive force, on which the light given out chiefly depends, due to something occurring at the crater surface.

Mr. BLAKESLEY asked whether Prof. Thompson's idea of the existence of a reversible part in the process was not a strong argument in favour of the direct-current arc.

Prof. AYRTON said that in two communications made to the Congress held at Chicago it was shown that even with direct-current arcs there was a certain length of arc for which the efficiency was a maximum. Mrs. Ayrton had quite recently found that the efficiency of arc-lamp carbons altered with time. Prof. Thompson's suggestion as to a thermic lag was a valuable one.

Prof. FLEMING, in his reply, said that when comparing two agents where there were so many variables, it was practically necessary to restrict the investigation. In their case they had kept the mean power constant and had left the other variables to take care of themselves.

XII. *On some Experiments with Incandescent Lamps.*

*By Sir D. SALOMONS.**

[Abstract.]

A LARGE electro-magnet is excited by means of a continuous current, while an alternating current is passed through an incandescent lamp. On bringing the lamp near the magnet the filament is set in vibration, which, if the lamp is brought sufficiently near the magnet, is sufficiently intense to break the filament. The number and position of the nodes formed

* Read February 28, 1896.

in the vibrating filament are found to be independent of the natural period of the filament, but depend on the frequency of the alternating current.

DISCUSSION.

Prof. S. P. THOMPSON asked whether it was not found that the number of segments into which the filament is divided depend to some extent on the natural period of the filament.

Prof. AYRTON said that the magnetic leakage was very large with the arrangement adopted, and he would like to know whether this method was a more sensitive one for mapping out the field than those ordinarily employed. In an instrument designed by Prof. Perry and himself, an alternating current was passed through a wire stretched between the poles of a magnet, and the tension of the wire was altered till the vibrations set up were a maximum. The natural period of the wire, and hence the frequency of the alternating current, was then deduced from the tension, &c. In Prof. Ewing's magnetic-curve tracer, on the other hand, the natural period of the stretched wire was made as different as possible from the period of the alternations which were to be observed, so that the natural vibrations of the wire did not influence the result. The author's arrangement appeared to him (Prof. Ayrton) to be intermediate between these two, and it would seem that the natural period of the filament would, to a certain extent, influence these results.

Prof. PERRY suggested that the lamp might be held in a very steady support, so that after the large vibrations due to the natural period of the filament had died out, the vibrations of the period of the alternations alone could be observed.

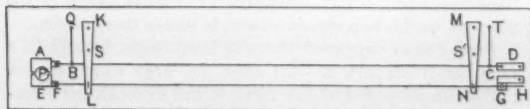
Sir D. SALOMONS, in his reply, said that the arrangement was not intended for making *measurements*. A lamp had been fixed in a steady clamp and the current passed for many hours, but the character of the vibrations remained unaltered. He had found the vibrating filament useful for microscopic work where any surface rather than a line of light was required.

XIII. *An Addition to the Wheatstone Bridge for the Determination of Low Resistances.* By J. H. REEVES, M.A.,
City and Guilds of London Central Technical College*.

WHILE it can be assumed with fair certainty that even in moderately equipped laboratories there will be found one sensitive galvanometer and a good set of resistance-coils arranged in the form of a Wheatstone bridge, it is by no means so common to find a convenient method for determining low resistances, such as, for example, a Kelvin bridge. The piece of apparatus which forms the subject of this paper is a comparatively cheap addition to the ordinary bridge, and enables the resistances of exact metre-lengths of even thick wires to be directly measured with almost as much ease as larger resistances can be determined with the ordinary Wheatstone bridge. The method also possesses the advantage that all the measurements are made in terms of a standard wire with fixed contacts, and, therefore, not subject to the wear which accompanies the frequent use of a slider; further, in the case of copper wires no temperature-correction is needed.

The apparatus is represented in fig. 1. On a mahogany

Fig. 1.



baseboard are stretched close to one another two wires—one, ABCD, being the standard of comparison, while the other, EFGH, is the wire to be tested.

A and E are two massive pieces of brass which can be joined together by a plug P. D and H are smaller pieces of brass with binding-screws attached.

The standard BC is permanently fixed to A and D, whilst

two clamps F and G fixed respectively to E and H form the means of fixing in its place the wire to be tested.

KL and NM are two brass springs which pass over but do not touch BC. They are provided with binding-screws at K and M, and with two knife-edges L, N exactly one metre apart, which press on the wire FG when in position, while two screws S, S' will raise the springs when required for inserting or removing this wire.

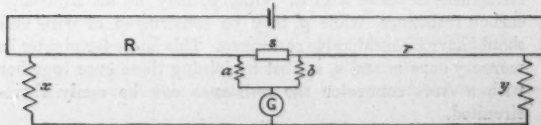
At points B, C in the standard are soldered two short lengths of wire terminating in binding-screws Q and T. The gauge of the standard wire, which is of copper, and the distance apart of B and C are so chosen that the resistance between B and C is 0.01 ohm at the temperature of the wire at the time of its final adjustment, which should be noted. The method of determining these points is given later on.

The arrangement is a variation of the Kelvin bridge, with this difference: that in the latter the measurements are made by varying the length between the knife-edges of sliders which press on the standards, of which several are required: whilst in the former only one standard resistance is employed, and the measurements are effected by the alteration of the other resistances in the arrangement. If the whole apparatus had to be bought, the Kelvin bridge would be the cheaper, but the new Addition utilizes the box of coils, which as before-mentioned is sure to be available, and only requires in addition one standard to be made and adjusted.

The theory of the Kelvin bridge is to be found in the text-books. In Gray's 'Absolute Measurements,' for example, vol. i. p. 359, it is proved that if fig. 2 represents the usual arrangement, and, if R, r, x, y, a, b, s are as marked, the following relationship must hold in order to obtain balance:—

$$\frac{R}{r} = \frac{x}{y} = \frac{a}{b}.$$

Fig. 2.



Further, it is shown that when s is small compared with a and b the accuracy of the equation

$$\frac{R}{r} = \frac{x}{y}$$

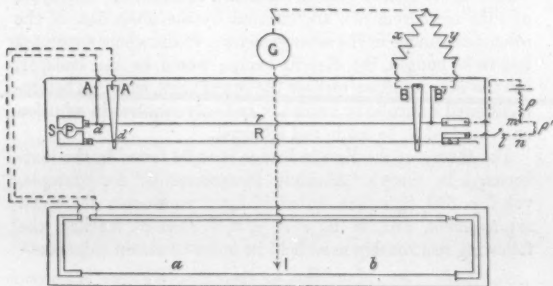
is not affected by a small want of equality between the ratios

$$\frac{a}{b} \text{ and } \frac{x}{y}.$$

In addition to the galvanometer and set of coils there is required a slide-wire bridge, which may be of a rough description but should have a resistance of an ohm or so. At the end of the paper it is shown how this latter and the Addition can be united in one.

The whole arrangement is joined together as shown in fig. 3, the dotted lines indicating the temporary connexions. By comparing this with figs. 1 and 2, no detailed description is necessary, as R, r, x, y, a, b, s are lettered to correspond.

Fig. 3.



The battery may with advantage be a storage-cell with two resistances in series with it. One, ρ , may be an adjustable carbon resistance, while ρ' may be constructed of wire and should have a considerable resistance. This latter terminates in mercury cups m and n , so that by joining these cups together with a short connector the resistance can be easily short-circuited.

The resistances x and y are resistances unplugged from the box of coils forming the ordinary Wheatstone bridge; y is the 1000-ohm coil of one of the ratio arms, and x is in the adjustable arm. If such a set of coils be not available, y may be any 1000-ohm coil, and x any box of coils containing resistances up to 5000 ohms. The two leads joining x and y to B and B' respectively should be as stout and short as possible. The resistances from d , d' to I form the $a b$ of fig. 2.

To make a measurement, insert the wire whose resistance is required in its proper clamps, taking care that it lies quite straight between them. Its diameter and material being known, the resistance of one metre of it can be approximately calculated. Calling the resistance R_1 , choose x_1 in the box so that

$$\frac{x_1}{1000} = \frac{R_1}{0.01}.$$

Remove the plug P, and, including ρ' in the battery circuit, obtain balance by shifting the slider I, the galvanometer being also shunted if necessary. The arrangement is now an ordinary Wheatstone bridge,

$$\therefore \frac{a + R_1}{b + r} = \frac{x_1}{y} = \frac{R_1}{r},$$

or

$$\frac{a}{b} = \frac{R_1}{r} = \frac{R}{r} \text{ approximately.}$$

Next insert the plug P, short-circuit ρ' so that now a strong current passes when the battery circuit is closed, and obtain balance by altering x . Let the new value be x_2 . Then, since approximately $\frac{a}{b} = \frac{R}{r}$, we have, by the formula quoted above,

$$\frac{R}{r} = \frac{x_2}{y},$$

i. e.,

$$R = \frac{x_2}{y} \times r \text{ very approximately.}$$

In any case this new value of R is very much nearer the

true value than the approximate value R_1 . If the value of x has to be but little altered, it may be taken as the true value of R , but if, on the other hand, the first estimate was considerably in error, and, in consequence, a large alteration has to be made in x in order to obtain balance, the much closer value thus obtained must be taken as the first approximation, and both experiments repeated.

Now, since the wires are so close to one another, their temperature can be called the same, and, if the tested wire be of copper, their temperature-corrections are the same. Therefore, if R be a certain fraction of r at one temperature, it will be the same fraction at any other. But in the actual bridge on which the experiments described later on were first made, this temperature was $17^{\circ}\cdot7$ C. Therefore, substituting numerical values in the last equation,

$$R = \frac{x}{1000} \times \cdot 01 = \frac{x}{100,000} \text{ ohms at } 17^{\circ}\cdot7 \text{ C.}$$

Of course, if R be not made of copper, the temperature must be noted and allowed for.

The accuracy and sensibility of this method depend on three conditions:—

- (1) The accuracy of the ratio $\frac{x}{y}$;
- (2) The accuracy of the value of r ;
- (3) The sensibility of the galvanometer.

As far as condition (1) is concerned, x and y being taken from a good box have an error of certainly less than 0·1 per cent.

Condition (2) is discussed later, and the effect of condition (3) can be best seen by reference to tests made in the laboratory of the Central Technical College with the apparatus arranged as in fig. 3, except that, at the start, the battery consisted of a single Daniell cell, the galvanometer being a four-coil reflecting one having a resistance of 687 ohms and a sensibility of about 400 scale-divisions per microampere.

TABLE I.

Tests made on high conductivity copper, $R = \frac{x}{100,000}$ ohms.

No. of wire. S.W.G.	Diameter. mils.	x .	Observation.	R.	Specific Gravity.	Specific Resistance.	Conductivity. Matthiessen's Standard.
22	28.07	4324	$\frac{1}{2}$ div. deflexion left.	0.04325	8.841	1.616	100.3
		4326	Small " right.				
		4328	$1\frac{1}{2}$ div. " right.				
20	35.5	2618	1 " left.	0.02621	8.918	1.572	103.1
		2620	Small " left.				
		2622	$\frac{1}{2}$ div. " right.				
18	48.8	1421	$\frac{1}{2}$ " left.	0.014215	8.912	1.602	101.1
		1422	$\frac{1}{2}$ " right.				
		802	3 " left.				
16	64.2	803	$\frac{1}{2}$ " left.	0.008032	8.907	1.572	103.1
		804	2 " right.				
		515	$2\frac{1}{2}$ " left.				
14	80.1	516	Small " left.	0.005163	8.888	1.566	103.4
		517	1 div. " right.				
		314	1 " left.				
12	103.2	315	1 " right.	0.003145	8.904	1.585	102.2
		291	1 " left.				
		292	$\frac{1}{2}$ " right.				
—	106.9	308	$1\frac{1}{2}$ " left.	0.003086	8.870	1.588	102.4
		309	1 " right.				
		309	2 " left.				
—	104.2	310	$\frac{1}{2}$ " right.	0.003098	8.891	1.580	102.6

Matthiessen's standard has been taken to be a specific resistance (resistance per cubic centimetre at 0° C.) of 1.620 International Microhms.

Towards the end of this table it will be seen that even a sensitive galvanometer barely allowed an accuracy of 0.1 per cent. The Daniell cell was therefore removed, and a small accumulator with a resistance of 2 ohms in series with it substituted. The last three wires were then again tested, and a marked improvement was at once seen, for now an alteration of 1 ohm in x produced a difference of deflexion of 10–15 scale-divisions, thus giving the interpolated figure correct to the first place given, *i. e.*, R correct to 1 part in 3000.

To further test the capabilities of this method, a strand cable composed of 7 copper wires, each of No. 16 S.W.G., was taken, the strands being flattened to pass under the springs, which had not been constructed for a wire of such a large diameter. The results, using the same cell and resistance, are shown in the next table.

TABLE II.

x .	Observation.	R.
117	6 divs. deflexion left.	0.001183
118	$1\frac{1}{2}$ " " left.	
119	3 " " right.	

Here only $4\frac{1}{2}$ divisions correspond with a change in x of 1 ohm : thus an error in reading of $\frac{1}{2}$ a division means an error in R of 0.1 per cent. The effect of using a still larger current was next tried, the 2-ohm coil being removed, and the two resistances ρ, ρ' of fig. 3 substituted. An ammeter was also included in the battery-circuit, so that the actual current could also be measured.

Now, on passing a strong current both wires heat, and if the wire r has the smaller diameter it will heat the faster. Therefore its resistance will also increase the faster, and hence the resistance of the wire R will apparently diminish, and *vice versa*. In order to see how much this diminution might amount to, the resistance ρ' was short-circuited, and ρ was adjusted until the battery-current was 6 amperes, and the results are shown in Tables III. and IV.

TABLE III.

With the current only continued long enough to measure the deflexion : current 6 amperes.

x .	Observation.	R.
118	3 divs. deflexion right.	0.0011812
119	23 " " left.	

TABLE IV.

After the current had flowed continuously for 3 minutes:
6 amperes as before.

x.	Observation.	R.
119	28 divs. deflexion left.	0.0011784
118	4 " " left.	
117	20 " " right.	

The difference between these two results is about $\frac{1}{4}$ per cent., while the deflexions are more than sufficient for an accuracy in R of 0.1 per cent.

A few days later these experiments were repeated with a current of 2 amperes. The slight discrepancy in the value of R was due to the fact that in the interval the cable, which was not quite straight, had been removed. On replacing it a slightly less length must have been included between the knife-edges. As, however, these experiments were being made solely to find the *sensibility* of the measuring arrangement, no attempt was made to straighten it. The results are shown in Table V., experiment (a) being for the short duration of current, and (b) the result after the current had flowed continuously for 3 minutes.

TABLE V.
Current 2 amperes.

Experiment.	x.	Observation.	R.
(a)	119	16 divs. deflexion left.	0.001178
	118	3 " " left	
	117	10 " " right.	
(b)	118	4 " " left.	0.001177
	117	9 " " right.	

From the above results it will be seen, firstly, that 13

divisions correspond with 1 ohm difference in x , thus making the fourth significant correct, in other words, an error in R is less than 0.1 per cent.; and secondly, the passage of a current of 2 amperes only produced in 3 minutes a difference of like amount.

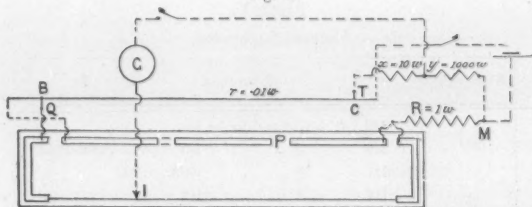
No larger cable was tried, nor was any wire smaller than No. 22 S.W.G., but between these limits an accuracy of 0.1 per cent. was easily obtained so far as the *sensibility of the method* was concerned.

The accuracy of the standard now remains to be considered. This can be best seen from a description of the method by which the correct length between the points B, C was determined.

In order that this length might be of a convenient amount to suit the dimensions of the base-board, it should have been made of copper wire No. 17 S.W.G.; but a piece of this gauge which was bought for the purpose proving unsatisfactory, a piece of No. 16 was taken and stretched till of the required diameter.

One, Q B, of the two short arms of fig. 1, was soldered on, the other, T C, being left loose. The whole was then screwed to a rough board, and the wire annealed by passing through it a current of about 50 amperes till it was too hot to touch and then allowing it to cool, the operation being repeated ten times. The arrangement represented in fig. 4 was next made.

Fig. 4.



By following out this figure, it will be seen to be the same arrangement as fig. 3, except that now the place of the plug is taken by the gap P. The standard was a 1-ohm coil of manganin wire, constructed by Messrs. Nalder, Bros. Its

terminals rested in mercury-cups from which the various leads were taken, the ends of all wires dipping in the mercury being freshly amalgamated. The leads joining the 10- and 1000-ohm coils together and to the binding-screws T and M respectively were short and stout.

The arm TC was then put approximately in position, and good contact at C was maintained by pressure. To avoid thermoelectric effects this pressure was made with a piece of wood, and in order to see if any such effects were present a commutator, not shown in the figure, was put in the battery-circuit. It was then found that if balance had been obtained for one direction of the current, a reversal produced no appreciable deflexion, thus showing that this piece of wood had entirely prevented such effects. As a precaution, however, the current was reversed each time balance was obtained, and in no instance was any deflexion caused by the reversal.

The experiments were then conducted in the same order as before. With the gap open, balance was obtained by moving the slider I. The gap was then closed and balance obtained by altering the position of the arm TC. The gap was next opened, and the slider moved till again no deflexion was noted. This motion was very slight, and the alteration, after closing the gap, was not found to have produced any observable want of balance. The correct position of C was thus determined.

In order to see with what degree of accuracy this position had been arrived at, the arm was shifted 1 millim. in both directions, and the want of balance was indicated by a deflexion on either side of zero of one division; hence, as the length between B and C was over one metre, the error in the position of C was well under 0.1 per cent.

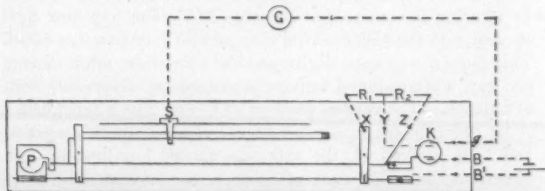
The arm TC was then soldered to the wire, and the tests repeated. No deflexion was observable with the ratio $\frac{10}{1000}$, but on adding a 1-ohm coil to the 1000, making the ratio $\frac{10}{1001}$, the want of balance produced nearly 2 divisions deflexion. The wire was then screwed in its proper place and there tested in a similar fashion with the same result, viz., with the ratio $\frac{10}{1000}$ no deflexion was noted, while with the

ratio $\frac{10}{1001}$ a deflexion of over 1 division showed the want of balance. A thermometer lying beside the wire indicated $17^{\circ}7$ C. Thus the resistance between B and C was 0.01 ohm at $17^{\circ}7$ C., with an error of less than 0.1 per cent.

Thus on all three conditions an accuracy of 0.1 per cent. was obtained. This simple apparatus is, therefore, capable of measuring the resistances of metre-lengths of wires between the limits of No. 22 S.W.G. and a stranded cable of 7 No. 16's (and probably over a still greater range) with an accuracy *throughout* of 0.1 per cent., an accuracy quite sufficient for all commercial purposes.

Although in the above simple form of The Addition a slide-wire bridge is necessary, it may happen that this latter is not available. At a slight additional cost this slide-wire can be made an integral part of the apparatus. In fig. 5 is represented a rather more elaborate design. The slide-wire is

Fig. 5.



easily recognized. The contact-piece of the slider S must be made adjustable so as to be able to press on either part of this wire. The circle K represents an ordinary Wheatstone-bridge key.

It will be seen that this design is very self-contained. All the connexions to be made consist of: a battery, with its supplementary resistance, to the binding-screws B, B', a galvanometer to g and S, and two resistances between X, Y and Y, Z. The former, marked R_1 , is an adjustable box of coils, and the latter, R_2 , is a single coil of 1000 ohms resistance.

DISCUSSION.

Prof. A. GRAY (communicated) said that the author had in his arrangement combined the fixed standard employed in Matthiessen and Hocking's modification of the ordinary bridge with the greater celerity of working arising from the smaller number of operations to be performed when the Kelvin bridge is used. Prof. Gray thinks that he, and probably others, have used a method similar to that of Mr. Reeves, but that the paper is of great utility since it shows how time may be saved and existing apparatus utilized.

Prof. AYRTON said that the advantage of the method described lay in the fact that it was independent of the resistances at the contacts. In Carey Foster's method, however, the coils had to be interchanged, and inaccuracy might be introduced owing to the varying resistance of the mercury contacts. Unless the mercury cups and the copper plates at the bottom were cleaned every day, and the contacts re-amalgamated, the resistance of the mercury cups was very variable. With regard to the question of Matthiessen's standard, it is to be remembered that the specific conductivity of copper has been steadily increasing. This increase was particularly noticeable in the copper prepared by the Elmore process, where, during the deposition of the metal, an agate burnisher is kept continually passing over the surface. Fitzpatrick had explained the rise in conductivity of copper by supposing that the density of the copper now supplied was greater than that of the copper used by Matthiessen, and this explanation seemed quite satisfactory. Mr. Reeves's experiments, however, have conclusively shown that this is not the true explanation; it was now possible to obtain copper in large quantity having a conductivity of 103 on Matthiessen's scale.

The Chairman (Prof. CAREY FOSTER) explained how, when using his method, the accuracy of the result depends not on the elimination of the small resistances at the mercury cups, but on the constancy of these resistances. Matthiessen and Dr. Russell found that the specific gravity of copper was apt to be low, on account of the presence of dissolved oxide,

and they were the first to pass hydrogen gas through the molten metal to remove this oxide.

Mr. APPLEYARD gave a simple diagrammatic sketch of the author's arrangement, and also pointed out that better results would probably be obtained with a galvanometer of 1 or 2 ohms resistance.

Mr. CAMPBELL said that it ought to be definitely settled whether Matthiessen's standard was the conductivity per unit volume or per unit mass. Since copper was always bought by weight, he, as a practical man, strongly advocated the adoption of the mass conductivity; further, in this case the measurement of the specific gravity would be avoided.

XIV. *On Kathode Rays.* By Herr PULUJ*.

[Abstract.]

HERR PULUJ sent for exhibition some Röntgen photographs taken by means of a form of Crookes tube which he had described in a memoir published in 1889. With this tube he has succeeded in obtaining impressions with exposures of only two seconds.

Herr Puluj considers that the particles of matter torn from the kathode, which convey negative electrostatic charges, by impact on the glass walls or on screens, equalize their electric charges, and in this process call forth not merely a disturbance of the material molecules, but also of their ether envelopes. Each portion of the glass or screen bombarded by the kathode stream becomes the starting-point of ether waves, which, according to their oscillation period and oscillation character, are either visible rays (phosphorescence) or invisible Röntgen rays. The oscillations of the invisible rays may take place in the longitudinal direction, but no convincing argument has up to now been brought forward to support this view.

* Read March 13, 1896

XV. *Note on Permeability to Röntgen Rays.**By Messrs. ACKROYD and KNOWLES*.*

[Abstract.]

THE Authors have exposed a plate on which a number of pieces of metal, oxides, and sulphates were placed to the Röntgen rays in order to see whether the permeability of bodies to these rays depends on the atomic or molecular weight of the body. In each case it was found that the opacity increased with the molecular weight.

DISCUSSION.

Mr. BLAKESLEY said that he considered the Röntgen rays to be the propagation of electrostatic strain through space. With reference to the non-refrangibility of these rays, he had observed in one of the photographs exhibited by Mr. Swinton a dark line at the edge of the shadow of a wooden pencil, which might have been due to the refraction of the rays by the wood. Mr. Blakesley has, however, found that this line is due to the varnish on the pencil. Some Röntgen photographs of quartz and ebonite rods not only did not exhibit these dark lines, but there was a very slight indication of a bright line just on the edge of the shadow, which would indicate that the refraction of these rays was less in the rods than in the surrounding medium.

Mr. EDSEER exhibited some photographs taken with Mr. Jackson's form of tube, in which a concave kathode is employed. Mr. Edser said that the whole of the tube on the kathode side of the anode plate phosphoresced, so that the Röntgen rays seem to partake of the character of diffused light.

Prof. AYRTON said Mr. Jackson had found that the kathode rays form a parallel beam, and do not first come to a focus and then again spread out.

The CHAIRMAN said some observations made by Mr. Porter agreed with those of Mr. Edser.

Mr. BLAKESLEY described the tube used by Puluj, in which a mica screen coated with green calcium sulphide is placed between the kathode and the anode.

* Read March 13, 1896.

Mr. GARDNER said that there seemed to be some confusion, for, when a concave kathode is employed, the kathode rays are brought to a focus and then again diverge. The phosphorescence on the inside of the glass had been shown by Lenard to be due to electricity travelling round the inside surface of the glass.

Mr. PIDGEON asked if any one had tried the effect of mounting the photographic film on a metal plate.

The CHAIRMAN said that Captain Abney had found that if the film was mounted on a ferrotype plate no action took place.

Prof. PERRY said he, for one, was of opinion that the Röntgen rays were undulatory. Prof. Larmor had given an explanation which seems to agree with the observed facts. This explanation supposes that the intermolecular spaces respond to vibrations of a certain frequency. The reason no refraction or diffraction effects had been observed was probably because of the extreme smallness of the wave-length of the undulation.

XVI. *Notes on the Electro-magnetic Theory of Moving Charges.* By W. B. MORTON, B.A.*

1. THIS subject has been brought into prominence recently by the use which Mr. Larmor has made of moving electrons in his dynamical theory of the æther. The matter was investigated in 1881 by Prof. J. J. Thomson†, who showed that a point charge moving so slowly that the electric displacement it carries is not sensibly disturbed generates magnetic force like a current element according to Ampère's rule; and by Mr. Heaviside‡, who investigated the matter more generally in 1889, and showed that in steady rectilinear motion at any speed less than that of light, the lines of displacement continue to be radial but are concentrated

* Read March 27, 1896.

† Phil. Mag. April 1881, July 1889; Recent Researches, pp. 16-23.

‡ Electrical Papers, ii. pp. 504-518; Electromagnetic Theory, i. pp. 269-274.

towards the plane perpendicular to the direction of motion. The displacement at distance r , in a direction making an angle θ with the line of motion is proportional to

$$\frac{1}{r^2 \left(1 - \frac{u^2}{V^2} \sin^2 \theta\right)^{\frac{3}{2}}},$$

where u is the velocity of the moving charge and V the velocity of light. The lines of magnetic force are circles round the line of motion.

2. This solution of course represents the state of affairs at a great distance from a small charged conductor of any shape. It would also give us the distribution of charge on a moving sphere if it were correct to assume that the lines of displacement meet the charged surface at right angles. This assumption was made by Prof. Thomson and, at first, by Mr. Heaviside, but the latter, quoting a suggestion of Mr. G. F. C. Searle, subsequently pointed out that when there is motion the electric force is no longer derived from a potential function, and as a consequence does not meet the equilibrium surface at right angles. Substituting the correct surface condition, he showed that the charged conductor, whose motion would give at all points the radial distribution found for a point charge, was not a sphere but a spheroid of certain ellipticity.

3. It seemed of some interest to inquire what the distribution of charge on a moving sphere would be. The surface-density at a point of the surface is now the normal component of the displacement at that point. By carrying the investigation a step further I have found that, if the conductor be a sphere or any ellipsoid, the ordinary static arrangement of charge is unaltered by the motion; *i.e.* the number of tubes of displacement leaving each element of the surface is unchanged, but the tubes no longer leave the surface at right angles. We may imagine that the motion has the effect of deforming the tubes, keeping their ends on the conductor fixed. The proof of this, involving a consideration of the general case, is here given and is followed by a note on the energy of a moving charge in a magnetic field.

4. Suppose we have any distribution of charge moving

with uniform velocity u parallel to the axis of z , and that the field has assumed its steady configuration. We shall denote $1 - \frac{u^2}{V^2}$ by k^2 , V being the velocity of light. Then since we have a steady state,

$$\frac{d}{dt} = -u \frac{d}{dz}.$$

Also, since each element of charge produces a magnetic field with no z -component, we have $\gamma = 0$ in the general case also. Using these two data, the equations connecting the displacement (f, g, h) and the magnetic force (α, β, γ) become

$$-\frac{d\beta}{dz} = -4\pi u \frac{df}{dz},$$

$$\frac{d\alpha}{dz} = -4\pi u \frac{dg}{dz},$$

$$\frac{d\beta}{dx} - \frac{d\alpha}{dy} = -4\pi u \frac{dh}{dz},$$

$$\frac{dg}{dz} - \frac{dh}{dy} = -\frac{u}{4\pi V^2} \frac{d\alpha}{dz},$$

$$\frac{dh}{dx} - \frac{df}{dz} = -\frac{u}{4\pi V^2} \frac{d\beta}{dz},$$

$$\frac{df}{dy} - \frac{dg}{dx} = 0.$$

These equations together with

$$\frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz} = 0$$

are satisfied by

$$f = -\frac{d\phi}{dx}, \quad g = -\frac{d\phi}{dy}, \quad h = -k^2 \frac{d\phi}{dz},$$

$$\alpha = 4\pi u \frac{d\phi}{dy}, \quad \beta = -4\pi \frac{d\phi}{dx},$$

where ϕ is any function satisfying

$$\frac{d^2\phi}{dx^2} + \frac{d^2\phi}{dy^2} + k^2 \frac{d^2\phi}{dz^2} = 0.$$

These results have been obtained by Prof. Thomson and Mr. Heaviside. The particular case of a point charge, e , is got by putting

$$\phi = \frac{e}{4\pi \sqrt{k^2(x^2 + y^2) + z^2}}.$$

Evidently in the general case ϕ must vanish at infinity.

5. Mr. Heaviside points out that $\phi = \text{constant}$ is the condition holding at a surface of equilibrium. The matter may be stated thus:—If we suppose the field to terminate at the surface of a conductor, inside which the vectors vanish, we must see that the “curl” relations of the field are not violated for circuits which lie partly inside the empty space enclosed by the conductor. In particular, if there is a vector whose line integral round every circuit in the field vanishes, the lines of this vector must meet the surface at right angles. Otherwise we should have a finite value for the integral round a circuit drawn close to the surface outside and completed inside. In other words, if a vector is derived from a potential function, this function must be constant over the surface. In the ordinary static case it is the electric force (X, Y, Z) which is so derived; but in the case of a steadily moving field it is the vector $(X, Y, \frac{Z}{k^2})$ which meets the surface at right angles.

6. Let $F(x, y, z) = C$ be the equation of the charged surface. Then $\phi(x, y, z)$ has to be constant over this surface and satisfy

$$\frac{d^2\phi}{dx^2} + \frac{d^2\phi}{dy^2} + k^2 \frac{d^2\phi}{dz^2} = 0.$$

Put $z = k\xi$, then ϕ is a function of x, y, ξ , which is constant when $F(x, y, k\xi) = C$, and which satisfies

$$\frac{d^2\phi}{dx^2} + \frac{d^2\phi}{dy^2} + \frac{d^2\phi}{d\xi^2} = 0.$$

Therefore if we regard (x, y, ξ) as Cartesian coordinates of a point, ϕ is the potential at external points of an electrostatic free distribution on the surface $F(x, y, k\xi) = C$. The components of electric force due to this distribution, at a point

$(x y \zeta)$ on the surface, are

$$-\frac{d\phi}{dx}, \quad -\frac{d\phi}{dy}, \quad -\frac{d\phi}{d\zeta}.$$

This force acts in the normal to the surface, and is proportional to the surface-density at $(x y \zeta)$, which we shall call σ' . Therefore

$$\left(\frac{d\phi}{dx}, \frac{d\phi}{dy}, \frac{d\phi}{d\zeta}\right) = -\Lambda\sigma' \left(\frac{dF}{dx}, \frac{dF}{dy}, \frac{dF}{d\zeta}\right) / \sqrt{\left(\frac{dF}{dx}\right)^2 + \left(\frac{dF}{dy}\right)^2 + \left(\frac{dF}{d\zeta}\right)^2}$$

But

$$\frac{d}{d\zeta} = k \frac{d}{dz};$$

therefore, denoting differentiation with respect to $x y z$ by subscripts 1 2 3,

$$(\phi_1, \phi_2, \phi_3) = -\Lambda\sigma' (F_1, F_2, F_3) / \sqrt{F_1^2 + F_2^2 + k^2 F_3^2}.$$

Now let σ be the surface-density at $(x y z)$ on the moving conductor $F(x y z) = C$, then equating σ to the normal component of $(f g h)$,

$$\sigma = \frac{fF_1 + gF_2 + hF_3}{\sqrt{F_1^2 + F_2^2 + F_3^2}};$$

or putting in the values we have found for $(f g h)$ in terms of ϕ ,

$$\begin{aligned} \sigma &= -\frac{\phi_1 F_1 + \phi_2 F_2 + k^2 \phi_3 F_3}{\sqrt{F_1^2 + F_2^2 + F_3^2}} \\ &= \Lambda\sigma' \sqrt{\frac{F_1^2 + F_2^2 + k^2 F_3^2}{F_1^2 + F_2^2 + F_3^2}}. \end{aligned}$$

Now the perpendicular from the origin on the tangent plane to $F(x y z) = C$ at the point $(x y z)$ is

$$p = \frac{x F_1 + y F_2 + z F_3}{\sqrt{F_1^2 + F_2^2 + F_3^2}},$$

and the perpendicular from the origin on the tangent plane to $F(x, y, k\zeta) = C$ at $(x y \zeta)$ is

$$p' = \frac{x \frac{dF}{dx} + y \frac{dF}{dy} + \zeta \frac{dF}{d\zeta}}{\sqrt{\left(\frac{dF}{dx}\right)^2 + \left(\frac{dF}{dy}\right)^2 + \left(\frac{dF}{d\zeta}\right)^2}}$$

$$\begin{aligned}
 &= \frac{x F_1 + y F_2 + k \zeta F_3}{\sqrt{F_1^2 + F_2^2 + k^2 F_3^2}} \\
 &= \frac{x F_1 + y F_2 + z F_3}{\sqrt{F_1^2 + F_2^2 + k^2 F_3^2}};
 \end{aligned}$$

$$\therefore \frac{\sigma}{\sigma'} = \frac{\Lambda p}{p'}.$$

If now $F(x y z) = C$ is an ellipsoid, then we know that $\sigma' \propto p'$, therefore also $\sigma \propto p$, that is the arrangement of charge on the moving ellipsoid is the same as if it were at rest.

7. Applying the above to the ellipsoid ($a b c$), we find that ϕ as a function of ($x y \zeta$) is the potential of a free distribution on the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{k^2 \zeta^2}{c^2} = 1;$$

$$\begin{aligned}
 \therefore \phi &= C \int_{\mu}^{\infty} \frac{d\lambda}{\sqrt{(a^2 + \lambda)(b^2 + \lambda)(c^2 + k^2 \lambda)}} \\
 &= C k \int_{\mu}^{\infty} \frac{d\lambda}{\sqrt{(a^2 + \lambda)(b^2 + \lambda)(c^2 + k^2 \lambda)}},
 \end{aligned}$$

where μ is given by

$$\frac{x^2}{a^2 + \mu} + \frac{y^2}{b^2 + \mu} + \frac{\zeta^2}{c^2 + k^2 \mu} = 1,$$

or

$$\frac{x^2}{a^2 + \mu} + \frac{y^2}{b^2 + \mu} + \frac{z^2}{c^2 + k^2 \mu} = 1.$$

Determining the value of the constant C so that the density at a point shall be $\frac{ep}{4\pi abc}$, we get

$$\phi = \frac{e}{8\pi} \int_{\mu}^{\infty} \frac{d\lambda}{\sqrt{(a^2 + \lambda)(b^2 + \lambda)(c^2 + k^2 \lambda)}}.$$

Putting $b = a$, $c = ka$, we get

$$\phi = \frac{e}{4\pi \sqrt{k^2(x^2 + y^2) + z^2}}.$$

Showing that, as Mr. Heaviside pointed out, the field of a point charge is given when the conductor is an oblate spheroid whose axes have the ratio $1:k$.

For a sphere the integral becomes

$$\phi = \frac{e}{8\pi k'a} \log \frac{\theta + k'a}{\theta - k'a}$$

where

$$k' = \sqrt{1-k^2} = \frac{u}{V},$$

and θ is given by

$$\frac{k^2(x^2+y^2)}{b^2-k'^2a^2} + \frac{z^2}{b^2} = 1.$$

To test the value of ϕ let us make k' approach zero, *i. e.* the motion becomes infinitely slow. θ is then $=r$.

$$\begin{aligned} \text{Then } \phi &= \frac{e}{8\pi a} \lim_{k'=0} \log \frac{(r+k'a) - \log(r-k'a)}{k'} \\ &= \frac{e}{8\pi a} \cdot \frac{2a}{r} = \frac{e}{4\pi r}. \end{aligned}$$

8. The mutual energy of a moving charge and external magnetic field has been given by Mr. Heaviside for the case of motion which is very slow compared with the velocity of radiation. It is $euA \cdot \cos(uA)$, where A is the circuital vector potential of the external field. Mr. Larmor, in the second part of his "Dynamical Theory" (Phil. Trans. 1895, p. 717), concludes that the same expression holds good for motion at any speed. He seems, however, to overlook the fact that in the general case the displacement-currents in the medium—being no longer derivable from a potential function—will make their appearance in the result as well as the convection-current eu .

If (FGH) is the vector potential, the part of the energy corresponding to the displacement currents will be

$$\int (F\dot{f} + G\dot{g} + H\dot{h}) d\tau,$$

which in the case we have been considering becomes

$$\begin{aligned} &-u \int \left(F \frac{df}{dz} + G \frac{dg}{dz} + H \frac{dh}{dz} \right) d\tau \\ &= u \int \left(F \frac{d^2\phi}{dx dz} + G \frac{d^2\phi}{dy dz} + k^2 H \frac{d^2\phi}{dz^2} \right) d\tau. \end{aligned}$$

But by a well-known transformation, when we take the integral through all space, we have

$$\begin{aligned} & \int \left(F \frac{d^2 \phi}{dx dz} + G \frac{d^2 \phi}{dy dz} + H \frac{d^2 \phi}{dz^2} \right) d\tau \\ &= \int \frac{d\phi}{dz} \left(\frac{dF}{dx} + \frac{dG}{dy} + \frac{dH}{dz} \right) d\tau \\ &= 0 \text{ since } (F G H) \text{ is circuital.} \end{aligned}$$

$\therefore$ The expression for this part of the energy reduces to

$$-u(1-k^2) \int H \frac{d^2 \phi}{dz^2} d\tau = -\frac{u^3}{V^2} \int H \frac{d^2 \phi}{dz^2} d\tau.$$

Therefore if the velocity u ceases to be negligible in comparison with V , we have a correction of the *second* order in the ratio $\frac{u}{V}$ in addition to the expression involving the convection-current simply. It also appears from the above that the force on the moving charge cannot, unless this term be neglected, be expressed in terms of the magnetic intensity at the charge, but will depend on the entire field.

XVII. *A Further Examination of the Edison Effect in Glow Lamps.* By J. A. FLEMING, M.A., D.Sc., F.R.S., Professor of Electrical Engineering in University College, London*.

§1. THE experiments described in this paper had for their object the further examination of an effect which can be produced in certain forms of electric incandescence lamps and to which attention was first drawn by Mr. Edison in 1884. This effect may be generally described as follows:—A carbon filament incandescence lamp having the ordinary horse-shoe loop carbon has a metallic plate placed in the exhausted bulb, the plate being carried on a platinum wire sealed through the globe, and fixed so as to stand up between the legs of the

* Read March 27, 1896.

horse-shoe (see fig. 1). If the lamp is set in action at the usual incandescence by a continuous current of the proper strength, and a suitable sensitive galvanometer is connected between the insulated metal plate and the *positive* terminal of the lamp, it will in general be found to indicate a current of some milliamperes flowing through it. The direction of this current is *from* the positive electrode of the lamp through the galvanometer *to* the insulated metal plate, or wire. When the same galvanometer is connected between the *negative* pole of the lamp and the middle plate, unless it is very sensitive, it indicates no current. This effect was very carefully examined

Fig. 1.

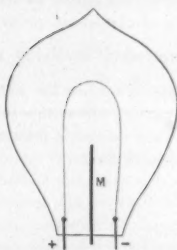
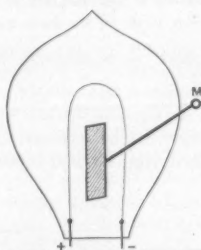


Fig. 2.



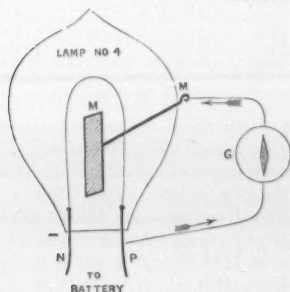
by Mr. W. H. Preece in 1885, and he subjected it to a systematic examination by the aid of a number of lamps having such metal plates placed in various positions*. By this observer a number of very interesting facts were collected, the result of which was to point out the general nature of the phenomenon. A sufficient number of new questions were, however, suggested by the information so acquired to invite further inquiry. Whilst confirming and re-examining the experimental results obtained by Mr. Preece, some facts that had previously escaped notice presented themselves, which it is the object of this paper to describe.

* "On a Peculiar Behaviour of Glow-Lamps when raised to High Incandescence," by W. H. Preece, F.R.S. Proceedings of the Royal Society, 1885, p. 219.

§ 2. The first experiments were made with a lamp of the form shown in fig. 2, similar to some used by Mr. Preece in his experiments. A metal plate, generally of aluminium, is supported on a platinum wire sealed through the bulb or glass receiver, the plate being so fixed that its plane is at right angles to the plane of the loop of the carbon, and as nearly as possible midway between the legs. The plate therefore projects between the legs of the horse-shoe carbon, and the carbon conductor arches over it without touching it. The plate is entirely insulated from the carbon.

The preliminary experiments with this normal type of middle-plate lamp consisted in determining the effective potential-difference between the third terminal and one or other of the two electrodes of the carbon filament when the lamp was subjected to varying steady electromotive forces sufficient to raise the temperature of the carbon from dull red to vivid incandescence, and in determining the magnitude of the current flowing in a circuit connecting the middle plate with one or other of the electrodes of the lamp.

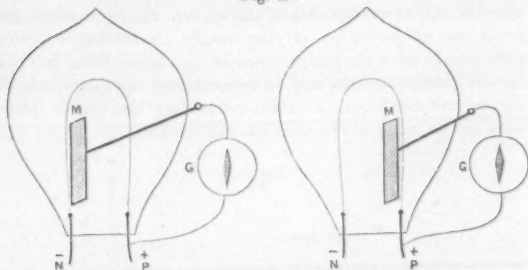
Fig. 3.



§ 3. *Experiment 1.*--An ordinary carbon filament electric lamp, having the horse-shoe shaped conductor, had a platinum plate (see fig. 3) about 2.5 centimetres long by 1.5 centimetres wide welded to a platinum wire, sealed through the side of the bulb. The plate was so placed as to project

between the legs of the carbon conductor, having its plane at right angles to the plane of the horse-shoe, and initially fixed about halfway between the two legs. This lamp will be described in the subsequent paragraphs as Lamp No. 4. Under a steady electromotive force of 40 volts, this lamp took 3.7 amperes when working at the normal temperature corresponding to about 3.5 watts per candle-power. When a milamperemeter having a resistance of 6372 ohms was joined between the base P of the *positive* leg of the carbon (see fig. 4) and the middle plate M, a current was found passing through the galvanometer from the terminal P to the plate M. This

Fig. 4.



current had a magnitude of about 3 milliamperes when the carbon was in the normal state of incandescence.

If the milamperemeter was connected between the *negative* electrode of the lamp and the middle plate, no current perceptible by this galvanometer was found. On replacing the milamperemeter by a more sensitive Elliott mirror galvanometer (resistance 7142 ohms), it was found that a small current passed through it, when joined in between the negative electrode of the lamp and the middle plate, but that this current had a magnitude hardly exceeding .0001 of a milliampere when the lamp was at its normal incandescence.

In order to avoid repetition, it may be here said that, unless otherwise stated, the terminal of the lamp in connexion with

the positive pole of the working battery will be spoken of as the positive electrode of the lamp; that in connexion with the negative pole of the battery as the negative electrode. For brevity's sake, the half of the carbon filament between the centre of the filament and the positive electrode will be called the *positive leg*, and the other half the *negative leg*.

§ 4. A preliminary series of experiments was made with lamp No. 4 by placing the lamp in a photometer and determining the watts per candle-power and the current taken by the lamp corresponding to various working electromotive forces, taken over the whole range of electromotive force from that necessary just to render the filament incandescent to the highest the lamp could with safety endure. In any subsequent experiments, the simple measurement of the potential-difference between the electrodes of the lamp enabled the rate of dissipation of energy in the filament and the watts per candle-power to be deduced. It may here be remarked that in the preliminary experiments some difficulties arose from the occlusion of residual gas by the middle metal plate, but finally this was overcome, and the vacuum in these experimental bulbs made and preserved as perfect as in good ordinary commercial lamps. The following results were then obtained with this lamp No. 4. The lamp was raised to various degrees of incandescence by varying the working volts by the aid of a rheostat in series with the lamp.

The milamperemeter was employed to measure the effective potential-difference between the positive electrode of the lamp and the middle plate and then, tabulating against the working volts of the lamp the current in milliamperes flowing through the galvanometer, the potential-difference between the middle plate and the positive electrode of the lamp was calculated from these figures. The results are given in the table below (Table No. 1).

TABLE NO. 1. LAMP NO. 4. *Milamperemeter.*

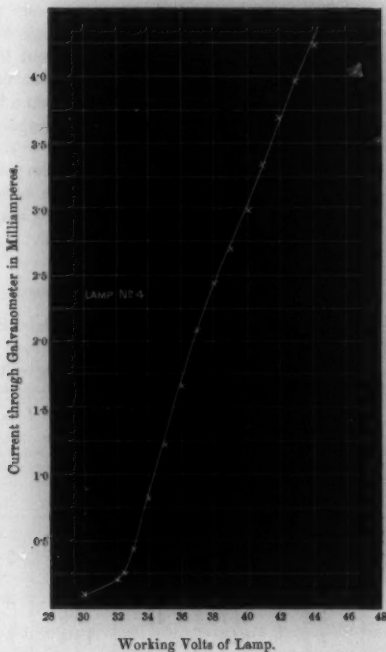
Table showing the volts between the middle plate and the positive electrode, and the current flowing through a galvanometer of 6372 ohms resistance connecting them, taken for various working voltages of the lamp.

Working volts of lamp.	Volts between plate and positive lamp electrode.	Current in milliamperes through galvanometer.	Working volts of lamp.	Volts between plate and positive electrode.	Current in milliamperes through galvanometer.
30	·54	·085	36	10·7	1·69
32	1·2	·190	37	12·7	2·01
32·5	1·6	·25	38	14·9	2·36
33	2·8	·44	39	17·1	2·71
33·5	4·7	·74	40	18·9	2·99
34	5·3	·84	41	21·4	3·37
34·5	7·1	1·12	42	23·4	3·71
35	7·8	1·23	43	25·2	3·99
			44	26·8	4·25

The results given in table No. 1 are plotted in curve No. 1, in which horizontal abscissæ represent to scale the working volts of the lamp and vertical ordinates the milliampere currents through the galvanometer. It will be seen that the curve representing the current from lamp electrode to plate takes a rather sharp turn upwards at a point corresponding to 33 working volts, and this occurs when the lamp is working at about 7·8 watts per candle-power. Beyond this point the curve is very approximately a straight line. Accordingly, at and beyond the volts at which the carbon filament becomes fairly well incandescent, the effective potential-difference between the middle plate and the positive lamp electrode is very nearly a linear function of the lamp voltage; and at the normal working volts, viz. 40 volts, this potential-difference between the middle plate and the positive electrode so determined is apparently about half that between the lamp terminals, the plate being nearly midway between the carbon legs. The results given in Table 1 are the mean of several observations, but it was noticed that when the lamp was maintained

at a steady voltage, the potential-difference between the middle plate and the positive electrode would often *jump suddenly* from one value to another. This effect renders it difficult to obtain the stable values of the plate and positive electrode potential-difference. Corresponding to any definite

TABLE No. 1.—CURVE No. 1.



steady voltage on this lamp, the current may have one or other of two values, but not always permanently preserving either; a galvanometer deflexion indicating say 10 volts between the plate and positive electrode of the lamp will

often slowly increase until after a few minutes it is 12 or 14 volts, yet all the time the working volts on the lamp are remaining perfectly constant. It will then often suddenly jump perhaps to 22 volts, and then slowly decrease to 19 volts, or so. This tendency of the potential-difference between the middle plate and positive lamp electrode to jump from a low to a high value, or *vice versa*, is most marked in lamps in which the plate is about half-way, and symmetrically placed, between the legs of the carbon. We shall speak of these two values as the high and low value of the current through the galvanometer, and defer until later a discussion of some other causes tending to make the current pass from a high to a low value or the reverse, as well as its possible explanation. In Table No. 2 are tabulated a set of observations on the same lamp No. 4, showing these double values which the potential-difference and current may have, and it may be here noted that in the previous Table No. 1, the higher values have been taken in those cases in which double values exist.

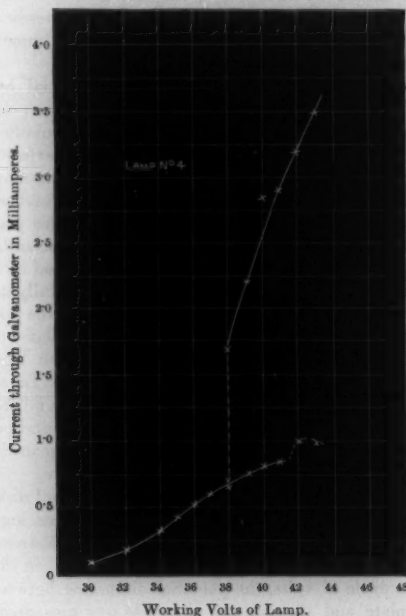
TABLE NO. 2. LAMP NO. 4. *Milamperemeter*.

Table showing the multiple values of the potential-difference between the middle plate and positive electrode of the lamp corresponding to various given working voltages.

Working volts of the lamp.	Volts between the middle plate and positive electrode of the lamp.	Current through the graded galvanometer in milliamperes.	Working volts of the lamp.	Volts between the middle plate and positive electrode of the lamp.	Current through the graded galvanometer in milliamperes.
30	·6	·095	39	4·3	·761
32	1·1	·174		14·0	2·22
34	2·0	·317	40	5·1	·809
35	2·6	·412	"	18·1	2·87
36	3·2	·507	41	5·3	·841
37	3·9	·618	"	18·4	2·91
38	4·3	·682	42	6·4	1·01
"	4·5	·714	"	20·0	3·17
"	11·0	1·74	43	6·2	·983
			"	22·0	3·49

These observations are plotted in Curve No. 2, in which the abscissæ represent the working volts of the lamp and the ordinates the current in milliamperes flowing through the galvanometer connecting the positive electrode and the middle plate. It is seen that corresponding to any working pressure

TABLE No. 2.—CURVE No. 2.



above 38 volts for this lamp, which is equivalent to 4.2 watts per candle-power, there are two possible values of the effective potential-difference between the middle plate and the positive electrode. As the working voltage of the lamp is gradually raised, the reading of the galvanometer inserted between the middle plate and positive electrode is also increased, but there

is a great tendency to jump from a certain low value to a higher one, and this occurs when the working pressure of the lamp is preserved steady. There is also an effect produced by the presence of a magnet near the lamp bulb. When the current is at the low value corresponding to any working voltage, the galvanometer reading does not seem to be perceptibly altered by bringing a magnet near the lamp, but when it is at its high value, the reading is sometimes increased for a little, showing a steady deflexion, and then immediately falls to its low value.

§ 5. *Experiment 2.*—The difference of potential between the middle plate and the positive electrode of the lamp depends to a considerable extent upon the position of the middle plate. Supposing the plate to be placed with its plane perpendicular to the plane of the carbon horse-shoe and then moved to various positions between the two legs of the carbon, it is found that the difference of potential between the plate and the positive electrode will have different values according to the position of the plate. This fact was elucidated by means of the same lamp No. 4 as used above. By carefully tapping the lamp, the supporting platinum wire carrying the platinum middle plate could be bent so as to displace the plate from its symmetrical position as regards the two carbon legs, and bring it nearer to one or other of the legs. In several different positions the current flowing through the milampere-meter, when connected between the middle plate and positive electrode, was measured, the lamp being kept meanwhile at the same working electromotive force.

Estimating as nearly as possible the fractional distances, the plate was placed at distances from the negative leg equal to $\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{9}{10}$ of the whole distance between the positive and negative legs, and the lamp being taken through a definite cycle of volts, the potential-difference between the middle plate and the positive electrode was measured with the milampere-meter. The results are collected in the following tables. The diagrams in fig. 6 represent the horse-shoe carbon loop and the middle plate M in various positions, the galvanometer G being inserted between the plate M and the positive electrode P. By the phrase "whole distance" in the following tables is meant the whole distance or width of the space between the positive and negative carbon leg.

TABLE NO. 3. LAMP NO. 4. *Milamperemeter.*

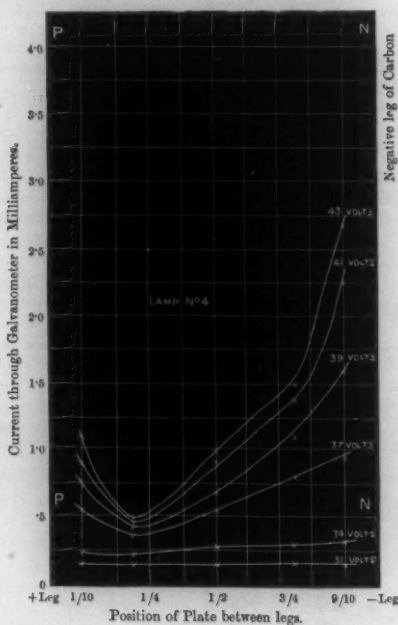
Table showing the potential-difference between the middle plate and positive electrode of the lamp at various positions of the plate and at various working voltages.

Working volts of the lamp.	Volts between the middle plate and positive electrode.	Current through the galvanometer in milliamperes.	Working volts of the lamp.	Volts between the middle plate and positive electrode.	Current through the galvanometer in milliamperes.
Middle plate at $\frac{1}{10}$ of whole distance from negative leg.			Middle plate at $\frac{3}{4}$ of whole distance from negative leg.		
31	·9	·142	31	·9	·142
34	2·2	·317	34	1·4	·222
37	6·0	·952	37	2·2	·349
39	10·2	1·61	39	2·5	·396
41	14·2	2·24	41	2·9	·460
43	16·8	2·66	43	3·0	·476
Middle plate at $\frac{1}{4}$ of whole distance from negative leg.			Middle plate at $\frac{9}{10}$ of whole distance from negative leg.		
31	·9	·142	31	·9	·142
34	1·8	·285	34	1·5	·238
37	5·1	·809	37	3·4	·539
39	7·0	1·11	39	4·8	·761
41	8·7	1·38	41	5·7	·904
43	9·5	1·50	43	6·8	1·07
Middle plate $\frac{1}{2}$ of whole distance from negative leg.					
31	·9	·142			
34	1·7	·269			
37	3·5	·555			
39	4·3	·682			
41	5·6	·888			
43	6·2	·983			

The results of this Table No. 3 are plotted in the curves No. 3. These curves are to be interpreted as follows:—The two vertical lines P and N represent the two legs of the

TABLE NO. 3.—CURVE NO. 3.

The middle plate was moved along into different positions between the two carbon legs indicated by the horizontal distances, and at each position the current between the middle plate and positive electrode of the lamp is represented by the vertical ordinate of a curve. The several curves correspond to different working volts on the lamp.



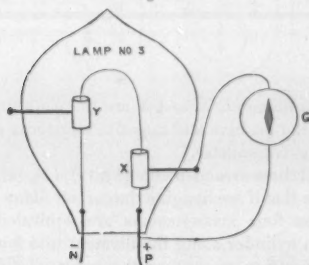
carbon horse-shoe. At various distances on the way from P to N the milliamper current through a galvanometer connected between the middle plate, placed at that point, and the

positive electrode of the lamp is represented by the magnitude of the vertical ordinate of each curve. For every one of the different voltages at which the lamp is worked, there is therefore a curve representing by its ordinates this current strength through a galvanometer inserted between the middle plate, placed at these positions, and the positive electrode of the lamp, and it is seen that there is a *minimum value* for this current at a position equal to $\frac{3}{4}$ of the whole distance between the legs reckoned from the negative leg.

Imagine the middle plate therefore connected through a galvanometer with the positive electrode of the lamp, and let the middle plate be first placed close to the positive leg and then moved continuously nearer towards the negative leg. The current through the galvanometer would first fall off as the plate receded from the positive leg, and after reaching a minimum at a point about $\frac{1}{4}$ of the whole distance between the legs reckoned from the positive leg, would rise up to a maximum when the middle plate was as nearly in contact with the negative leg as possible without actually touching it.

§ 6. *Experiment 3.*—In order to explore more thoroughly the action of the different portions of the incandescence carbon conductor in producing this effect, a lamp was taken having

Fig. 5.

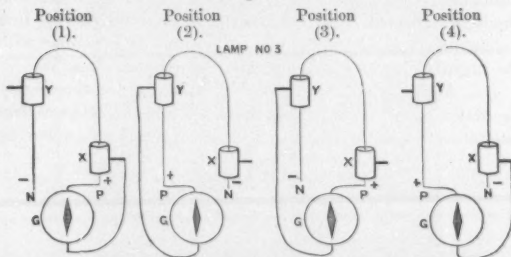


a horse-shoe shaped carbon, and a pair of small platinum cylinders, held on platinum wires sealed through the glass, so placed as to embrace without touching the carbon conductor. One of these cylinders, X, was placed so as to embrace the

carbon near the bottom of the leg, and the other, Y, near the spring of the arch (see fig. 5). These small cylinders had a length of about 12 millims. and a diameter of about 8 millims. so that the distance from the carbon filament to the inner surface of the cylinder was about 3 or 4 millims. The lamp had a rather thick carbon, and at an electromotive force of 48 volts took a current of 1.32 amperes to raise it to its normal incandescence of 18.8 candles, corresponding to 3.3 watts per candle-power. This lamp will be alluded to as Lamp No. 3.

It is obvious that there are four possible arrangements in which a current can be obtained between an embracing cylinder and a positive electrode of the lamp. These are illustrated in fig. 6, in which the horse-shoe shaped line stands

Fig. 6.



for the carbon filament, X and Y are the platinum cylinders, P and N are the positive and negative electrodes of the lamp, and G is the galvanometer.

We will call these arrangements (1), (2), (3), (4), as figured. It will be seen that if we imagine the carbon filament straightened out, these four arrangements are equivalent to being able to slide a cylinder along the filament into four positions, and in each position measuring the potential-difference between the cylinder and the positive end of the carbon. We are thus able to place an embracing collecting-plate at four different places along the carbon conductor, and determine the potential-difference between this embracing cylinder and

the positive electrode of the lamp. A series of experiments was made with lamp No. 3, in which the working volts of the lamp were raised to various values, and in each case the potential-difference between one of the cylinders X or Y and the positive electrode of the lamp was observed as before by means of the milamperemeter. The results are given in Table 4.

TABLE NO. 4. LAMP NO. 3. *Milamperemeter.*

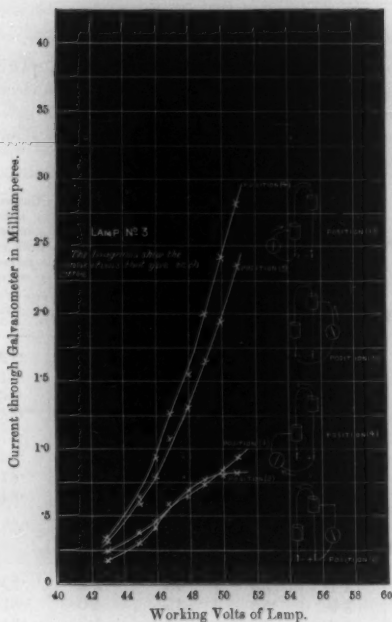
Table showing the potential-difference between a platinum cylinder embracing the carbon and the positive electrode of the lamp, and the current flowing through the galvanometer in milliamperes ; for the four positions shown in fig. 6.

Working volts of the lamp.	Volts between cylinder and positive electrode.	Current in milliamperes through the galvanometer.	Working volts of the lamp.	Volts between cylinder and positive electrode.	Current in milliamperes through the galvanometer.
Position (1).			Position (2).		
43	1.6	.25	43	1.2	.19
45	2.4	.38	45	2.0	.31
46	2.9	.46	46	2.9	.36
47	3.7	.58	47	3.7	.58
48	4.2	.67	48	4.2	.67
49	4.7	.74	49	4.9	.78
50	5.2	.82	50	5.1	.81
51	5.9	.93	51	5.3	.84
Position (3).			Position (4).		
43	2.1	.33	43	2.2	.35
45	3.9	.62	45	4.0	.63
46	4.9	.78	46	5.9	.94
47	6.8	1.08	47	8.0	1.26
48	8.2	1.30	48	9.8	1.55
49	10.4	1.65	49	12.6	1.99
50	12.3	1.95	50	15.1	2.38
51	14.9	2.36	51	18.0	2.85

These observations are plotted in Curve No. 4.

This table of observations shows us that taking the lamp at any definite voltage, the potential-difference between the positive electrode and a cylinder embracing the carbon filaments is greatest when that cylinder is as low down near

TABLE NO. 4.—CURVE NO. 4.



the foot of the negative leg of the carbon as possible. If the cylinder is placed near the top of the negative leg that potential-difference becomes less. If the cylinder is taken near the top of the positive leg or near the base of the positive leg it is least, and it seems to be a minimum when the cylinder

surrounds the top of the positive leg, and is as far as possible away from the foot of the negative leg. We have here a confirmation of the fact observed with respect to lamp No. 4, viz.: that the potential-difference between the positive lamp electrode and the metal plate held somewhere near the incandescent carbon conductor is greatest when the plate is as near as possible to the foot of the negative leg or negative electrode.

In this lamp No. 3, in which in all positions the cylinder employed is placed very near some point on the incandescing conductor, the current through the galvanometer joining the positive electrode and the cylinder has never been observed to jump or to take double values as in the case of lamp No. 4.

If a sensitive galvanometer is connected to the two insulated cylinders X and Y, and if in addition there is a battery in series with this galvanometer, then no current can be detected in such an arrangement even when the battery has an electromotive force of 120 volts, whether the lamp carbon is incandescent or not. Just as in the case of lamp No. 4, the current obtained by connecting either cylinder with the negative electrode of the lamp is excessively small.

In order to obtain curves showing the mode of variation of the effective difference of potential between one or other of the metal cylinders X and Y and the positive electrode of the lamp, a set of observations was made on the lamp when submitted to various working voltages, and at the same time the milliamperemeter was connected first between the positive electrode and cylinder X and then between the positive electrode and cylinder Y, with the following tabulated results (p. 204).

These results, when plotted out in curves in which horizontal ordinates represent the working volts of the lamp and the vertical ones the milliampere currents going through the galvanometer, in both cases have the same general form as curve No. 1 plotted for the case of lamp No. 4.

TABLE NO. 5. LAMP NO. 3. *Milliamperemeter.*

Table showing the potential-differences between the positive electrode of the lamp and the two cylinders X and Y respectively, and the corresponding currents through the galvanometer.

Working volts of the lamp.	Volts between cylinder X and the positive electrode P of the lamp. Position (1).	Milliampere current through the galvano- meter connect- ing X and P.	Volts between cylinder Y and the positive electrode P of the lamp. Position (3).	Milliampere current flowing through the galvanometer connecting Y and P.
34	·1	·015	·1	·015
35	·15	·023	...	
36	·20	·031	·3	·045
37	...		·4	·060
38	·6	·095	·7	·11
39	·8	·126	·9	·14
40	1·05	·166	1·1	·17
41	1·3	·206	1·6	·25
42	1·8	·285	2·0	·31
43	2·1	·333	2·8	·44
44	2·6	·412	3·5	·55
45	3·0	·476	4·5	·77
46	3·6	·571	5·8	·92
47	4·2	·659	7·2	1·12
48	4·9	·791	8·8	1·39
49	5·6	·889	10·9	1·72
50	6·2	·984	13·6	2·15
51	...		15·1	2·39
52	...		18·6	2·95

§ 7. *Experiment 4.*—The magnitude of the current found on connecting any galvanometer between one of the cylinders and the positive electrode of the lamp was found to be dependent to some degree on the perfection of the vacuum. Lamp No. 3 when first made had not a very perfect vacuum. A series of measurements was, however, made with it, and the same repeated after re-exhaustion. The results are tabulated together below.

TABLE NO. 6. LAMP NO. 3. *Elliott Galvanometer.*

Table showing the relative values of the potential-difference between cylinder Y and the positive electrode for good and imperfect vacua in the case of lamp No. 3.

Working volts of the lamp.	Very high vacuum in the lamp.		Imperfect vacuum in the lamp.	
	Watts per candle-power taken up in carbon.	Volts between cylinder Y and positive electrode.	Watts per candle-power taken up in carbon.	Volts between cylinder Y and positive electrode.
40	6.75	2.02	9.1	4.17
41	5.75	2.89	7.9	7.25
42	5.30	3.62	6.5	8.39
43	4.90	4.54	5.9	9.80
44	4.50	6.08	4.3	11.96
45	4.15	7.50	4.0	13.76
46	3.84	9.25	3.7	15.08
47	3.55	10.33	3.4	16.21
48	3.30	13.2	3.2	18.28
49	3.07	16.3	3.0	22.05
40	2.90	18.4	2.8	22.99
51	2.74	21.8	2.6	23.93
52	2.58	24.4	2.4	28.26

The imperfection of the vacuum is indicated by the higher watts per candle-power absorbed at low voltages, and we see that at any given working pressure the potential-difference between the positive electrode and the cylinder Y embracing the top of the negative leg is *greater* when the vacuum is imperfect than when it is very good. The presence of residual air tends to bring down the potential of the embracing cylinder more nearly to that of the carbon at the point adjacent to it.

§ 8. *Experiment 5.*—A series of observations was next made in which the potential-difference between the middle plate and the positive electrode was determined by the aid of a condenser. If a condenser of capacity C in microfarads is charged to a potential of V volts and discharged through a ballistic galvanometer, we can determine the ballistic constant

of the galvanometer. A second observation of a like nature in which the "throw" of the same galvanometer is observed when the same condenser is charged by contact with two points concerning which we require to know the potential-difference, gives us the means of calculating the electrostatic potential-difference in volts. A condenser of .987 microfarad capacity carefully determined was charged to a potential of 54 volts and discharged through a certain ballistic galvanometer having a needle whose periodic time of vibration was about three seconds. The resulting "throw" of the galvanometer was $5^{\circ} 30'$. Hence a discharge of $54 \times .987 = 53.3$ microcoulombs through the galvanometer produces a "throw" of $5^{\circ} 30'$. Neglecting a very small correction for the logarithmic decrement, in this case not of importance, we have for the ballistic constant R the value

$$53.3 = R \sin \frac{1}{2}(5^{\circ} 30') = R \times .04798 ;$$

hence

$$R = 1110.$$

The same condenser was then connected between the middle plate and positive electrode of lamp No. 4 and then discharged through the same ballistic galvanometer. The lamp was subjected to a working pressure of 39 volts as determined by a corrected voltmeter attached to the electrodes of the lamp. The charge of the condenser was sent through the ballistic galvanometer, and a "throw" of 4° obtained. If v is the potential-difference between the middle plate and positive electrode of the lamp, we have the following equation for v in terms of the ballistic constant and angle of "throw":

$$\begin{aligned} .987 v &= 1110 \sin 2^{\circ} \\ &= 1110 \times .0349 \end{aligned}$$

or

$$v = \frac{111 \times 349}{987} = 39 \text{ nearly.}$$

The potential-difference between the middle plate and the positive electrode as determined by this method is therefore *exactly the same* as the potential-difference between the positive and negative electrodes of the lamp. In other words, when the filament is brought to full incandescence, the middle metal

plate is brought to the same potential as the negative electrode of the lamp. This observation was repeated with several other lamps having middle plates in various positions and of various forms, and always with the same result, viz., that the potential of the middle plate when insulated is brought down nearly to that of the negative electrode.

§ 9. *Experiment 6.*—In order to confirm the results obtained by the condenser method and to eliminate all the conditions which necessarily exist when we attempt to measure potential-difference galvanometrically, an electrostatic method of measuring the potential-difference at any instant between the metal plate and the positive electrode of the lamp was next used. For this purpose a Kelvin multicellular electrostatic voltmeter was employed to determine the potential-difference between the positive and negative electrodes of the lamp and between the positive electrode of the lamp and the middle plate, with the following results:—

A lamp (No. 4) having the plate fixed between the carbon legs was raised to various working voltages and the potential-differences above mentioned taken.

TABLE NO. 7. LAMP NO. 4. *Kelvin Electrostatic Voltmeter.*

Working volts of the lamp.	Static potential-difference in volts between middle plate and positive electrode of the lamp.
41	41
58.1	58.7
61.0	61.5

These observations confirm conclusively the previous results. The insulated metal middle plate is in this case brought to the *same potential* as the base of the negative leg of the carbon; and hence, on measuring electrostatically the potential-difference between that metal plate and the positive electrode of the lamp, we find it to be the same as the potential-difference between the two electrodes of the lamp.

§ 10. *Experiment 7.*—In order to see if this was the case

when the metal collecting-plate had a very small surface placed at some distance from the negative electrode of the lamp, the lamp called No. 1 was employed. In this lamp a platinum wire threaded through the turns of a double spiral 100 volt carbon lamp (see § 11). The lamp was raised to various working voltages, and the electrostatic voltmeter employed to measure at the same time the static potential-difference between the positive electrode of the lamp and the platinum wire, with the following results:—

TABLE NO. 8. LAMP NO. 1. *Electrostatic Voltmeter.*

Working volts of the lamp.	Static potential-difference in volts between platinum wire and positive electrode of the lamp.
62	53
79	75.5
97	85
118	107

The figures in the above Table No. 8 show that when the surface of the collecting-plate is very small and is placed some distance from the base of the negative leg of the carbon it is brought down only to the potential of some point (probably the nearest point) on the carbon conductor, and that therefore the potential-difference between the plate and positive electrode of the lamp is somewhat less than the potential-difference between the working terminals of the lamp. At the same time, however, the electrostatic voltmeter shows no measurable potential-difference between the negative terminal of the lamp and the platinum wire, and the most sensitive galvanometer between these points gives no indication of any current.

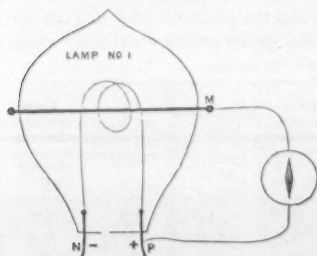
By means of the electrostatic voltmeter it was, however, ascertained that in those cases in which the metallic plate presented considerable surface (several square centimetres) and was placed so that some portion of it was not removed by more than a centimetre or two from the base of the negative leg of the carbon, it was brought down almost immediately

to the potential of the negative terminal of the lamp. If the middle plate is placed at a little distance from the carbon loop then, on testing by the condenser method, it is found that the plate is not instantly brought down to the potential of the negative terminal, but that some few seconds have to elapse before this is the case.

§ 11. A series of experiments was then undertaken in order to determine the effect of varying (1) the surface, and (2) the position of the metal plate in the bulb, and in these experiments the plate was sometimes of platinum and sometimes of aluminium. In all cases the vacuum was a very perfect one, any occluded gases in the plates being got rid of by special means.

Experiment 8.—A normal 100-volt carbon-filament lamp, having a carbon filament coiled in a spiral of two turns (see fig. 7) had a short stout platinum wire ($\cdot 024$ inch diam.)

Fig. 7.



sealed across the bulb so as to thread through, without touching, the spirals of the carbon. The lamp at 100 volts took 1.54 amperes and gave an illumination of 40 candles, equivalent to a power absorption of 3.9 watts per candle-power. The vacuum was very good. This lamp will hereafter be called Lamp No. 1. As before, no current could be detected by a galvanometer when joined up between the platinum wire and the negative electrode, but when the galvanometer was connected between the platinum wire and the positive elec-

trode of the lamp a current of some milliamperes was found passing through it. As in the case of lamp No. 4, this lamp was characterized by a great tendency to change suddenly the value of the current flowing through the galvanometer when the working volts on the lamp were kept perfectly constant. In the first series of observations the milliamperemeter was employed to measure the current flowing between the positive electrode of the lamp and the platinum wire when it was connected between these points, and at and beyond a working-pressure of 90 volts or so the galvanometer would often jump suddenly from one reading to another, when the lamp working volts were kept perfectly constant.

In the following table, No. 9, are collected the results when the working pressure of the lamp was gradually raised from 80 to 100 volts :—

TABLE NO. 9. LAMP NO. 1. *Milliamperemeter.*

Table showing the potential-difference between the positive electrode and the platinum wire, and the current flowing through the galvanometer connecting them, for various voltages of the lamp.

Working volts of the lamp.	Volts between platinum wire and positive electrode.	Milliampere current through the galvanometer.	Working volts of the lamp.	Volts between platinum wire and positive electrode.	Milliampere current through the galvanometer.
80	·1	·016	92	1·7	·270
"	·2	·032	"	2·4	·381
82	·3	·048	93	1·9	·302
83	·2	·032	"	2·6	·413
84	·4	·064	94	3·2	·509
86	·4	·064	"	9·4	1·49
"	·7	·114	95	4·9	·78
87	·5	·080	96	5·8	·92
88	·8	·127	97	7·1	1·13
"	·9	·142	98	8·3	1·32
90	1·0	·159	99	8·9	1·41
"	1·3	·206	100	8·6	1·37
91	1·4	·222	"	10·3	1·64

These figures show that at any definite working electromotive force of the lamp the current between the positive electrode and the middle plate has very variable values, and that it suddenly changes from one value to another without any apparent reason, the working volts of the lamp remaining constant all the time.

If the surface of the collecting-plate is large, say several square centimetres, the potential-difference existing between it and the positive electrode is not found to be so much reduced by attempting to measure it with a galvanometer of about 6000 ohms resistance as it is when the collecting wire presents, as in this lamp No. 1, only a small total surface of about one square centimetre.

§ 12. *Experiment 9.*—A horse-shoe carbon filament, taking 1.3 ampere of current at a working-pressure of 42.5 volts, had a middle plate made of a long piece of platinum wire bent up in a zigzag shape so as to form a rectangular-shaped grating (see fig. 8). The object of this was to ascertain whether a middle plate offering a surface pierced with many apertures was as effective in producing the current as a solid

Fig. 8.

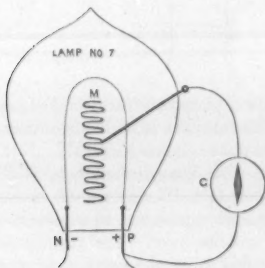


plate of about the same general outline. Practically it was found that this was the case.

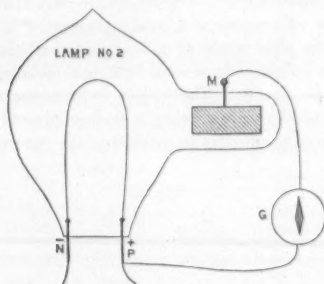
The magnitude of the currents obtained at various working voltages are of the same magnitude approximately as in the

case of a lamp like No. 4, that is to say some 3-4 milliamperes at full incandescence.

§ 13. A set of experiments was then undertaken with the object of examining the special effect of varying the position of the middle plate, and a series of lamps was used in which platinum or aluminium plates held on platinum wires were placed in the lamp bulb, or in tubes opening into it, in various positions. These lamps are generally 50-volt lamps of usual type, and had single horse-shoe shaped filaments.

Experiment 10.—A lamp-bulb had a side tube blown on it (see fig. 9) and a plate about 6 centims. long and 1.5 centims. wide welded to a platinum wire was sealed into it. The

Fig. 9.



platinum plate was placed vertically and edgewise in the side tube and the side tube was in such a position that the plane of the platinum plate coincided with the plane of the horse-shoe filament. This lamp, called henceforth No. 2, when worked at 48 volts took 1.3 amperes of current and gave a light of 17.5 candles, equivalent to a power-consumption of 3.55 watts per candle-power. The vacuum was very good. In the case of this lamp the current between the positive electrode of the lamp and the platinum plate was found to be numerically very much smaller at the usual working pressure of the lamp than was found to be the case in those lamps in which the middle plate was placed between the carbon legs or in the form of a cylinder embracing the carbon.

The current obtained at any definite working voltage was considerably greater when the leg of the carbon nearest the plate was the *positive* leg than when it was the *negative* leg. A series of observations were taken using the lamp at different voltages and measuring with the Elliott galvanometer the potential-difference between the platinum plate and positive electrode of the lamp, and these results were as tabulated below in Table No. 10.

TABLE NO. 10. LAMP NO. 2. *Elliott Galvanometer.*

Table showing the potential-difference between the platinum plate and the positive electrode of the lamp at various working voltages. Positive leg of carbon nearest the platinum plate.

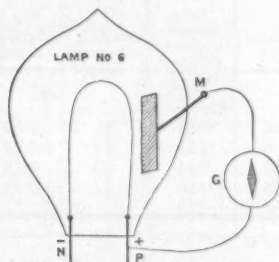
Working volts of the lamp.	Watts per candle-power.	Volts between platinum plate and positive electrode.	Working volts of the lamp.	Watts per candle-power.	Volts between platinum plate and positive electrode.
43	6.26	.088	48	3.55	.199
44	5.80	.111	49	3.32	.236
45	4.42	.144	50	3.12	.289
46	4.10	.156	51	2.94	.352
47	3.80	.172			

If we compare together the results obtained with this lamp No. 2, in which a plate is placed edgewise on and outside the carbon loop, with the results obtained in the case of lamp No. 3, in which the plates embraced the carbon in the form of cylinders, we see the difference produced by the change of position of the plate. Both these lamps, No. 2 and No. 3, are 48-volt lamps when working at normal incandescence. Referring to Table No. 7 in § 7, we see that for lamp No. 3 at 48 volts the voltage difference of the positive electrode and the platinum cylinders was respectively 13.2 and 18.3 volts as measured with the Elliott galvanometer, and this indicated a current of about 1.3 and 1.8 milliamperes flowing through the resistances from the positive lamp electrode to the metal plate; but in the case of lamp No. 2, at 48 volts the potential-

difference between the platinum plate and the positive lamp electrode was only $\cdot 2$ volt, and this corresponded to a current of $\cdot 03$ milliampere nearly. Accordingly the current is greatly diminished when the collecting-plate is placed edgewise to and somewhat outside the loop of the carbon. At normal incandescence the current between the positive lamp electrode and the middle plate when joined by the galvanometer is about $\cdot 03$ or $\cdot 04$ milliampere when the *positive* leg of the carbon is nearest the middle plate, but only about $\cdot 02$ or $\cdot 03$ milliampere when the negative leg is nearest the plate.

§ 14. *Experiment 11.*—In order to compare the previous results just given with those obtained when the collecting-plate was placed broadside to and yet outside the carbon loop,

Fig. 10.



a lamp was made as in fig. 10 in which an aluminium plate was held on a platinum wire just outside one leg of the carbon and with its plane perpendicular to the plane of the horse-shoe.

The aluminium plate was 5 centims. long and 1 centim. wide, and distant from the nearest leg of the carbon about $\cdot 5$ centim. This lamp therefore differed from lamp No. 4 in having the plate outside the carbon loop rather than between the legs. It may be noticed that in this lamp the current here obtained by joining the positive lamp electrode to the plate through a galvanometer was slightly greater when the

leg nearest the plate was *negative* than when it was the positive leg, whereas in the case of lamp No. 2 it is just the reverse. This lamp exhibited also the same effect as lamp No. 4, in that the current flowing between electrode and plate is very liable to "jump" from one value to another even when the lamp is kept at constant volts. The following tabular result of the observations shows this. This lamp is called Lamp No. 6 and was a 50-volt horse-shoe carbon lamp, taking 1.33 amperes of current at a working electromotive force of 50 volts.

TABLE No. 11. LAMP No. 6. *Milampere-meter.*

Table showing the potential-difference between the positive electrode of the lamp and the aluminium plate, and the current in milliamperes flowing through a galvanometer connecting them. Vacuum good.

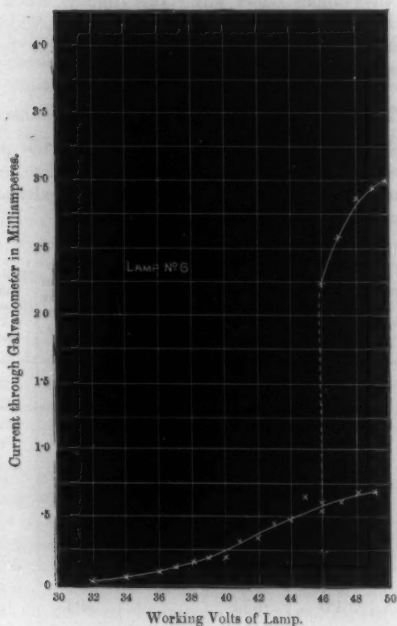
Working volts of the lamp.	Potential-difference of the positive electrode and plate in volts.	Milliampere current through the galvanometer.	Working volts of the lamp.	Potential-difference of the positive electrode and plate in volts.	Milliampere current through the galvanometer.
32	·1	·016	46	3·8	·601
34	·3	·047	"	3·5	·555
36	·6	·095	"	14·1	2·23
37	·8	·126	47	3·8	·602
38	1·0	·158	"	16·2	2·57
39	1·2	·190	48	4·1	·650
40	1·7	·221	"	17·7	2·80
41	2·0	·317	49	4·2	·666
42	2·3	·364	"	18·3	2·90
43	2·8	·444	50	19·5	3·03
44	3·1	·491	"	20·0	3·17
45	4·0	·634	"	4·1	·650

The results in Table No. 11 are plotted in curve No. 5.

This table shows that when the lamp is kept at a constant voltage the current through the galvanometer jumps from one value to another. The fluctuation of the current takes

place when the negative leg of the carbon is the one farthest from the plate. When the leg adjacent to the aluminium plate is the negative one then the current is steady at any definite voltage of the lamp.

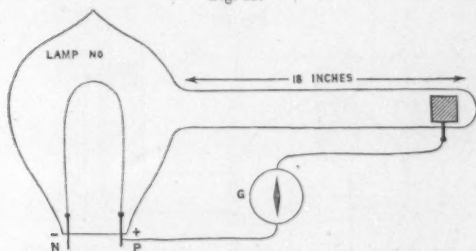
TABLE NO. 11.—CURVE NO. 5.



§ 15. From the above experiments it is clear that the current obtained when a galvanometer is connected between a metallic plate and the positive electrode of the lamp is greater in proportion as the collecting-plate is larger and in proportion as it is brought into close proximity to the base of the

negative leg of the carbon. Also that a plate so placed is brought down to the potential of the negative electrode. It seemed desirable to see how far the removal of the collecting-plate to a great distance from the negative leg would influence these results, and experiments were accordingly tried with a tube of the form shown in fig. 11.

Fig. 11.

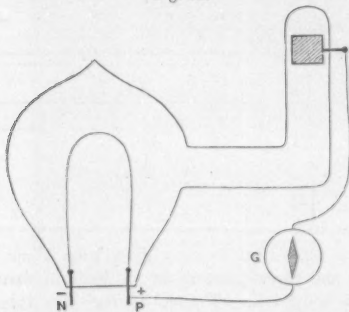


Experiment 12.—In this case a glass tube about eighteen inches long and three-quarters of an inch in diameter was attached to a lamp bulb. The end of the glass tube farthest from the bulb was closed and an aluminium plate welded to a platinum wire was sealed in near this closed end. The plate had a length of about 3 centimetres and a width of about 1 centimetre. The tube formed an extension of the bulb-space, and accordingly this arrangement formed a device by which a metal plate could be removed to a distance of some eighteen inches from the incandescent conductor contained in the bulb. On placing this lamp on a circuit and bringing the carbon to normal incandescence and connecting the terminals of the Elliott galvanometer respectively to the aluminium plate and the positive electrode of the lamp, a very small current was found to be passing through it, not, however, exceeding one ten-thousandth of a milliampere. When the galvanometer was joined in between the aluminium plate and the negative leg of the carbon no current whatever could be detected with this galvanometer, which was sufficiently sensitive to show one hundred-thousandth of a milliampere. We thus find that the removal of the plate to a distance of some

eighteen inches from the incandescent conductors practically extinguishes the phenomenon.

Experiment 13.—Another similar bulb was provided having a side tube blown on it of half the length, viz. about 9 inches long. At the end of this tube was placed a small aluminium plate as before, and the tube was bent up about the middle at right angles (see fig. 12). When the carbon conductor in

Fig. 12.



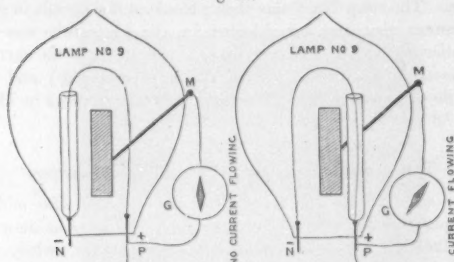
the bulb, which was that of an ordinary 50-volt 16 candle-power lamp, was rendered incandescent by being connected to a circuit of appropriate electromotive force, and the Elliott galvanometer connected in between the aluminium plate and the positive electrode of the lamp, a current of not more than about one twenty-thousandth of a milliamperere was detected. The fact that the "Edison effect" was extinguished when the collecting-plate was placed at the extremity of an elbow-tube was first observed and recorded by Mr. Preece.

§ 16. The effect of position and size of the plate having been examined, the next step which naturally suggested itself was to determine the effect of the different portions of the incandescent conductor in the production of it.

Experiment 14.—A lamp like No. 4 was provided, but in which one leg of the carbon was enclosed in a glass tube of the size of a quill. The glass tube was sealed on to the platinum wire and extended nearly up to the bend of the carbon (see fig. 13). This lamp, called No. 9, was placed on

the circuit in such a manner that the shielded leg was the *positive* leg, and a series of observations taken as usual of the current flowing through the milamperemeter when connected between the middle plate, placed between the carbon legs,

Fig. 13.



and the positive electrode of the lamp. The results are as tabulated below in Table No. 12. The lamp took 1.25 ampere of current at a working electromotive force of 42 volts.

TABLE NO. 12. LAMP NO. 9. *Milamperemeter.*

Table showing the potential-difference between the middle plate and positive electrode of the lamp, and the current flowing through a galvanometer connecting them, when the *positive* leg of the carbon is shielded in a glass tube.

Working volts of the lamp.	Potential-difference of middle plate and positive electrode in volts.	Milliampere current through galvanometer.	Working volts of the lamp.	Potential-difference of middle plate and positive electrode in volts.	Milliampere current through galvanometer.
36	.2	.032	45	6.9	.936
38	.3	.048	46	7.5	1.19
40	.95	.12	47	9.7	1.53
41	1.2	.19	48	12.4	1.96
42	1.8	.29	49	15.1	2.39
43	3.1	.49	50	19.1	3.03
44	4.2	.66	51	23.1	3.66

So far the results are quite normal, and if the results in the above Table No. 12 are compared with those in Table 2 for a similarly constructed lamp with no tubular shield, we find that the magnitude of the current flowing from the positive electrode to the middle plate is in the two cases very much the same. The lamp No. 9 was then placed on the circuit in such a manner that the leg shielded by the glass tube was the *negative* leg, and a similar series of observations of the current between the positive leg (now the uncovered leg) and the middle plate was made. The results were as follows in Table No. 13:—

TABLE NO. 13. LAMP NO. 9. *Milampere-meter.*

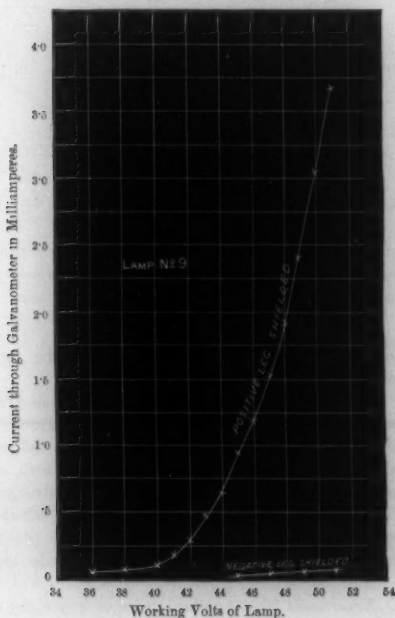
Table showing the potential-difference between the middle plate and positive electrode, and the current flowing through a galvanometer connecting them, when the *negative* leg of the carbon is shielded in a glass tube.

Working volts of the lamp.	Potential-difference of middle plate and positive electrode in volts.	Milliampere current through galvanometer.
45	·10	·016
47	·15	·024
49	·25	·039
51	·30	·044

On comparing the results in Table No. 13 and the previous one No. 12 we see what an immense reduction in the current flowing between the positive electrode and the middle plate is produced by shielding the *negative* leg. Hence the action in virtue of which the current is produced is greatly interfered with by enclosing or covering up the negative leg of the carbon. In this particular case at 51 volts, the current between the positive electrode of the lamp and the middle plate when the *negative* leg is covered up is only $\frac{1}{3}$ of that which it is when the *positive* leg is covered up. The results in Tables Nos. 12 and 13 are plotted together in Curve No. 6.

§ 17. The question remained to be settled whether the nature of the tubular screen in any way affected the results, and the glass tube was accordingly replaced by a metal (aluminium) tube and a lamp (No. 10) taken in which one of

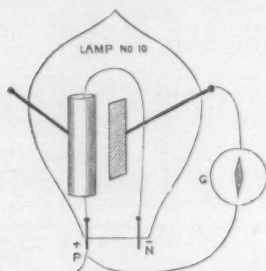
TABLES NO. 12 AND 13.—CURVE NO. 6.



the carbon legs (see fig. 14) was surrounded by an aluminium cylinder extending nearly the whole length of the leg, and also a middle plate of aluminium was placed between the legs. Both the plate and the cylinder were held on platinum wires sealed through the glass. The lamp took 1.25 amperes of current at 41.5 volts.

Experiment 15.—The lamp was placed on a circuit so that the leg shielded by the aluminium cylinder was the *positive* leg. The milamperemeter was then connected between the positive electrode of the lamp and the middle plate, and the usual measurements made. It was found that the current “jumped” a good deal, and that high and low values of the

Fig. 14.



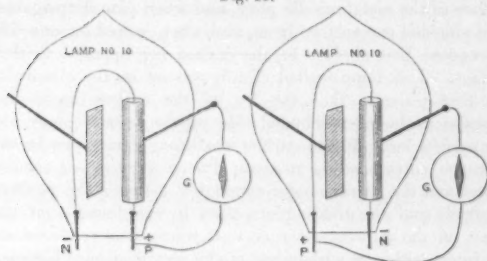
galvanometer current occurred, even when the terminal voltage of the lamp was kept perfectly constant.

The lamp was then reversed on the circuit so that the shielded leg was the *negative* one, all other arrangements remaining the same. The current now between the positive electrode and the middle plate was practically zero, at any rate too small to be measured with this galvanometer. Hence we see that shielding the negative leg, whether by glass or a metallic cylinder, entirely cuts off the production of a current between the positive lamp electrode and the middle plate.

§ 18. *Experiment 16.*—Another series of experiments was made with the lamp No. 10 in which the galvanometer was connected between the positive electrode of the lamp and the aluminium cylinder (see fig. 15), the leg inside the cylinder being either the positive or the negative leg. In this case the middle plate remained unused and insulated and acted as a shield between the cylinder and the carbon leg which was not contained in the cylinder.

It was found that when the cylinder surrounds the negative leg and its surface is, therefore, as much exposed to it as possible the current is a maximum, but that when it includes

Fig. 15.



the positive leg the current is greatly diminished, both by reason of the fact that it opposes less surface to the negative leg, and also because the middle plate acts as a shield between it and the negative leg of the carbon.

§ 19. *Experiment 17.*—In order finally to demonstrate that the negative leg of the carbon loop was the chief active agent in this production of a current between the middle

Fig. 16.

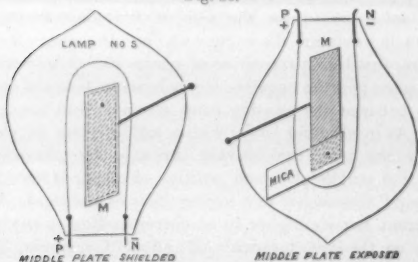


plate and the positive electrode, a lamp like No. 4 was taken having a metal middle plate between the legs, and this middle plate had attached to it a mica screen (see fig. 16) a little

larger than the plate, and so fixed by a loose rivet that it could be shaken in front of the plate so as to shield one side of it, or shaken on one side so as to fully expose the plate. This device was in fact a removable shield attached to one surface of the metal middle plate, and when placed up against it, it shielded one surface from, and when jerked on one side it exposed that surface to, the carbon leg opposite to that surface. This lamp (called No. 5) was set on the circuit in the first place so that the leg of the carbon horse-shoe opposite to the mica-shielded side of the middle plate was the *positive* leg. If the carbon was brought up to an incandescence corresponding to about 3.5 or 4 watts per candle-power and the galvanometer connected between the positive electrode and the middle plate, then it was found that the effect on the galvanometer current which was produced by the interposition or withdrawal of the screen of mica between the positive leg and the plate was not very great. It reduced the current through the galvanometer from about .44 milliampere to .38 milliampere. If, however, the current flowing through the lamp carbon is reversed in direction so that the mica screen is interposed on that side of the middle metal plate which faces the *negative* leg, the result is very different. When the screen is down, the current flowing through the galvanometer from the positive electrode to the middle plate being as before .44 milliampere, the interposition of the mica screen on the side of the plate facing the negative leg reduced the current at once to zero. We find therefore, that the interposition of a mica screen between the middle plate and the negative leg reduces to zero the current flowing between the positive lamp electrode and the middle plate. As in all other lamps with a middle plate set exactly between the legs, the current through the galvanometer joining the middle plate and positive electrode is very liable to "jump" from a low to a higher value or *vice versa*. When the current has its higher value corresponding to any given voltage on the lamp terminals, the effect of screening is less marked, and although the interposition of the mica screen on the side facing the negative leg has an effect of reducing the current flowing through the galvanometer connected

between the middle plate and positive electrode, it is not by any means reduced to zero.

§ 20. The foregoing experiments afford proof that the production of the current through a galvanometer joined between the positive electrode of the lamp and a metal plate placed somewhere in the vacuous bulb, is an effect due chiefly to the *negative* leg of the carbon, and that shielding the negative leg by enclosing it in a glass or metal tube, or covering with a mica screen that surface of the plate which is exposed to the negative leg, either quite prevents or greatly reduces the production of this current. The experiments also have shown that the magnitude of the current flowing through the galvanometer is increased by bringing the plate near to the base of the negative leg, or, better still, making the metal plate in the form of a cylinder and making this cylinder surround the *negative* leg near its base; and correspondingly it is diminished by removing it from the negative leg to a considerable distance, or by shielding this collecting-plate from the radiation from the negative leg of the carbon. The experiments with the condenser have also given evidence that when an insulated metal plate is sealed into a lamp, this plate is brought down either instantly or in a very short period of time to the potential of the *negative* leg near its base or to that of the *negative electrode* of the lamp. In looking for an explanation of these facts we are assisted by our previous knowledge that in carbon incandescence lamps, when working at an efficiency equivalent to 3 to 4 watts per candle-power, there is a gradual loss of carbon from all parts of the conductor. We know also that the carbon molecules which are projected from the conductor are thrown off into a space so highly vacuous that their mean free path is of a length comparable with, or greater than that of the dimensions of the glass bulb. The existence of molecular shadows in incandescence lamps* affords evidence that from intensely heated portions of the carbon conductor carbon molecules are projected in straight lines and move freely forward until they impinge against the glass. Commercial experience informs us that at and above a tem-

* See J. A. Fleming, 'Philosophical Magazine,' August 1885, p. 141.

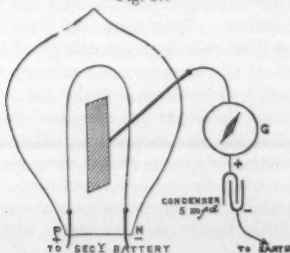
perature corresponding to 3 watts per candle-power this loss of carbon becomes very rapid and thins away the filament in one place, or generally reduces the diameter of the carbon conductor. Hence we have every reason to believe that when in a normal state of incandescence the carbon conductor in a lamp is throwing off in all directions carbon molecules, and that in the vacuum usually obtained the mean free path of these projected molecules is comparable with the dimensions of the vessel containing the conductor. The whole of the experiments which are detailed here seem to be capable of consistent interpretation if we may justifiably make the hypothesis that these carbon molecules or atoms so projected from the conductor when intensely heated by the current flowing through it are all *negatively* charged. Some of the observed facts seem to point to the conclusion that the molecules projected from the incandescent conductor, whether they are portions of the conductor itself or molecules of the residual gases, respectively carry away negative charges proportional in magnitude to the potential of the conductor at the point from which they are thrown off. They may, therefore, be looked upon as condensers of small but definite electrostatic capacity charged to the potential (negative) of that part of the incandescent conductor at which they separate from it. We have then in addition to explain how it comes to pass that there are few or no projected molecules charged positively. Two suggestions may be made on this point: either the radiation of matter is wholly confined to that half of the conductor at a negative potential or the incandescent carbon molecule thrown off from the heated conductor cannot retain a positive charge. There is much to lead to the conclusion that from *all parts* of the incandescent carbon conductor there is a constant radiation of matter carrying a negative electric charge. The nearer down to the negative electrode of the carbon we select our point of observation the greater is this molecular charge found to be. It will be convenient to denote this conveyance of electric charge by moving charged molecules by the term *molecular electrovection*. We can then state the hypothesis thus—from all portions of the negative leg of the carbon loop a process of molecular electrovection is going on when

the conductor is incandescent, the molecular charge being negative, and equal in potential to that of the point on the conductor from which it is projected.

§ 21. On the assumption that a molecular shower of negatively charged atoms was being projected against the middle plate when the conductor of the lamp was incandescent it was considered probable that a positively charged conductor connected to the middle plate would be discharged, and this was found to be the case.

Experiment 18.—A lamp of the form of No. 4, having a middle metal plate placed between the carbon legs, had its middle plate connected to one terminal of the Elliott galvanometer. The other terminal of the galvanometer was connected to one terminal of a condenser of 5 microfarads capacity. The other terminal of the condenser was connected by a wire to the gas pipes of the laboratory. The lamp was actuated by secondary batteries (see fig. 17) not very well

Fig. 17.



insulated. If the condenser was charged to a potential of 50 volts so that the plate next the galvanometer was *positively* charged, then this positive charge was *instantly discharged* when the carbon was rendered incandescent. If, however, the plate of the condenser in connexion with the middle plate through the galvanometer was charged *negatively* the condenser was not discharged when the lamp was illuminated by rendering its carbon incandescent. It is a very striking experiment to see a condenser charged with this

amount (250 microcoulombs) of electricity instantly discharged when its positive coating is brought into connexion with the middle plate of such a lamp. The discharge may be brought about either by joining up the positive side of the condenser to the middle plate first, and then rendering the carbon of the lamp incandescent by switching on the lamp, or the lamp may be first of all illuminated and then the junction of the condenser effected. In both cases the middle plate when positively electrified is instantly discharged.

It was found that if the lamp carbon is rendered incandescent by a highly insulated secondary battery, then in order to produce the discharge, the plate of the condenser not in connexion with the middle plate, and which is *negatively* charged, must be somewhere connected with the battery circuit. It does not matter, however, whether the wire from the negative side of the condenser is in connexion with the positive or the negative pole of the secondary battery actuating the lamp; all that is necessary is that the negative side of the condenser should be in conducting connexion with the circuit of the incandescent carbon. The experiment may be interpreted by considering that this negative charge of the condenser can escape out of the incandescent conductor and discharge across the highly vacuous space to the positively electrified cool middle metal plate; but that a positive charge cannot be discharged out of the hot conductor, or, which amounts to the same thing, a negative charge cannot discharge across from the cool metal plate to the incandescent carbon which is positively charged. We have then a unilateral conductivity exhibited by this highly vacuous space bounded by two electrodes one of which is incandescent and the other of which is cold. Negative electricity is discharged at once out of the hot surface but not out of the cold, and a negative discharge can take place from hot to cold but not *vice versa*. When the discharge of a charged condenser is effected by connecting the positive plate, through a galvanometer, with a metal plate sealed into the lamp and the negative plate with the lamp circuit, and then switching on the lamp, there is a curious instant of delay before the discharge begins. When the metal plate is placed very near the negative leg of the carbon the discharge of the condenser is complete

in one instant. This the case when a lamp of the type No. 4 (fig. 18) is used. If, however, we employ a lamp of the type No. 2 (fig. 10), in which the metal plate is at some distance from the negative leg of the carbon, the discharge of the condenser is long drawn out and the electric charge in it is as it were tapped off slowly and not in one short sharp discharge.

Moreover this effect of discharging a condenser takes place only when the carbon is above a fair red heat. At brilliant incandescence and when the carbon is above a temperature corresponding to 3 watts per candle-power, the discharging power of a lamp of the type of No. 4 is very great. A condenser of 10 or 20 microfarads capacity charged to 50 volts is discharged instantly if its positive plate is connected to the metal plate placed not far from the negative end of the incandescent carbon conductor.

The foregoing results were confirmed with lamps of other types. Using, for instance, a lamp like No. 6 with the aluminium plate placed outside the carbon horse-shoe and near the leg, the same discharging power for positive electricity was found. It was not dependent on the direction of the current through the lamp carbon, although it seemed a little more vigorous when the leg nearest the plate was the negative leg. As above observed, the rate of discharge was much reduced when employing a lamp having the metal plate placed edgewise on to the carbon and some way from it, as, for instance, when employing a lamp of the form of No. 2.

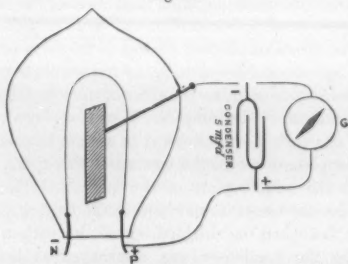
§ 22. *Experiment 19.*—A series of experiments was in this case also tried to determine the effect of shielding the negative leg of the carbon. The lamp No. 9 was employed, in which one leg of the carbon was enclosed in a glass tube connecting the positive plate of a charged condenser through a galvanometer with the middle plate of the lamp, and the negative plate of the condenser somewhere to the battery circuit; it was found that when the shielded leg of the carbon was the positive leg the condenser was discharged as before. If, however, the shielded leg or leg enclosed in the glass tube was made the negative leg, which could be done by reversing the current through the carbon conductor, then the condenser was not discharged when its positive plate was connected with the middle plate.

The same fact was less perfectly exhibited by employing the lamp with the middle plate having a removable mica shield on one side. We are thus able to assure ourselves that the active agent in producing this discharging effect upon a positively charged body connected to the middle plate is the negative leg of the carbon conductor. The experiments were varied in many ways, but all pointed to the conclusion that if a charged condenser is connected to two terminals, one of which is a metal plate and the other a carbon conductor, both enclosed in a high vacuum but yet separated from each other by an inch or so of distance, the condenser is discharged instantly when the carbon terminal is rendered highly incandescent, provided that the negative plate of the condenser is in connexion with it.

§ 23. If the condenser is left in contact with the middle plate under some circumstances, not only is it discharged if previously charged but is charged again in an opposite direction.

Experiment 20.—A condenser of 5 microfarads capacity perfectly discharged has its poles or terminals connected for one instant, one with the middle plate of No. 4 lamp and the other with the positive electrode of the lamp (see fig. 18).

Fig. 18.



On removing it and testing it with the galvanometer G it is found that the condenser plate in connexion with the middle plate of the lamp has received a negative charge and the other plate of the condenser a positive charge.

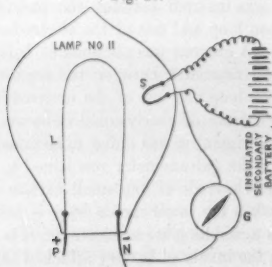
If, however, the condenser is connected between the negative electrode of the lamp and the metal middle plate of the lamp, on insulating and testing it we find it has not the slightest charge.

It is very astonishing to see how instantly a condenser of very large capacity is charged when one pole of the condenser is connected to the middle plate and the other to the positive electrode of the lamp.

§ 24. In considering the behaviour of the heated carbon electrode and the cool metal plate in their respective powers of discharging the positive or negative charge of the condenser, it seemed that the fundamental fact was the power of the heated surface to discharge negative electricity out of itself. Hence arose the question, how far the observed facts would be modified if the middle metal plate itself could be also heated. One way by which this might have been done would have been to have rendered this plate incandescent by heating it by radiant heat concentrated by means of a powerful mirror or lens. Some experiments tried in this way were not satisfactory, and consequently a method was adopted in which a middle plate of carbon could be rendered incandescent electrically.

Experiment 21.—A vacuum tube was provided with two carbon conductors (see fig. 19), one the ordinary carbon

Fig. 19.



filament *L* of a 50 volt lamp, and the other the small carbon *S* of a 4 volt lamp. The smaller carbon was sealed in the

usual way through the glass and placed so as to stand symmetrically between the legs of the larger carbon loop.

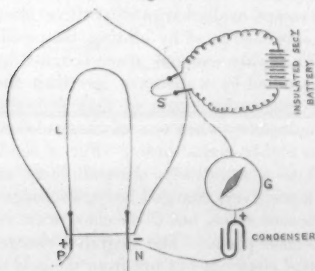
The smaller carbon could be rendered incandescent by an insulated battery of fifteen secondary cells, appropriate resistance being introduced. The larger carbon also could be rendered incandescent by the proper electromotive force. If the smaller carbon was kept cold and employed simply as a third electrode or middle plate, all the phenomena previously described as happening with metal middle plates of aluminium or platinum took place. If the small (cold) carbon loop is connected through a galvanometer with the positive electrode of the larger carbon loop when this last is rendered incandescent by a current, we find as usual a current of a few milliamperes passing through the galvanometer from the positive electrode of the larger carbon to the small carbon. If the small carbon (still cold) is connected through the galvanometer to the negative electrode of the larger carbon we get no current. This is the normal effect, and it is the same for a cold carbon conductor used as a middle plate as for a metal middle plate.

Experiment 22.—The next experiment consisted in making this small carbon incandescent by an insulated secondary battery, appropriate resistance being inserted so that the carbon was brought to the normal condition of temperature as indicated by its incandescence. When this was done the galvanometer was inserted between the positive electrode of the large carbon loop and one of the electrodes of the small carbon loop. A current was obtained as before. On connecting the galvanometer between the *negative* electrode of the large carbon loop and one of the electrodes of the small carbon loop, a current of nearly equal value was now obtained. In this last experiment it was found to be immaterial whether the terminal of the galvanometer was joined to the positive or to the negative electrode of the small carbon loop. Hence we find that when the small carbon loop is not incandescent and is used as a middle plate or electrode, it is brought down together with the insulated battery attached to it to the same potential as the negative end of the large incandescent carbon, and we get as usual a current through a galvanometer connected between the positive electrode of the large incandescent

carbon and any point on the small cold carbon, and no current between the negative electrode of the large hot carbon and the small cold one. On rendering the smaller carbon loop incandescent this is all changed. The smaller carbon, now hot, is not brought down to the potential of the negative ends of the larger carbon, and we get a current through a galvanometer connected between either positive or negative electrode of the large hot carbon and any point on the circuit of the smaller equally hot carbon.

§ 25. *Experiment 23.*—With this same vacuum tube having double carbons, further experiments were performed on the discharging power of the hot and cold electrodes for positive and negative electricity. The two carbons could be rendered incandescent either simultaneously or singly by two sets of insulated secondary batteries attached to each respectively. For the sake of distinction we shall speak of the large carbon loop in this bulb as the L loop and the smaller one as the S loop. A condenser of 5 microfarads capacity (see fig. 20)

Fig. 20.



was employed, which was charged to a potential of about 50 volts. When the positive plate of this charged condenser was attached to the carbon L and the negative side to the carbon S, then on making L incandescent by its own insulated battery and keeping S cold, the condenser was found not to be discharged when insulated and tested by a galvanometer. If, however, the same charged condenser was

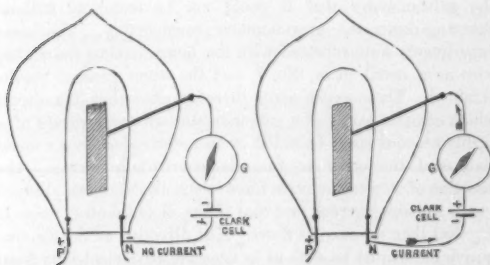
connected in the same way to the two carbons and the carbon S, to which the negative side of the condenser was attached, was made incandescent, the condenser was instantly discharged. If the direction of the charging of the condenser was reversed the same rule was found to hold good. The condenser was discharged if the *negatively* charged plate of the condenser was connected to the *incandescent* carbon loop, but not if it was connected to the cold carbon loop. Beginning with the condenser charged and connecting it in between the two carbon loops, neither of them being incandescent, then the condenser was discharged instantly if that loop to which the *negatively* charged side of the condenser was attached was rendered incandescent, but not discharged if the loop to which the positive side of the condenser was connected was rendered incandescent. If both loops were rendered incandescent simultaneously the condenser in any case was discharged, but apparently at an accelerated rate. These experiments show again that if two carbon electrodes are sealed into a high vacuum, negative electricity escapes very freely out of either electrode if it is rendered incandescent, but that the escape or discharge of positive electricity is not in the same way facilitated by heating the positive electrode. Accordingly a highly vacuous space bounded by two carbon electrodes separated by a distance less than the mean free path of the gaseous molecule at that pressure, presents a unilateral conductivity when one of these electrodes is cold and the other highly incandescent. For if the hot electrode is connected to a negatively charged body and the cold electrode to a positively charged body, discharge takes place across the vacuous space, but if the charges are reversed then no discharge takes place. The negative charge can escape from the heated electrode but not from the cold one.

§ 26. *Experiment 24.*—The question of the apparent unilateral conductivity of the vacuous space bounded by a hot and a cold electrode was then further examined by the aid of the lamp No. 6 formerly used.

In this lamp an aluminium plate is sealed into the vacuum and placed just outside the carbon horse-shoe. If a sensitive galvanometer (the high resistance Elliott galvanometer) is joined up between the metal plate and the negative electrode

of the lamp, then, as in other cases when the lamp is in action, no current of a magnitude much greater than $\cdot 0001$ of a milliampere is detected. If a single Clark standard cell is inserted in the galvanometer circuit (see fig. 21) with its negative pole attached to the middle plate and its positive pole to the galvanometer terminal, the current is barely if at all increased. In this case the negative pole of the Clark cell is in connexion with a cold metal electrode and the *positive* pole is in connexion through the galvanometer with the

Fig. 21.



incandescent carbon electrode, and under these circumstances the galvanometer detects no current flowing. The position of the Clark cell is now reversed, and it is joined up so that its positive pole is in connexion with the middle plate and its *negative* pole in connexion, through the galvanometer, with the incandescent carbon electrode. It is then found that a considerable current of some few milliamperes in magnitude is flowing through the galvanometer. The direction of this current in the ordinary way of speaking is *from* the negative electrode of the lamp through the galvanometer *to* the metal plate sealed into the bulb. We thus find that a negative current of electricity can be made to flow across the vacuous space between the incandescent carbon and the metal plate *from* the hot carbon *to* the cooler metal plate, but not in the reverse direction. The space presents an apparently unilateral conductivity.

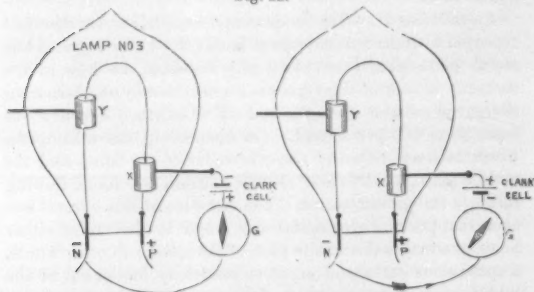
§ 27. *Experiment 25.*—The same experiment was repeated, only using instead of a Clark cell an insulated secondary battery of 25 small cells. When the secondary battery (see fig. 21) was connected with its negative pole to the metal plate and its positive pole through the galvanometer to the negative electrode of the carbon, no current greater than that found with the Clark cell similarly arranged was found; but if the secondary battery was reversed and joined up with its positive pole to the middle plate and its negative pole through the galvanometer to the negative electrode of the incandescent carbon, then so strong a current flowed through the galvanometer that it could not be measured without shunting-down the galvanometer considerably. The same experiments were repeated with the lamps having the zigzag wire as a metal plate, No. 7, and the same general results obtained. These experiments therefore show that in a circuit which consists partly of a galvanometer-wire and partly of a highly vacuous space bounded by two electrodes—one a metal plate and the other an incandescent carbon surface,—the insertion of an electromotive force in one direction can produce a very sensible current, but that if the electromotive force is reversed then no current flows. The direction of the electromotive force must be such as to urge negative electricity from the hot surface to the cold across the vacuous space.

§ 28. *Experiment 26.*—In order to make use of different parts of the incandescent conductor as the electrode opposed to the metal plate, recourse was had to the lamp No. 3, with metal (aluminium) cylinders embracing without touching the carbon at two different places. These cylinders, as before, we will call X and Y (see fig. 22). Cylinder X was the one near the base of the positive leg of the carbon, and cylinder Y was the one near the top of the negative leg.

When the galvanometer was connected between the negative electrode of the lamp and the cylinder X surrounding the lower part of the positive leg, no perceptible current was found to be passing when the carbon was rendered incandescent. On inserting a single Clark cell in series with the galvanometer so that the negative pole of the cell was in connexion with the cylinder X and the positive pole of the cell through the galvanometer in connexion with the negative electrode of the lamp,

hardly any perceptible current was found to be passing (see fig. 22). The Clark cell was then reversed, connecting the positive pole of the cell to the cylinder X and the negative pole through the galvanometer to the negative electrode of the lamp. On bringing the lamp into action, a considerable current of several milliamperes was found to be passing in such a direction that a current of positive electricity was flowing across the vacuous space from the metal cylinder to the hot carbon, or a negative current from the hot carbon to the cooler

Fig. 22.



metal cylinder. On switching-off the lamp, there was a curious "kick" or "throw" of the galvanometer, indicating a sudden rush of current in the same direction as the steady current which the cell had been sending. These effects occurred also when the cylinder Y was employed, and the galvanometer with or without the cell in series joined in between the negative electrode of the lamp and the cylinder Y embracing the top part of the negative leg of the carbon (see fig. 22); but in the last case the steady current sent by the cell across the vacuous space between the cylinder and the hot carbon was only about a quarter as great in magnitude as when the cylinder X was employed. There was the same kind of "kick" of the galvanometer on breaking the lamp-circuit. These experiments evidently showed that the highly vacuous space between the hot carbon traversed by its own current, which rendered it incandescent, and the insulated cylinder possessed a sort of unilateral

conductivity, negative electricity from a separate source of small electromotive force being able to be forced through it from the hot carbon surface to the cooler metal surface, but not in an opposite direction.

§ 29. In the above-recorded experiments the carbon conductor was rendered incandescent by a unidirectional or continuous current in a highly perfect vacuum. In seeking for an hypothesis to connect them together, it became essential to ascertain how the effects would be modified if the vacuum was imperfect and if the current was alternating instead of continuous.

Experiment 27.—The fundamental experiment was therefore repeated with the normal type of lamp (No. 4), having a middle metal plate placed symmetrically between the legs of the carbon. A lamp of this type was set in action by an alternating current of suitable strength and of which the frequency was some 80 to 100 per second. On connecting the milampere-meter between *either* of the electrodes of the lamp and the middle plate, a *continuous* electric current was found flowing through the galvanometer. The direction of this current was such that positive electricity was found to flow from either lamp terminal to the middle plate of the lamp. In other words, a continuous current of negative electricity flowed out of the middle plate to one or other of the two terminals of the lamp, viz. to that terminal to which the other extremity of the galvanometer was joined. Hence, since in this case each leg of the carbon becomes in rapid succession positive *and* negative when the lamp is operated with an alternating current, the unilateral effect observed of a current flowing between the middle plate and the positive leg, when the current through the carbon is a continuous current, is here found to exist equally between the middle plate and *both* terminals of the lamp. This is only what might have been expected. The potential of the middle plate is then not the same as that of the base of either leg of the carbon, but something between the two depending upon the position of the plate.

§ 30. *Experiment 28.*—The effect of lowering the vacuum was also the subject of experiment. In a lamp with a highly perfect vacuum no current greater than about '0001 milli-ampere is observed when a very sensitive high-resistance

galvanometer is joined up between the negative electrode of the carbon and the insulated middle plate, and, as we have seen, the experiments with the electrostatic voltmeter showed that the plate was brought down under these circumstances to the potential of the base of the negative leg of the carbon. If, instead of employing a very perfect vacuum, a bad one is produced by imperfectly exhausting the lamp, then it is found that under these conditions the Elliott galvanometer indicates a not inconsiderable current of something approaching to a milliampere when joined in between the negative electrode of the lamp and the middle plate. Hence, when the vacuum is imperfect the equality in potential between the middle plate and the negative electrode is not maintained.

The direction of the current in this last case is such as to show that negative electricity is flowing through the galvanometer from the negative electrode of the lamp to the middle plate. In other words, negative charge is carried over from the plate to the positive leg of the carbon across the imperfectly vacuous space; and the means by which this is effected is the residual air. This seems to afford proof that the normal effect of the molecular electrovection of negative electricity from the negative leg is due to carbon molecules, and that the presence of residual air exhibits itself, when present beyond a certain amount, in producing an effect which the carbon molecular electrovection cannot produce.

§ 31. *Experiment 29.*—It seemed very desirable to ascertain if the effect of molecular electrovection exists in the case of an incandescent platinum wire rendered vividly incandescent in a highly perfect vacuum. A bulb was accordingly constructed similar in every way to lamp No. 4, but having a platinum-wire horse-shoe conductor and a platinum middle plate. When this wire was rendered highly incandescent by a continuous current, a sensitive galvanometer (the high-resistance Elliott) showed a current of about one five-thousandth of a milliampere when connected between the positive electrode of the incandescent wire and the middle plate, but little or no current when connected between the negative electrode and the middle plate. This molecular *electrovection* current was thus very much less in magnitude than that observed in the case of the carbon filament lamps,

but it is in the *same direction*. We are, however, enabled to state that at a condition of vivid incandescence just short of fusion a platinum wire *in vacuo* exhibits the same effects as a carbon filament, and that it can disturb the electrical condition of an insulated metal plate near it sealed into the same vacuum and tends to bring it down towards the potential of the negative end of that platinum wire.

On the hypothesis that all these effects are due to a scattering of negatively charged molecules from the incandescent conductor, we must affirm that the same process goes on in a platinum wire rendered incandescent in a vacuum, only that the radiation of matter is far greater in the case of the incandescent carbon than it is in the case of the incandescent platinum.

§ 33. If a lamp is selected having an insulated plate fixed between the legs of the carbon filament, it is found that under certain conditions the electric conductivity of the vacuum space between the plate and the negative leg is much affected by the presence of a magnetic field. If a galvanometer, preferably a movable coil galvanometer, having a resistance of about 500 or 600 ohms, is connected between the middle plate and the negative leg it will show but little current passing when the lamp is incandescent at normal temperature. If the volts on the lamp terminal are raised so that the filament is brought into a state of incandescence corresponding to about 2.5 or 3 watts per candle, then the galvanometer will show a small current passing through it. If then a horse-shoe magnet is held so as to create a magnetic field the direction of which is across the space between the plate and the negative leg, the current indicated by the galvanometer immediately decreases considerably. This happens irrespective of the direction of the field so long as it is across the direction of the line joining the negative leg and the middle plate. This indicates that the presence of this transverse field increases the resistance of the rarefied gas. The galvanometer current responds to the presence of the magnet in a manner which shows that the resistance to the flow of the current through the gas is increased by creating a magnetic field at right angles to the line of the current. The general fact that gaseous resistance

is increased by such a transverse magnetic field has been already noted and described by Professor J. J. Thomson. The behaviour of bismuth as regards electrical resistance in a magnetic field is strikingly similar.

The "jumping" of the current from one value to a higher, which has been already mentioned, appears to be due to something equivalent to a sudden change in the resistance of the space between the negative leg and the middle plate when the lamp is in action and at high incandescence. The fact of sending a small current through this space seems to effect a change in the qualities of the rarefied gas as a conductor which makes it conduct better. There are certain after-effects in some cases which are strongly similar to the polarization of electrodes observed in the case of liquid electrolysis, and which seem to point to the validity of the view that gaseous conduction is effected by a similar process.

The experiments also confirm the opinion of Professor J. J. Thomson that gases, or at least certain gases in a rarefied condition, are very good conductors, and they show that the greatest part of the obstacle to conduction through a vacuum-tube is at the electrodes and may be largely removed by heating the kathode to incandescence.

DISCUSSION.

Prof. S. P. THOMPSON said he would like to have some information as to the state of exhaustion of the lamps; whether this was such as is found in ordinary commercial lamps, or whether it more nearly approached that used by Crookes. A great change in the conductivity, &c., took place at an exhaustion slightly greater than that ordinarily found in incandescent lamps. It would be of interest to vary the size of the kathode, and to investigate whether the magnitude of the effects observed depended on the fall of potential per unit length along the filament. Another point was whether the position of the plate for which the effect was a minimum was the same for all lamps, or whether it changed with the volts and the length of the filament employed. Again, did the minimum occur at a certain

fraction of the distance between the positive and negative leads, or, as was the case in some of the phenomena observed by Crookes, at a definite distance from either of the leads? These points might be investigated by means of a lamp with a straight filament, where the fall of potential per unit length along the filament could be made the same as with the loop-shaped filament, but the fall of potential per unit length in the vacuum would be different. The author's proposed experiment of heating the kathode by concentrating on it the rays of a lamp did not seem to him (Prof. Thompson) to differ materially from Crookes' experiment in which an incandescent wire, heated by a current, was used as the kathode.

Mr. SKINNER said that the heating of the kathode by means of a "burning-glass" could easily be carried out.

Mr. BLAKESLEY pointed out that it would be quite possible to produce an increase of the current by means of a magnet.

Mr. SEARLE said that Prof. J. J. Thomson had shown that a magnet affected the conductivity of a gas.

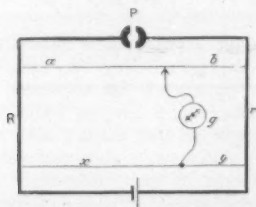
Prof. FLEMING, in his reply, said that no doubt the effects were largely dependent on the vacuum in the lamps. The lamps employed were exhausted to the ordinary commercial vacuum. Since it was found that the "treating" was more worn off the negative leg of the filament, and that a screen placed between the legs of the filament was more blackened on the side turned towards the negative leg, it would appear that the particles of carbon were shot off from the negative leg, and hence perhaps the charge was carried by these carbon molecules.

XVIII. *Adjustment of the Kelvin Bridge.**By* ROLLO APPELYARD*.

In a recent paper by Mr. J. H. Reeves† an alternative method of adjusting the ratios of the resistances in the Kelvin bridge was described; the following remarks may be regarded as a note upon that paper.

The measurement of a resistance by the arrangement adopted by Mr. Reeves involves two operations. It may be shown, however, that for certain purposes the two adjustments can be combined mechanically, and balance effected by a single test.

Fig. 1.



Consider the conductors R , r , a , b , x , and y in fig. 1. It is required to compare R with r . In the accepted form of the Kelvin bridge a , b , x , and y have fixed values, such that $ay = bx$, and balance is obtained by the one operation of varying R or r . Mr. Reeves prefers to keep R and r fixed; and, consequently, he has first to find provisional values of a , b , x , and y such that $ay \approx bx$; and then to balance again, using these values, to find $R : r$; with the plug, this time, inserted at P .

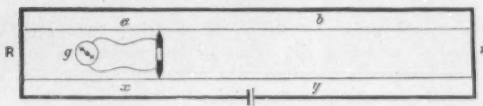
But fig. 1 suggests that $a + b$, and $x + y$, or convenient parts of them, may each be formed into a slide-wire, and that these two slide-wires may be placed parallel to one another, with a double sliding-contact between them, as

* Read April 24, 1896.

† *Supra*, p. 166.

shown in fig. 2. Then, in whatever position the slider may be, the fundamental ratio $ay = bx$ is always maintained, and

Fig. 2.



the first condition of the Kelvin bridge is mechanically fulfilled. The one adjustment consists in moving the double slider along the bridge until there is no deflexion of the galvanometer at g ; in which case

$$\frac{a}{b} = \frac{x}{y} = \frac{R}{r}.$$

Since writing the above, I have referred to the original paper of Lord Kelvin (*Proc. Roy. Soc.* vol. xi. p. 313, 1861), and find that he proposes the use of parallel slide-wires for his auxiliary conductors; I have no doubt he had in view some such apparatus as that which I have here suggested. A Kelvin bridge with a single slide-wire was used by Matthiessen and Hockin in their differential method; it is described by Clerk-Maxwell in '*Elec. and Mag.*' vol. i. p. 406 (1873).

DISCUSSION.

Mr. REEVES said that apparently the author had completely missed the object of his (the speaker's) paper. For the object there aimed at was to make use of such sets of resistance-coils as are always to be found in any laboratory. In the author's arrangement it would be necessary to carefully calibrate the two wires, and also, since the resistances used must necessarily be small, to determine the resistance of the contacts.

Prof. AYRTON (communicated) said the author's suggestion was ingenious, but did not obviate the necessity for much of Mr. Reeves' "addition." Further, Mr. Reeves' proposal to employ ordinary resistance boxes was not made because such resistances are absolutely necessary, but because, since

they are to be found in every laboratory, their use saves the expense of such a wire resistance, accurately calibrated, as Mr. Appleyard employed.

Mr. APPLEYARD, in his reply, said that his instrument was designed for use in a factory, where the time saved in making a series of tests was of more importance than the cost of the instrument.

XIX. *The Effect of Wave Form on the Alternate Current Arc.* By JULIUS FRITH, 1851 Exhibition Scholar*.

In the paper by Dr. Fleming and Mr. Petavel, recently read before the Physical Society, on the Alternate Current Arc†, I think too little attention was paid to the wave form of the alternate current used.

It is known that if the arc is allowed to exert a preponderating influence at all on the alternate current circuit, it alters the wave form of both the current and the P.D. in a very marked degree.

As an illustration of the change produced in the wave form by the character of the external circuit, I give some curves for alternate current arcs taken from a paper which I read before the Manchester Literary and Philosophical Society in March 1894.

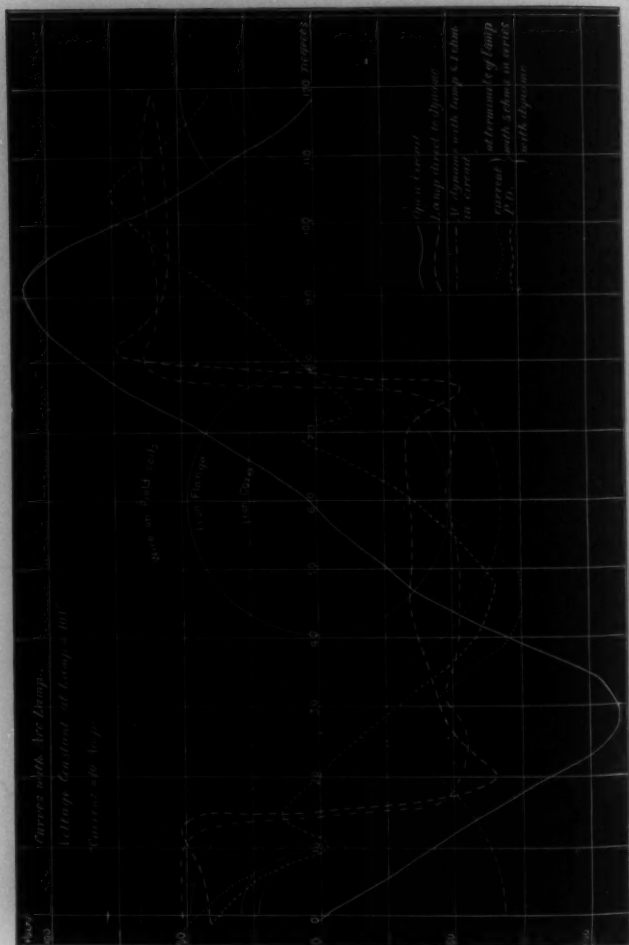
Here is shown, first the E.M.F. curve of the machine on open circuit, which is rather more peaked than a sine curve and involves the third harmonic largely. Next is shown the curve obtained under the same conditions, but with an arc lamp taking 10 amperes at 40 volts joined direct to the machine.

The lag recorded is due to the self-induction of the machine, which was a "Wilde" alternator, the armature of which contained iron.

It will also be noticed that the first ordinates of the curves are not quite equal to the last ordinates; this is due to the

* Read April 24, 1896.

† *Supra*, p. 115.



curve being slightly asymmetrical. The whole period is only one third of the revolution of the alternator (shown as 120 degrees in the figure). In one complete revolution this lack of symmetry would of course disappear.

It is seen that the arc alters the wave form from a peaked curve to a very flat topped curve, changing sign almost instantaneously, and with two small maxima, which occur respectively just before and just after each reversal. The next curve shows the effect of adding a resistance of 1 ohm in series with the arc. This smooths out the two maxima, but otherwise does not affect the shape materially.

The current and P.D. curves with 5 ohms in series with the arc are next shown. Both waves now assume much more the form of the E.M.F. curve on open circuit, except at the zero line. Here the P.D. curve crosses the line twice in each direction, and the current curve runs parallel to the zero line for some way before crossing it.

These curves show, I think, the great effect that the arc has on the wave form, and also how this effect can be destroyed by placing resistance in series with the arc.

In a paper by Rössler and Wedding, which appeared in the 'Electrician' for August 31st, 1894, it was proved that an alternate current arc is more efficient, that is, gives a higher candle-power for the same electric power consumed, when the alternating current feeding it has a flat-topped than when it has a peaked wave form.

Rössler was, however, mistaken, I think, in assuming that the machine was making that curve. Indeed, this mistake runs through the whole of this otherwise most valuable paper. Rössler took three machines giving, as he thought, wave forms from the extremely peaked to the extremely flat wave, and determined the efficiencies of the same arc lamp for each of them. Whereas exactly the same results might have been obtained from one machine alone on causing it to give a higher voltage by increasing the field excitation and then absorbing the excess of voltage in resistance, exactly as in the case of the Wilde alternator above referred to.

I understand that in the experiments described by Dr. Fleming and Mr. Petavel there was always a resistance amounting to 7 ohms outside the arc, and hence a wave form

was forced upon the arc which, as Rössler has proved, is not the most efficient one, and which the arc would convert into a form better suited to itself if it had been allowed to do so, as it is in commerce. This consideration must affect, not only the efficiency, but also the curves which Fleming and Petavel obtained for the variations in the luminous intensity.

Further, this action of the arc in modifying the wave form may throw some light on the discrepancy between the efficiency of alternate current arcs as determined in the laboratory and that stated to be obtained in practice.

When an arc is run in the laboratory, a large resistance is almost certain to be put in series with it to ensure that degree of steadiness which is essential to exact measurement, and hence the arc cannot alter the wave form. In the commercial use of arcs, on the contrary, the circumstances are widely different. In this case, for economic reasons, the arc must form a large percentage of the total "reactance" of the circuit, and therefore can easily alter the wave to the form required for the greatest efficiency.

It is interesting to note that the wave form giving the best result for the arc is almost exactly the opposite to that giving the best efficiency for transformers. In the former case a flat-topped wave is best, while for the maximum efficiency of transformers an exceedingly peaked wave is best, as lately found by Dr. Rössler.

This points to the building of alternators for use with transformers in such a way as to give peaked wave-forms.

In the case of the arc the building of machines to give the most efficient wave form is not so necessary, since, generally, the arc itself has the power of automatically converting *any* wave form into the one best suited to its requirements. Nevertheless, when the arc has to run in series with a large resistance it is of the utmost importance for obtaining the best efficiency that the machine should give a flat-topped wave.

City and Guilds of London Central Technical College,
April 2, 1896.

DISCUSSION.

Mr. BLAKESLEY said it seemed as if the more nearly the alternate current resembles a direct current, *i.e.* the longer in each period the current remains constant, the greater is the efficiency of the arc.

Mr. PRICE asked what was the cause of the reaction of the arc on the wave form?

Mr. TREMLETT CARTER asked whether previous observers' results were vitiated by this action of the arc on the wave form?

Prof. AYRTON (communicated) considered the author's suggestion of great importance, as bearing on the efficiency of the alternate current arc.

Prof. S. P. THOMPSON said that the dynamo employed by the author was one in which there was a large quantity of iron in the armature, so that the self-induction was large. Was it not on account of this large coefficient of self-induction, which would tend to keep the current constant, that the arc was able to alter the wave-curve? If an arc is connected to the mains of a supply station in which a number of machines in parallel are feeding a number of lamps, would the arc still be able to affect the wave form of the current?

Mr. TREMLETT CARTER asked if the author had tried the effect of replacing the arc by a resistance such that it would absorb the same volts as did the arc, and comparing the curves for the current and impressed P.D. with those obtained with the arc.

The AUTHOR, in his reply, said that the effect of the self-induction of the machine was shown in the curves. Current curves had not been taken with the arc straight on the machine. The current and self-induction were the same for all the curves, the voltage of the machine being increased by increasing the field when resistance was placed in series with the arc. When, as is commonly the case, special machines are used to supply arcs, and the load consists solely of arcs, the arcs could alter the character of the wave form. If the arc is replaced by a resistance, the wave form is of the same type as is obtained for the E.M.F. of the machine on open circuit.

XX. *On the Magnetic Field due to an Elliptical Current at a point in its plane within it.* By J. VIRIAMU JONES, M.A., B.Sc., F.R.S., Principal and Professor of Physics in the University College of S. Wales and Monmouthshire, Cardiff*.

§ 1. IN a communication presented to Section A of the British Association† at Oxford in 1894, giving an account of measurements made to determine the value of the International Ohm in absolute measure by the method of Lorenz, I referred to a small error consequent on the fact that my standard coil is wound on a cylinder, the section of which at right angles to the generating lines is not a circle but an ellipse of small excentricity.

In considering the effect of this ellipticity on the value of the resistance calculated from the observations, it must be noted that the ordinary formula implies that the coil is circular. This formula is

$$R = Mn,$$

where R = the resistance in absolute measure,

M = the coefficient of mutual induction of the standard coil and disk circumference,

n = the number of revolutions of the disk per second.

But we are primarily concerned with the balance of the electromotive force between the ends of the resistance when the current is passing through it, and the electromotive force between the points of contact of the brushes on a radius of the rotating disk when the same current is passing through the standard coil; and this balance gives us, in the general case, the formula

$$R = 2\pi n \int_{a_0}^{a_1} r H dr, \quad \dots \dots \dots (A)$$

where a_0 and a_1 are the distances from the centre of the disk of the points at which the internal and external brushes are

* Read May 22, 1896.

† Report of Electric Standards Committee, Appendix II., Brit. Assoc. Report, 1894.

applied, and H is the magnetic-field intensity at a point on the radius through these points of contact at a distance r from the centre when unit current is passing through the standard coil. [If the coil is circular and coaxial with the disk, this formula simplifies into the formula first mentioned.]

§ 2. In the case of the coil used in my observations, the dimensions of which are given below, the excentricity of the elliptical section is so small that the value of the integral (A) differs only by a small quantity from the value it would have for a coil otherwise similar but of circular section with radius equal to the arithmetic mean of the semiaxes of the elliptical section; and to a first approximation we may assume that the percentage correction to be applied to the value of the integral for the circular coil to obtain its value for the elliptical coil is the same as the percentage correction to be applied to its value for the circle in which the mean plane cuts the circular coil to obtain its value for the ellipse in which the mean plane cuts the elliptical coil. It will be sufficient for our purpose, therefore, to calculate the latter percentage correction.

§ 3. Let H_e be the value of the field intensity at a given point in the disk due to unit current in the ellipse, and H the value of the field intensity at the same point due to unit current in the circle coplanar and concentric with the ellipse and of radius c equal to the arithmetic mean of its major and minor semiaxes.

Then we have, in this case,

$$\begin{aligned} A &= 2\pi n \int_{a_0}^{a_1} r H_e dr \\ &= 2\pi n \int_{a_0}^{a_1} r H dr + 2\pi n \int_{a_0}^{a_1} r (H_e - H) dr \\ &= n M_c + 2\pi n \int_{a_0}^{a_1} r \sigma dr \\ &= n (M_c + B), \end{aligned}$$

where M_c = the coefficient of mutual induction of the circle and disk circumference,

and $\sigma = H - H_e$.

§ 4. To obtain the value of B , we must first find an

expression for the intensity of magnetic field (H_e) due to an elliptical current at a point in its plane within it in terms of the semi-axes of the ellipse and the coordinates of the point.

Let the equation to the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

and let ξ, η be the coordinates of the point in question.

The intensity of the magnetic field at the point (ξ, η) may be expressed by the formula

$$H = \int_0^{2\pi} \frac{d\theta}{\rho},$$

where ρ, θ are the polar coordinates of a point on the ellipse referred to the point (ξ, η) as origin.

Forming the polar equation to the ellipse with the point (ξ, η) as origin, solving for $\frac{1}{\rho}$, and substituting in the above equation, we have

$$H = \frac{ab}{a^2b^2 - b^2\xi^2 - a^2\eta^2} \int_0^{2\pi} d\theta \sqrt{f^2 \cos^2 \theta + 2h^2 \cos \theta \sin \theta + g^2 \sin^2 \theta},$$

where

$$f^2 = b^2 - \eta^2,$$

$$g^2 = a^2 - \xi^2,$$

$$h^2 = \xi\eta.$$

Let ϕ^2, χ^2 be determined by the equations

$$\left. \begin{aligned} \phi^2 + \chi^2 &= f^2 + g^2 \\ \phi^2 \chi^2 &= f^2 g^2 - h^4 \end{aligned} \right\}$$

and we have

$$\begin{aligned} H &= \frac{ab}{\phi^2 \chi^2} \int_0^{2\pi} d\theta \sqrt{\phi^2 \cos^2 \theta + \chi^2 \sin^2 \theta} \\ &= \frac{4ab}{\phi^2 \chi^2} E(\phi, \chi), \end{aligned}$$

where

$$E(\phi, \chi) = \int_0^{\frac{\pi}{2}} d\theta \sqrt{\phi^2 \cos^2 \theta + \chi^2 \sin^2 \theta},$$

an elliptic integral of the second kind. The value of $E(\phi, \chi)$ for any values of ϕ and χ may be readily calculated by finding

their arithmetico-geometrical mean (v. Cayley, 'Elliptic Integrals,' chap. xiii.).

§ 5. In the case of my standard coil

$$a = 10.5419 \text{ inches,}$$

$$b = 10.5340 \text{ inches,}$$

and the angle made by the radius of brush contact with the major axis is approximately 55° .

I have calculated the values of H_c and H_e for points on this radius distant 1 inch, 2 inches, 3 inches, &c., from the centre, with the following results:—

r .	$H_c \div 2\omega$.	$H_e \div 2\omega$.	$\sigma \div 2\omega$.	$r\sigma \div 2\omega$.
0	.094896	.094896	.000000	.000000
1	.095542	.095542	.000000	.000000
2	.097551	.097552	.000001	.000002
3	.101142	.101144	.000002	.000006
4	.106764	.106768	.000004	.000017
5	.115252	.115260	.000008	.000040
6	.128235	.128250	.000015	.000089
7	.149170	.149198	.000028	.000199

$$\sigma = H_e - H_c.$$

§ 6. We are now in a position to calculate the value of B . In the apparatus I used

$$a_0 = .0585,$$

$$a_1 = 6.4949.$$

By numerical integration between these limits we have

$$\int_{a_0}^{a_1} \frac{r\sigma}{2\omega} dr = .000163,$$

and

$$B = 2\omega \int_{a_0}^{a_1} r\sigma dr = .00643.$$

Also we have

$$a_1 = 6.4949,$$

and

$$c = 10.5377,$$

and hence

$$M_c = 94.014,$$

and therefore

$$B + M_c = .0000684,$$

or the required correction is .00684 per cent.

And by the argument contained in § 2 this may be taken to be to a first approximation the correction to be applied in the case of the coil. Now the coefficient of mutual induction of the coil and disk, calculated on the assumption that the section of the coil at right angles to the generating lines is a circle of radius c , is equal to 16613·75. Adding the calculated percentage correction to this value, we have finally for the apparatus used by me

$$R = n(16613\cdot75 + 1\cdot14) = 16614\cdot89 n.$$

The value of the International Ohm in absolute measure previously given by me in the paper to which I have referred was

$$\cdot99976 \times 10^9 \text{ absolute units.}$$

The value corrected for the coil ellipticity is

$$\cdot99983 \times 10^9 \text{ absolute units.}$$

DISCUSSION.

Prof. SILVANUS THOMPSON said that this paper was of interest, not only on account of the application which others might make of the author's method, but also in that the correction, when applied to Prof. Jones' results, brought the international ohm more nearly into accord with the true ohm.

Mr. J. J. WALKER said he considered that the paper was better suited to the Mathematical Society. The integration which the author reduced to elliptic integrals might be more easily performed by another method.

Prof. AYRTON said that Prof. Jones' value for the true ohm was now 106·302 c.m. of mercury.

XXI. *Dielectrics.* By ROLLO APPLEYARD *.

SOME experiments upon the change of resistance of certain dielectrics with the duration of the testing-current, and with the testing-voltage, were described in a paper † which I read before the Physical Society two years ago. In continuation of this research some further tests have been made, the principal object being to determine the effect of *temperature* upon the dielectric resistance. For this purpose, mica and paraffined paper, in the form of condensers, have been chosen.

The resistances are measured by the "direct deflexion" method, and are expressed in megohms pro microfarad. The testing-voltage is the same throughout all the tests (450 volts), and each measurement is computed from the galvanometer-reading noted after the testing-current has been applied for one minute.

Two paraffin-paper condensers, each of one microfarad, and eight mica condensers, each of half a microfarad, were enclosed, separately, in water-tight cases, and submerged in a tank of water, the temperature of which could be continuously controlled. The platinum thermometers ‡ used for determining the temperature within the dielectric of the condensers have already been described.

Connexion between the condensers and the testing-apparatus was made by wires insulated with india-rubber; these were twenty-five feet long. The connexions within the condenser cases, where the wires were soldered to the respective sets of tin-foil sheets, were carefully sealed with paraffin wax. The ends of the leads were bared and cleaned in the usual way, for about six inches. Two such insulated wires were connected to each condenser, one to each set of tin-foil sheets.

* Read May 22, 1896.

† "Dielectrics," Proc. Physical Soc. xiii. p. 155, 1895; Phil. Mag. Oct. 1894, p. 396.

‡ A "Direct-reading" Platinum Thermometer. Proc. Physical Soc. March 1896; and Phil. Mag. Jan. 1896.

Surface Leakage.

Suppose a current to enter one of these condensers at one of its insulated leads, which we will call A ; and the second lead, B, put to earth. It is clear that there are at least four paths—(a), (b), (c), (d)—for the current between A and earth :—

- (a) Surface leakage at the end of A where the current enters the lead.
- (b) Surface leakage at the end of A within the condenser.
- (c) Surface leakage over the dielectric within the condenser.
- (d) Through the dielectric of the condenser.

The leads can generally be so chosen that there is practically no conduction *through* their dielectrics. When measuring the resistance of the condenser by the “direct deflexion” method, the current to which the galvanometer responds is that due to the sum of the currents through (a), (b), (c), and (d). We have to determine what the deflexion would be if all the current went by way of (d). This can be done, approximately, by taking a preliminary test, with the end B disconnected from earth, and carefully cleaned ; noting the deflexion after the current has been applied at the end A for one minute. The deflexion thus obtained must be deducted from the deflexion observed during the second operation, which consists in putting the end B to earth, and again applying the current and noting the deflexion after one minute.

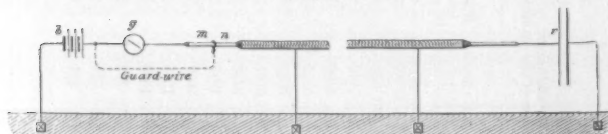
It sometimes happens that we want to separate (a) from (b) and (c) ; that is to say, we require to know what part of the preliminary deflexion is due to the exposed lead, and what part is due to internal leakage at the condenser. For this purpose I have applied a suggestion made to me by Mr. W. A. Price *, and have used a “guard-wire” to eliminate the leakage at (a). The method, which was used for its first trial in connexion with these experiments, consists in winding

* This method has since been described by Mr. Price. See ‘Electrical Review,’ vol. xxxvii. no. 941, p. 702 (1895). In the appendix to the present paper I have evaluated the possible errors, and shown that they are negligible.

one or two turns of bare wire around the cleaned end of A, at some point not very far from the middle of the cleaned portion, intercepting the path of leakage. This wire is then connected to the testing-battery, as shown in fig. 1.

The efficacy of the "guard-wire" for the present tests was proved by the following experiment. A copper earth-wire was given several turns around the insulated lead A, just above the cleaned ends, to facilitate leakage; and the preliminary test was then made in the usual way. The observed deflexion, due to (a), (b), and (c), was 4 divs. The cleaned end was then smeared with a mixture of glycerine and blacklead, and the resulting deflexion was far off the

Fig. 1.



scale; (a) had been increased enormously. A "guard-wire" was then applied at the middle of the smeared portion of the lead, and the deflexion was at once restored to its original value of 4 divs.

In practice it was always possible to avoid leakage (a) by giving a little care to the cleaning of the lead; the "guard-wire" was therefore used only as a check, and if it produced no change in the preliminary deflexion it was concluded that the preliminary deflexion was due to the effects of (b) and (c) alone, and that there was no leakage (a).

Results.

The results of the tests are given in Table I., which requires little explanation. The "Capacity," "Resistance," and "Electrification" of paraffin-paper and mica are there tabulated at the various temperatures.

Capacity.—As standards of capacity two condensers were used. These were mica of 1 and 0.5 microfarad capacity respectively. They were submerged in water and kept always

TABLE I.

Date.	Temperature.		Paraffin-paper.			Mica.		
			Mean values of two condensers.			Mean values of eight condensers.		
	F°.	C°.	Capacity. mfd.	Resistance. Megohms pro mfd.	Elec. p. c. between 1m and 2m.	Capacity. mfd.	Resistance. Megohms pro mfd.	Elec. p. c. between 1m and 2m.
11th March, 1896	32.8	0.4	0.08	17740	33	0.50	31286	40
12th " "	44.25	6.8	.08	11510	30	.50	28520	46
13th " "	54.5	12.5	.08	7216	28	.50	28427	47
26th Aug., 1895	65.7	18.7	.09	4196	24	.50	—	—
17th March, 1896	67.4	19.7	.07	3825	25	.50	25415	37
28th Aug., 1895	68.1	20.1	.08	3622	25	.50	22000	42
23rd " "	72.6	22.6	.08	2981	24	.50	22327	44
22nd " "	75.0	24.4	.08	2436	25	.50	—	—
20th " "	80.4	26.9	.08	1947	25	.50	16272	43
19th March, 1896	84.2	29.0	.07	1659	26	.50	16030	36
30th Aug., 1895	89.6	32.0	.07	1231	26	.50	17010	43
20th March, 1896	94.3	34.6	.08	1056	27	.50	18457	30
2nd Sept., 1895	100.1	37.8	.06	792	28	.50	15270	41
20th March, 1896	107.4	41.9	.05	611	32	.50	10340	33
3rd Sept. 1895	109.9	43.3	.05	551	32	.50	10521	38

at the same temperature, 27°C. , which is just above the maximum air temperature. The object of using two standards was to avoid shunts and to simplify the work.

In the paraffin-paper condensers there appears to be an irregular variation of capacity with temperature. It is too small to affect them in practical telegraphy; but it is sufficiently large to prohibit their use as standards. We must suppose that the softening of the wax, and the consequent approach or recession of the tinfoil sheets, is responsible for part of these changes, to which must be added the variation of the specific inductive capacity of the dielectric. It is curious that with paraffin-paper condensers the capacity *diminishes* as the resistance diminishes. With gutta-percha or india-rubber core the result is nearly always the reverse of this.

Owing to the vibration of the building in which the tests were made, the capacities could not be measured with a greater accuracy than 1 per cent.; but within this limit it is safe to say that throughout the whole range of temperature mica retains a constant capacity.

Temperature Coefficients.

It is usual to assume* that the resistance R_{τ} of a dielectric at any temperature τ is represented by an equation of the form

$$R_{\tau} = R\alpha^{\tau},$$

and that similarly, at some other temperature θ ,

$$R_{\theta} = R\alpha^{\theta};$$

so that

$$R_{\tau} = R_{\theta}\alpha^{\tau-\theta},$$

or

$$\log R_{\tau} = \log R_{\theta} + (\tau - \theta) \log \alpha. \quad (1)$$

From this equation, taking the fifteen observations of Table I. two at a time, in alternate order, and putting them successively as R_{τ} and R_{θ} in (1), we find as a mean value

$$\log \alpha = \bar{1} \cdot 96344. \quad (2)$$

The temperature θ may be given some standard value, say 20°C. ; and from a curve drawn from the fifteen observed

* See Appendix II.

values of resistance and temperature the corresponding resistance R_{20} may be obtained. The resistance at 20°C. , found in this way, was, for paraffin-paper,

$$R_{20} = 3670 \text{ megohms;}$$

equation (1) in this case becomes

$$\log R_{\tau} = 3.56467 + (\tau - 20)(1.96344), \quad . \quad . \quad (3)$$

from which may be deduced the resistance corresponding to any temperature τ . It is interesting to see how nearly the results calculated from (3) agree with the observed values, as can be done by drawing the two curves which represent the observed and calculated resistances respectively.

The agreement between these two curves is so close, considering the conventions which always have to be adopted in tests upon dielectrics, that it has been suggested to me that a simple logarithmic curve may perhaps represent the true state of affairs, the observed results being assumed more or less at fault. This seems improbable for the following reason:—In equation (1) α is really the ratio of two resistances at an interval of 1°C. , and it is presumed* that this ratio, *i. e.*

$$\alpha = \frac{R_{\tau}}{R_{\tau+1}},$$

is constant for all values of τ . This might be true for some ideal dielectric whose structure was perfectly definite at all temperatures, and which made invariable contact with its electrodes under all conditions; but there is no reason why α should be constant for a substance having the instability of wax.

Temperature Tables.

One use of these results is to enable the resistances of condensers to be reduced to the resistance at some standard temperature. A similar correction has always to be applied for gutta-percha and india-rubber cores. It is usual to draw up a table, assuming α constant over a given range, and to supply a correcting logarithm for each degree of temperature. This may be sufficiently accurate for small ranges of temperature; but I would suggest that a better way is to work from the *observed* results directly, making no assumptions

* See Appendix II.

whatever as to α . In order to do this, the "observed" results should be plotted as a curve, coordinating resistances and temperatures. The length of the ordinate corresponding to standard temperature on the "observed" curve is then to be taken as unity, and the lengths of the ordinates at every successive degree are to be expressed in terms of this unit. A table can then be drawn up giving a correcting divisor corresponding to each degree. With the aid of a slide-rule such a table can be completed in little more than an hour; and it is certainly more expeditious and less arbitrary than the ordinary tables of dielectric corrections. As an example, I have given in Table II. the divisors for paraffin-paper, from 33° F. to 110° F., deduced directly from the "observed" curve.

TABLE II. Paraffin-paper.

(To reduce the dielectric resistance at any temperature Fahr. to the corresponding dielectric resistance at 60° F., divide the observed resistance by the number opposite the given temperature.)

Temp. Fahr.	Divisor.	Temp. Fahr.	Divisor.	Temp. Fahr.	Divisor.	Temp. Fahr.	Divisor.	Temp. Fahr.	Divisor.
33	3.19	49	1.67	65	0.775	81	0.335	97	0.169
34	3.07	50	1.62	66	.733	82	.319	98	.159
35	2.96	51	1.55	67	.699	83	.306	99	.153
36	2.87	52	1.46	68	.656	84	.292	100	.148
37	2.76	53	1.39	69	.625	85	.281	101	.141
38	2.66	54	1.33	70	.595	86	.270	102	.137
39	2.57	55	1.27	71	.562	87	.259	103	.133
40	2.48	56	1.20	72	.528	88	.247	104	.126
41	2.39	57	1.15	73	.501	89	.234	105	.119
42	2.30	58	1.09	74	.477	90	.225	106	.114
43	2.19	59	1.04	75	.450	91	.216	107	.108
44	2.09	60	1.00	76	.432	92	.207	108	.105
45	2.00	61	0.944	77	.411	93	.198	109	.101
46	1.91	62	0.901	78	.389	94	.189	110	.099
47	1.82	63	0.858	79	.375	95	.180		
48	1.76	64	0.820	80	.355	96	.175		

The resistances of *mica* condensers at the different temperatures are also given on Table I.; the curve drawn from them is not so smooth as that which represents the behaviour of paraffin-paper condensers; at the colder temperatures the resistance of the mica is so high that the preliminary deflexion, corresponding to (a), (b), and (c), is by no means small

as compared to the second deflexion, due to (a), (b), (c), and (d) together. The results given are the best that could be obtained under the circumstances. In some ways it might have been advisable to connect the eight mica condensers in parallel; the deflexions to be observed would then have been greater, but the number of tests would have been fewer, and there would have been less chance of comparison one with another.

Electrification.—In Table I. the “Elec. p.c.” is given for paraffin-paper and mica, at the various temperatures. This is derived from the galvanometer deflexions taken respectively at one minute and two minutes after the application of the testing current, thus :—

$$\text{Elec. \% between } 1^{\text{m}} \text{ and } 2^{\text{m}} = \frac{1^{\text{m}} \text{reading} - 2^{\text{m}} \text{reading}}{1^{\text{m}} \text{reading}} \times 100.$$

Paraffin-paper seems to acquire a minimum Elec. p.c. at about 20°C .; the reason is not very apparent.

Charge and Discharge.

The following figures (Table III.) may be useful as indicating the difference between the deflexions obtained in capacity tests, using “charge” and “discharge” deflexions,

TABLE III.

	Charge.	Discharge.	Charge (repeated).
Paraffin-paper	196	196	195.5
Mica	223	223	223
Gutta-percha	208	214	$\left\{ \begin{array}{l} 207 \\ 210 \\ 210 \end{array} \right\}$
India-rubber.....	198	200.5	$\left\{ \begin{array}{l} 201 \\ 197 \\ 198.5 \end{array} \right\}$

respectively, for paraffin-paper, mica, gutta-percha, and india-rubber. The deflexions represented by “charge” are not so uniform as the “discharge” readings, that is to say, a repe-

tion generally gives a different reading, owing in part perhaps to some of the last charge having remained. This, however, is scarcely to be observed with mica. "Discharge" readings can generally be reproduced accurately.

Resistance during Melting.

If the temperature of paraffin-wax is raised much higher than is indicated in Table I., the change of state, as melting is approached, is characterized by a corresponding decrease in resistance. In order to examine this effect, a special condenser consisting of two horizontal circular plates, 11 in. diameter, was used. The lower plate was provided with a projecting rim to retain the wax, and was rather larger than the top plate. Three small wafers of thin ebonite sheet (16 mils) were used as distance-pieces. Melted wax was then poured in, filling the space between the plates.

There are so many disturbing influences affecting the resistance during the change of state, connected with the latent heat of solidification, the unequal melting, the absence or not of "electrification," and the contractions and expansions of the wax, that only qualitative results can be given.

Heating.—Starting at about 20° C. below the melting-point, the resistance rapidly diminishes until actual melting begins; there is then a definite fall to something like one third of what the resistance was just before melting. If heat is still applied to the condenser, the resistance keeps steady until the melting is complete; after which the resistance again diminishes steadily. So far as I am able to discover there is no "electrification" while the wax is in the melted condition.

The *spark-resisting power* of the melted wax is at least one third that of the solid. The *melted* wax in this particular condenser broke down under 1200 steady volts; the insulation heals, as in the case of oil, to break down again in a few seconds.

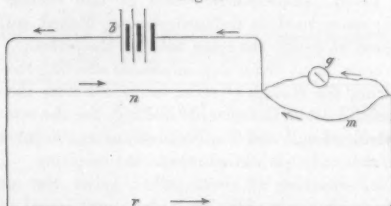
Cooling.—A converse process occurs during cooling; and as the cooling is more uniform than the heating the effects can be more easily noted. The resistance of the melted wax gradually increases until the first crystals appear, at which

stage it remains more or less constant until the process is nearly complete, when it very rapidly increases to about three times the value it had during the partial crystallization; it then behaves as solid wax.

APPENDIX I.

When an extra wire (fig. 1) is added from the testing-battery to some point near the middle of the cleaned end of the lead, intercepting the path of leakage (a) at that end, the circuits can be represented by fig. 2. The (a) leakage is here $m+n$, and the "guard" wire is supposed to come between m and n . Let b , g , and r denote the resistances of

Fig. 2.



the battery, galvanometer, and the dielectric resistance of the condenser, respectively; let E be the E.M.F. of the testing battery, and G the current through the galvanometer when the "guard" wire is applied. It is clear that m shunts the galvanometer, and n shunts the battery; and we have to find the error which this introduces into the results. From fig. 2, by Kirchhoff's laws, we have

$$G = \frac{E}{(b+g+r) + \frac{1}{mn}(nrg + brm + brg + bmg + nbg)} \quad \dots (1)$$

Let

$$\kappa = \frac{1}{mn}(nrg + brm + brg + bmg + nbg), \quad \dots (2)$$

then κ may be regarded as equivalent to a resistance added to the simple circuit $b+g+r$.

To evaluate κ , we will put $m=n=r$; which means that the whole leakage, $m+n$, is assumed to be only twice the resistance of the dielectric of the condenser, and corresponds to a worse lead than any I have employed throughout these tests. In this case, (2) reduces to

$$\kappa = g + b + \frac{3bg}{m},$$

which has now to be compared with $(b+g+r)$ in (1).

Taking the very lowest observed value of r , which was 5×10^8 ohms, and putting $g=8000$, and $b=200$ ohms, we have

$$\frac{\kappa}{b+g+r} = \frac{1}{50,000},$$

so that κ is quite negligible in equation (1), and the error introduced by the "guard" wire is practically *nil*.

APPENDIX II.

Change of Dielectric-resistance with Temperature.

Let the resistance at some fixed temperature be R ; and let R_1, R_2, R_3 , &c. be the resistances at $1^\circ, 2^\circ, 3^\circ \dots$ above R . The assumption is that the resistance diminishes $\frac{1}{n}$ th of itself for every 1° rise of temperature. In this case,

$$R_1 = R - \frac{R}{n},$$

$$n = \frac{R}{R - R_1} \quad \dots \dots \dots (1)$$

and

$$R_2 = R_1 - \frac{R_1}{n}.$$

Or, putting in the value of n ,

$$R_2 = \frac{R_1^2}{R}.$$

Hence

$$R_1 = R \left(\frac{R_1}{R} \right), \quad R_2 = R \left(\frac{R_1}{R} \right)^2, \quad \&c. ;$$

and, g  n  rally,

$$R_i = R \left(\frac{R_1}{R} \right)^i \quad \dots \dots \dots (2)$$

If τ and θ are any two values of t , we have

$$R_\tau = R_\theta \left(\frac{R_1}{R} \right)^{\tau - \theta}; \quad \dots \quad (2)$$

or,
$$\log \left(\frac{R_1}{R} \right) = \frac{\log R_\tau - \log R_\theta}{\tau - \theta}.$$

By putting observed values of R_τ , R_θ , τ , and θ in this last expression, the value of $\left(\frac{R_1}{R} \right)$ may be found, and substituted in (3), which then becomes the working formula. In the paper, $\left(\frac{R_1}{R} \right)$ is represented by α .

Comparison of α for Centigrade and Fahrenheit Scales.—Let R_1 be the resistance 1° C. above that at which R is measured. Then, by the preceding,

$$\alpha_c = \left(\frac{R_1}{R} \right),$$

and (2) becomes

$$R_t = R \alpha_c^t.$$

But 1° F. may be expressed as $\left(\frac{5}{9} \right)^\circ$ C., so that

$$R_{\left(\frac{5}{9} \right)} = R \alpha_c^{\left(\frac{5}{9} \right)}, \quad \dots \quad (4)$$

and $R_{\left(\frac{5}{9} \right)}$ will be the resistance at 1° F.

Hence
$$R_{\left(\frac{5}{9} \right)} = R \alpha_f. \quad \dots \quad (5)$$

Thus, from (4) and (5),

$$\alpha_f = \alpha_c^{\left(\frac{5}{9} \right)},$$

or,
$$\log \alpha_f = \left(\frac{5}{9} \right) \log \alpha_c.$$

DISCUSSION.

Prof. AYRTON said he could bear witness to the extreme value of Mr. Price's device, as it completely did away with the necessity for the extreme care previously required to prevent errors due to surface leakage. He regretted that he had not had an opportunity of comparing the author's numbers with some obtained some years ago by Prof. Perry and himself (Prof. Ayrton).

XXII. *On New Instruments for the Direct Measurement of the Frequency of Alternating or Pulsating Electric Currents.*

By ALBERT CAMPBELL, B.A.*

IN many experiments with alternating currents it is a very great convenience to be able to draw the supply from an ordinary lighting circuit, but the value of such a source of current is often lessened by the fact that the frequency of alternation is usually variable, the limits of variation being in some cases very wide indeed. It was for the purpose of getting rid of this uncertainty that, about a year ago, I devised the two instruments described below.

Most methods of measuring frequency depend either on Impedance measurements or on the production of Resonance (or Synchronism). The former I avoided, for unfortunately the wave-form as well as the frequency affects impedance, and I aimed at an instrument which would give trustworthy readings for any kind of pulsating current whatever.

Of the Resonance (or Synchronism) methods two are familiar. The first consists in running a small synchronous motor and measuring its speed by a speed-counter or indicator. This is a rather laborious way, and not very accurate if only a tachometer is used. In the second method† a stretched wire carrying the alternating current is placed in a constant magnetic field; by varying the stretching-weight the wire is tuned until it is set into strong vibration, and then the frequency is calculated from the tension, length, and mass of the wire. The original arrangement has been simplified, I believe, by Mr. Alexander Russell, of Faraday House, who uses a steel wire thrown into vibration by an iron-cored choking-coil placed near it. In both arrangements some uncertainty is introduced by the wire passing over a bridge or pulley.

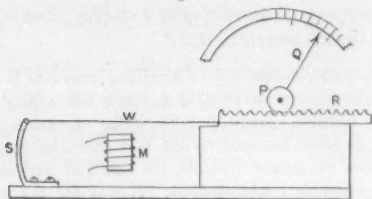
In my first type of instrument I developed the last-

* Read May 22, 1896.

† Due to Professors Ayrton and Perry.

mentioned arrangement. The working parts of the instrument are shown in fig. 1:—

Fig. 1.



A steel wire, *W*, is fastened at one end to a spring, *S*, and at the other to a rack, *R*, sliding in guides. Near the wire is fixed an electromagnet *M* with a laminated core; this magnet is excited by the given alternating current. By turning the pinion *P* the wire can be tightened until it is thrown into vigorous vibration by the magnet. The pointer *Q* attached to the pinion then shows directly the frequency on a suitably graduated scale. Many varieties of this arrangement may be used; for example, the pointer may be fastened to *S*, or other kinds of magnifying-gear may replace the rack-and-pinion.

With careful use I believe that this type is accurate to within less than 0·2 per cent., but it is not so convenient as the second type, which I now proceed to describe. This more practical instrument is shown diagrammatically in fig. 2.

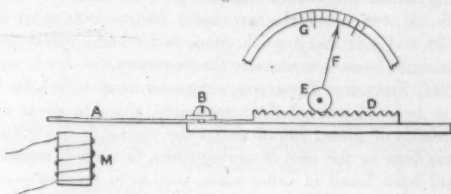
Here *M* is a choking-coil fixed near a steel strip *A**, which can be moved back and forward through the clamp *B* by a rack-and-pinion, not shown. *A*'s change of length is magnified by the rack-and-pinion *D* and *E* and the pointer *F*. The protruded length of *A* is altered until the alternating field due to *M* causes the maximum resonance, and the frequency is read off on the scale *G*.

By proper choice of the dimensions of the vibrator and the

* I find, since writing the above, that in 1889 Professors Ayrton and Perry suggested the use of a variable magnetic tongue near an alternating electromagnet. Their idea, however, was never embodied in a direct-reading practical instrument.

rack-and-pinion a very extended scale may be obtained. I have constructed one instrument in which the pointer goes more than twice round the circumference from 40 periods per sec. to 150 periods per sec., and at the middle part of the scale the accuracy of reading is within 0.3 per cent.

Fig. 2.



The electromagnet M is usually of fine wire, and has a non-inductive resistance in circuit with it, so that the current taken is very small.

The reading is usually taken at the point of maximum resonance. This is observed by means of the sound given out or by watching the variation in amplitude either directly or by mirror, light-spot, and scale. Sometimes I fix near the vibrator an adjustable piece, against which it rattles or jars when the resonance is sufficient. This jarring piece may be made part of an electric circuit, including a lamp or an indicating instrument, in such a way that the circuit becomes broken when the greatest resonance occurs.

The instrument can be used as a speed-indicator for machinery by attaching to the rotating shaft a suitable commutator to make and break an electric circuit carried to the instrument, which may thus be at any distance.

If we keep to simple English we might call such an instrument a "Wave-teller," but to make the meaning clearer I propose to name it a "Frequency-teller."

In conclusion I may mention that since there is very little inertia in the moving parts the readings can be taken rapidly, and thus the instrument can be made to follow fairly quick changes in frequency.

DISCUSSION.

Mr. WATSON said he thought that in the case of the spring there would be a considerable temperature correction, and he suggested a method by which this might be compensated.

Mr. BLAKESLEY asked if the author had found that the spring became magnetised and thus gave the octave.

Mr. CARTER asked whether elastic fatigue influenced the results, and said that a synchronous motor and a speed indicator could be used to measure the frequency.

Prof. SILVANUS THOMPSON suggested that it might be better to employ a polarized apparatus, since, to avoid the impression of forced vibrations on the spring, it was better, as was done in the case of tuning-forks, to make it massive. It had been found in other cases, such as in Hughes's telegraph and the telephone, that better results were obtained with polarized apparatus. He (Prof. Thompson) had used a telephone, placed anywhere near a magnet traversed by the periodic current, together with a tuning-fork, which gave beats with the note produced by the telephone, to measure frequencies. The variations in frequency ordinarily met with in practice were much greater than was generally suspected.

Mr. BLAKESLEY said he considered that the advantage of the author's instrument over a telephone and tuning-fork was that it was continuously variable over a large range.

Mr. ENRIGHT asked if the author had been troubled by the spring or wire breaking into overtones. In some experiments, in which rather long wires were used, he had been troubled in this way.

Prof. AYRTON said that he did not think that it was possible to get the wire or spring to respond to the octave unless the alternating current contained a component of the frequency of the octave. In fact, he had himself used such a stretched string as a wave-analyzer. He had used a telephone to prove that the note given by a hissing alternate-current arc corresponded in frequency to that of the current. In the instrument used by Prof. Perry and himself a polarized

arrangement was always employed, since the alternating current was passed either through a wire in a constant magnetic field, or through an electromagnet which acted on a wire through which a constant current was passed.

The AUTHOR, in his reply, said that the instrument responded, though feebly, to the octave, and this response might be made use of to check the accuracy of the scale. The total temperature correction would always be less than 0.1 per cent. and therefore may be neglected.

In the instruments shown the vibrator used was not a polarized one for three reasons :—

- (a) in order to get a note whose vibration frequency should be *twice* the frequency measured, thus avoiding very low notes which are not so easy to observe accurately.
- (b) to avoid the use of permanent magnets in alternating magnetic fields.
- (c) to render unnecessary the use of an auxiliary direct-current circuit (for exciting electromagnets).

As to the question of elastic fatigue in the steel strip, I have never had the vibrator running for long enough time to detect this. In ordinary use the vibrator remains at rest except when an observation is taken.

XXIII. *Some Experiments with Röntgen's Rays.*

*By SILVANUS P. THOMPSON, D.Sc., F.R.S.**

THE following is a brief narration of points observed by me during the past three months, and which are now brought before the Physical Society.

1. Many experiments have been made to observe polarization of x -rays, but no trace has been found. Tourmalines of several colours, and thicknesses varying from 0.1 millim. to 6 millim., have been used. Andalusite, mica, calc-spar cut in slices parallel to the axis, epidote, and ripidolite have been tried without result. The method pursued has chiefly been to cut the slice of crystal into three parts, laying two of them upon the third, one of the upper parts having its axis parallel to the axis of the under part, while the other had its axis laid at right angles. In this way equal thicknesses of crystal were compared side by side. Thinking that results might be obtained from dichroic crystals containing a metal of considerable atomic weight, slices were tried of crystals of sulphate of nickel, sulphate of nickel and potassium, sulphate of cobalt and potassium, sulphate of cobalt and ammonium, fluosilicate of cobalt, and fluosilicate of nickel, but no trace of polarization was seen.

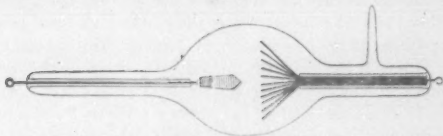
Another method consisted in comparing the opacity of tourmaline in a direction parallel to the axis with that of an equal thickness in a direction at right angles to the axis. Not the slightest difference was observed either in the photographic shadows or with the use of a luminescent screen of barium platinoeyanide.

2. For several weeks in the months of February and March experiments were made with many different forms of bulb to determine the source of the x -rays and the form of tube most favourable to their production. In common with the conclusions of so many other observers, it was found that the effective source was in every case a surface against which the cathodic discharge was directed. A form of tube which gave results superior to those of any Crookes tube at the time in

* Read June 12, 1896.

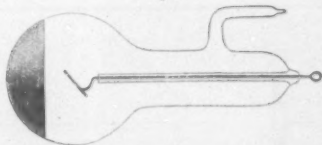
the market is shown in fig. 1, in which the kathode consisted of a number of iron wires spread out from a centre, and the antikathode, which also served as anode, was a spade-shaped piece of iron or platinum. Another form of which many

Fig. 1.



bulbs were tried was provided with an external kathode of foil, and an internal anode projecting to about the centre, terminating in a small spade of platinum-foil as antikathode (fig. 2). This also gave good results, but was liable to be pierced

Fig. 2.



at high stages of exhaustion. With this form various experiments were tried as to the influence of the material of the antikathodic surface. Glass was found to work quite well, but to be more troublesome than metal. A phosphorescent enamel made by fusing together two parts of a soft lead-glass with one part of Balmain's luminous paint was also tried. The result of these experiments was to show that, contrary to the opinion then current that the presence of much phosphorescence or fluorescence was promotive of the production of the x -rays, the x -rays were most freely emitted when the conditions were such as to waste as little as possible of their energy in internal fluorescent effects: that in fact a metal surface was preferable to a surface of glass, enamel, or porcelain for receiving the impact of the cathodic discharge.

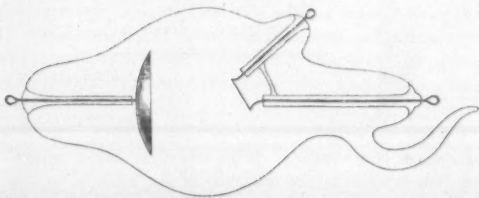
At about this time Mr. Jackson's perfected form of focus-

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tube was brought out, a tube which for photographic purposes was found superior to any other form, and has not yet been surpassed.

3. Observing in some of these experiments that the metal spade used as antikathode became red-hot, a special tube was constructed for me by Mr. Gardiner to test the question whether the high temperature of the antikathode was, or was not, prejudicial to the emission of x -rays. This special tube (fig. 3) was furnished with an antikathode of platinum-foil mounted so that while serving both as antikathode and as anode it could be heated by passing a current through it from an auxiliary battery. Observing the activity of the tube by means of a luminescent screen of platinocyanide of potassium, it was found that the heating of the antikathode, so far from being disadvantageous, decidedly promoted the emission of x -rays, and increased the continuity and brilliancy of the luminescence. Various amounts of current were sent

Fig. 3.

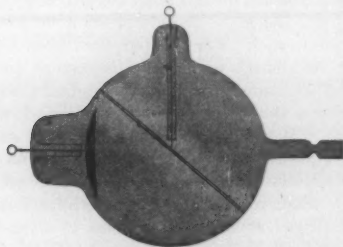


through the platinum, the most effective result being obtained by currents which heated the surface to visible redness. Whether the effect is a direct one or an indirect owing to the driving out of occluded gases is not yet determined.

4. When watching with the luminescent screen the emission of x -rays within bulbs connected with the pump, some observations were made of direct importance with regard to the state of exhaustion that is best. The degree of vacuum which suffices for the production of cathodic shadows is known not to be sufficient for the production of x -rays. It is also known that when evacuation is pushed very far the internal resistance of the bulbs rises very high, so that they

become almost non-conductive. If a bulb is exhausted, and heated during exhaustion, and the vacuum pushed almost to non-conductivity, and if a little air is again admitted and the tube again exhausted, the high degree of vacuum is again very soon reached, probably because during the first exhaustion the gases absorbed upon the walls of the bulb were mostly removed. After three or four repetitions of this process the transition from the low state of vacuum to the high state is exceedingly rapid. If a bulb in such a condition is examined by the luminescent screen while the pump is at work, scarcely any trace of x -rays can be noticed so long as the vacuum is such that the resistance is low. A pair of discharging-points arranged as a shunt to the tube serves as an approximate gauge. Kathodic shadows can be seen when the resistance is so low that the discharge-points do not spark even when placed 3 millimetres apart. When the resistance rises so that the spark-points must be put 20 or 30 millimetres apart x -rays begin to be given off; and are given off both from the back and from the front of the antikathode. The bulb, as seen upon the adjacent screen, shows two pale luminous regions divided by a fine oblique black line which is in the plane of the antikathode (fig. 4). If the pump goes on

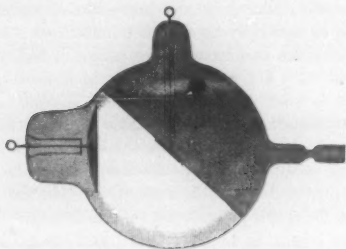
Fig. 4.



working, in two or three seconds, or while only a few cubic centimetres of mercury pass through the pump, the phenomenon changes. The luminosity behind the antikathode dies out, and that in front of the antikathode increases; so that there is seen simply a bright anterior region ending at the

oblique plane of the antikathode, beyond which all is dark (fig. 5). This oblique delimitation can also be seen in the yellow phosphorescence upon the walls of the bulb. This sudden transition occurs after the resistance of the bulb has passed its minimum. The brightest luminescence occurs

Fig. 5.



when the spark-length exceeds 40-50 millimetres. The luminosity does not fall off much even at very small angles to the plane of the antikathode, proving that the emission of x -rays does not follow Lambert's law of the cosine by any means. Experiments on this point are still in progress.

5. The phenomenon of diselectrification by x -rays is very readily demonstrated. For this purpose I have found a very convenient instrument to be an electroscope consisting of two strips of aluminium leaf (which is lighter than gold leaf) suspended in a thin-glass jar entirely covered with a fine metal gauze. It is charged with a dry pile, and a metal cap is then placed over the charging knob, so that it is entirely electrostatically screened from external electrical influences. Positive and negative electrifications are both readily discharged, even at the distance of several feet from the bulb.

6. On the first announcement of the diselectrifying properties of x -rays, I attempted to obtain electric dust-figures as shadows of metallic objects by applying the x -rays to discharge electrified surfaces of glass or ebonite upon which mixed powders of red lead and sulphur were then dusted.

These were obtained almost at the first trial; but to produce them satisfactorily requires a little care.

The object whose shadow is to be obtained—a pair of scissors, for example—is laid upon a thin sheet of aluminium placed to stand on four feet at the height of about 20 millim. over the sheet of ebonite or varnished glass upon which the shadow is to be thrown. This sheet of ebonite is first carefully diselectrified by passing it over an alcohol flame, and then laid upon an earthed sheet of foil upon the table. The aluminium tray with the scissors upon it is placed over the ebonite. A guard-box of lead with a rectangular hole in its top is placed over all. Then the aluminium tray is charged electrically by a small influence-machine which has one pole put to earth and the other connected to the aluminium tray. In this state of things the ebonite plate lies in an electrostatic field, but is not electrified upon its upper surface. The x -rays are now caused to fall upon the aluminium tray, through which they pass save when obstructed by the metallic object, and, discharging the tray, virtually carry down the electrification upon the surface of the ebonite in straight lines, leaving the shadowed portions unelectrified. The influence-machine is disconnected, the aluminium tray removed, the sheet of ebonite lifted off the table, and the mixed powders are forthwith dusted over its surface, revealing the shadow. Both positive and negative shadows can be obtained. Several alternate dispositions are possible.

These observations were made early in February before the announcement by M. Righi of some similar cases of production of shadows by x -rays.

7. I have also made some observations upon the reflexion of x -rays. The production of diffuse reflexion by solid bodies is very easily observed; but hitherto I have no clear evidence of specular reflexion. Air unfortunately itself sets up diffusion, behaving as a semi-opaque fluid. If ordinary experiments on the reflexion of light had to be carried on in dense smoke or in milky water, a similar diffusion would interfere with specular reflexion. In one set of experiments a V-tube made of lead pipes set at right angles, and open at the bottom, was used, the x -ray source being made to shine down one limb, while a shielded photographic plate was placed at the

upper end of the other. The surfaces to reflect x -rays were placed at the open lower ends at 45° to the lines of incidence and of presumed reflexion. Reflexion of a sort was indeed obtained when surfaces of metal and of glass were placed across the bottom of the tubes. But an effect was also obtained even when nothing was placed across the open bottom. It seems exceedingly doubtful whether true specular reflexion has been observed in any case.

DISCUSSION.

Prof. DU BOIS said that Galitzine had found that Röntgen rays were polarised by tourmaline, a special form of developer being employed. The behaviour of tourmaline to light-waves presents some curious features; for if the wave-length is increased, a point is at length reached where the ordinary and extraordinary rays are equally absorbed. For greater wave-lengths the ordinary conditions are reversed. If the Röntgen rays are not homogeneous, the contradictory results obtained by different observers might be due to the fact that they were working with rays which were differently absorbed by tourmaline.

Mr. SWINTON said he had tried the effect of heating the kathode, and had obtained results similar to those which were obtained by the author. Mr. Swinton further said that he had found that the blue luminescence sometimes observed depended on the size of the kathode. With tubes in which the kathode was almost a complete hemisphere, it was impossible to eliminate this blue luminescence.

Mr. APPLEYARD suggested the performance of the experiments under the surface of a dielectric.

Prof. GRAY said he had obtained some indication of regular reflection, but nothing definite.

The AUTHOR, in his reply, said that it had been found that if the Röntgen rays are reflected from a surface of sodium *in vacuo* the amount reflected is a minimum for normal incidence, and increases at oblique incidence. Comparing this behaviour with that of ultra-violet light, it supports the idea that the Röntgen rays consist of transverse vibrations.

XXIV. *On the Measurement of very large and very small Alternating Currents.* By ALBERT CAMPBELL, B.A.*

AIR-CORE transformers, although quite inefficient for ordinary lighting circuits, are yet much more valuable for testing purposes than most people are aware of. By the help of such transformers it is possible to extend almost indefinitely the ranges of many ordinary measuring instruments. If the secondary of an ironless transformer be kept in open circuit the secondary volts are accurately proportional to the primary P.D. if the frequency is constant, and hence by using an electrostatic voltmeter on the secondary we can transform either up or down, and thus measure voltages above or below the range of the electrostatic instrument. Of course the arrangement would have to be calibrated; this might sometimes be done by taking a reading for which both the primary and secondary voltages lay within the range of the voltmeter used.

The above way of using an air-core transformer was suggested to me some time ago by Mr. Hugh Erat Harrison, of Faraday House. I hear since from Mr. Mather that it has been also used at the Central Technical College †.

If we attempt to measure *current* (in the primary) by observing the voltage on an open-circuit secondary, we find that for a given primary current the readings depend also on the *frequency*. It therefore occurred to me that the secondary circuit *should be closed* and should besides be *highly inductive*. In practice this was found to work with perfect success, the primary current being proportional to the secondary current, which latter was measured by an ammeter.

To find the most favourable conditions, let

I_1, I_2 be the primary and secondary currents respectively;

* Read June 12, 1896.

† Since writing the above I have listened with interest to the paper on the "True Resistance of the Electric Arc," by Messrs. Frith and Rodgers. Their beautiful application of an air-core transformer to measure a small alternating current superimposed on a large direct current might, I think, be also employed to separate a *large* alternating current from a much *smaller* direct current.

R_2 the resistance of the secondary circuit ;

N its inductance ;

M the mutual inductance between primary and secondary.

Let $p = 2\pi n$, where n is the frequency of alternation.

Here M and N are constant, while R_2 may be variable with temperature.

At first let us assume that the P.D. follows a simple sine curve. Then

$$\frac{I_1}{I_2} = \frac{\sqrt{R_2^2 + p^2 N^2}}{Mp} \dots \dots \dots (1)$$

Now let the inductance of the secondary circuit quite swamp its resistance, so that we may neglect the term R_2^2 in comparison with $p^2 N^2$. In this case we obtain

$$\frac{I_1}{I_2} = \frac{N}{M}, \dots \dots \dots (2)$$

i. e. *the ratio of the primary current to the secondary current is independent of the frequency of alternation and the resistances of the coils.* We shall see below to what extent this valuable result can be realized in practice. It is clear that in the above case, since $\frac{pN}{R_2} = \infty$, the secondary current I_2 is in exactly opposite phase to I_1 .

Now suppose the primary P.D. to be no longer a pure sine function of the time but to follow any periodic curve. The curve of primary current can then be decomposed into a number of sine curves of various frequencies ; each of these will produce a secondary of opposite phase, and the ratio of each primary component to its secondary will $= \frac{N}{M}$.

Accordingly the total secondary current curve will be of the same wave-form as the primary and in opposite phase to it, while *the ratio of current transformation will be the same as for a pure sine-curve alternating current**.

* In this I have assumed that the Fourier series for the wave-form contains no constant term, i. e., that there is no constant component in the current. If there is such a component, nothing corresponding to it will appear in the secondary circuit, and the ratio of current transformation will be by no means the same as if the current were purely alternating. In fact an air-core transformer affords a good means of separating out the purely alternating part of any periodic current.

Unfortunately, in practice we cannot quite attain to the condition $\frac{Np}{R_2} = \infty$, but we can get near enough to make the error quite small. In the following table are given (approximately), for different values of $\frac{R_2}{N}$, the values of the ratio $\frac{I_1}{I_2}$ for frequencies of $n=80 \sim$ per sec., and $n=40 \sim$ per sec., if the ratio $=1$ when $n=\infty$.

TABLE I.—Transformation Ratios.

$\frac{R_2}{N}$	$n=\infty$	$n=80$	$n=40$
100	1.00	1.02	1.08
50	1.000	1.005	1.02
25	1.000	1.001	1.005
10	1.000	1.0002	1.0008

By this it will be seen that if $\frac{R_2}{N}=50$, the error in the transformation ratio from $n=80$ to $n=\infty$ becomes insignificant; and that if $\frac{R_2}{N}=25$ the ratio is approximately constant down to $n=40$.

In practice it is easy to make $\frac{R_2}{N}=50$, but to get $\frac{R_2}{N}=25$ would require a rather lavish expenditure of copper wire.

Instead of finding the ratio of a particular transformer (with ammeter) once for all, a simple way is to find it each time for the frequency to be used, the strength of the primary current being chosen so as to lie within the range of the instruments available.

The following instance will make the method clearer. Suppose we want to measure currents up to 1000 amperes with an electro-dynamometer and a Kelvin balance, each up to 100 amperes. Let the transformer have a "current ratio" of about 10 when the balance is the only load on the secondary. The electro-dynamometer is inserted in the primary circuit, and a primary current of (say) 98.2 amperes is found to produce

(say) 10 amperes in the secondary. If the electro-dynamometer be now taken out of circuit, any primary current up to 1000 amperes can be measured by reading the balance in the secondary and multiplying by 9.82.

In an exactly similar way very small alternating currents may be measured.

In order to avoid errors due to stray magnetic fields the transformer may be wound with double coils placed side by side with their axes parallel and reversed in the well-known manner. It might also be well to have the position of one or more turns of the primary or secondary adjustable with a scale to show the proper position for each particular frequency.

For the purposes of calibrating at any frequency a second primary of a different number of turns from the first may be wound on the transformer, and the relationship of the two primaries may be determined once for all.

For example, if the first primary is for 1000 amperes the second may have 100 times the number of turns and be suitable for 10 amperes. The correction for any particular frequency or wave-form may then be found experimentally by using the second primary only.

I may mention that with very simple instruments I have used an air-core transformer to measure currents between 1000 and 2000 amperes.

[Added June 12th, 1896.]

In order to get a larger increase of secondary voltage a well-known method is to connect up a transformer with the primary and secondary in series as in fig. 1. By a somewhat similar method the ratio of current transformation may sometimes be increased. The connexions in this case are shown in fig. 2.

Transformers with Iron Cores.

I have recently investigated the case of transformers with iron cores whose secondary circuits were made highly inductive. This was done by short-circuiting the secondary in each case through a Kelvin 100-ampere balance.

Two transformers were thus tested—an iron-ring trans-

former with very small magnetic leakage, and a small transformer with open iron circuit. The first of these had a

Fig. 1.

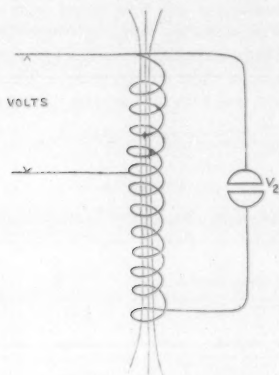
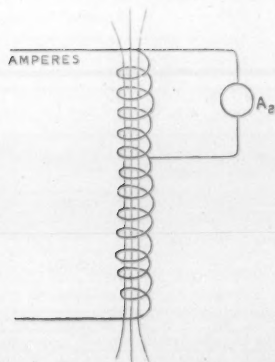


Fig. 2.



primary of 764 turns of No. 16 S.W.G. wire, and over this a secondary of 48 turns of 7/16 wire. The core of the second

was a short bundle of iron wires, over which were wound the primary of 38 turns of No. 16 S.W.G. wire, and over it a secondary of about 3000 turns of No. 26 S.W.G.

The ring transformer was first tested with a Kelvin 10-ampere balance as secondary load, the frequency being 84 ~ per second. The balance had a resistance of about 0.55 ohm and an inductance of about 0.0016 henry. The results given in Table II. show that with this amount of resistance in the secondary the $\frac{I_2}{I_1}$ is by no means constant.

TABLE II.

Iron-ring Transformer. Secondary Load 10-ampere Balance.

I_1 Primary current.	Ratio $\frac{I_2}{I_1}$.
0.0365 amp.	8.12
0.0764	9.28
0.1205	10.12
0.201	11.48
0.336	12.28

TABLE III.

Iron-ring Transformer. Secondary Load 100-ampere Balance.

I_1 Primary current.	Ratio $\frac{I_2}{I_1}$.
1.0	15.695
1.5	15.67
2.4	15.67
4.1	15.73
6.0	15.69

Similar measurements were then made with a 100-ampere balance as secondary load. From Table III., which gives

some of the actually observed ratios, it will be seen that *the ratio of current transformation is practically constant.*

With almost the same arrangement of secondary circuit experiments were made to find to what extent the ratio $\frac{I_2}{I_1}$ was affected by change of frequency. Table IV. gives the results of these and shows that *the ratio is almost independent of frequency through the range tried.*

TABLE IV.
Ring Transformer.

Frequency, ~ per sec.	$\frac{I_2}{I_1}$	Error from 84 ~ per second.
40	15.48	0.7 per cent.
57	15.52	0.45 per cent.
84	15.59	

The results in Table V. of a similar experiment with the transformer of open magnetic circuit described above show how different its behaviour is.

TABLE V.
Transformer with open Iron Circuit.

~ per second.	$\frac{I_2}{I_1}$	Error from 84 ~ per second.
44	28.1	24.3 per cent.
84	37.1	

It seems clear, therefore, that iron-ring transformers may in many cases be used in a similar way to that described above for air-core transformers; but care must be taken to have the resistance of the secondary circuit sufficiently small.

In conclusion I beg to thank Messrs. Lovell, Macalister, Sankey, and Norman for their kind help in some of the experiments.

DISCUSSION.

Mr. BLAKESLEY said that the author's arrangement could only be used for measuring the *current* in the primary. Only if the self-induction of the primary is large and its resistance small, the secondary circuit being open or of large resistance, can the voltmeter of the secondary coil give indications proportional to the P.D. of the primary. The reading will then be the P.D. of the primary multiplied by fraction representing the ratio of the coefficient of mutual induction to that of self-induction in the primary. This will be true independently of the period. But if the frequency is to be kept constant as well as all the other properties of the circuits, of course all effects in both circuits will simply be proportional to the P.D. of the primary, but in that case there is no necessity to make the self-induction of the secondary great, so that the advantage of doing so is not obvious.

XXV. *On the Use of Bare Wire for Resistance-coils.* By F. W. BURSTALL, M.A., formerly Scholar of St. John's College, Cambridge*.

I. Introduction.

It is perhaps somewhat surprising that the form of resistance-coils as used in the ordinary box has, during the last twenty years, changed less than any other kind of electrical instrument. The most serious defects of the present form of resistance-box are, firstly, the difficulty of ascertaining the temperature of the wire forming the resistance, and, secondly, that the method of short-circuiting any required number of coils by means of divided brass blocks and taper plugs leads to contact-resistances which are often of very variable amount. A resistance-coil, the wire of which is covered with silk wound very closely and then covered with paraffin, of necessity conducts heat very badly, and therefore the temperature, as shown by a thermometer placed in the box, gives but little information as to the temperature of the resistance-wire. I have endeavoured to overcome the temperature difficulty by using manganin

* Read June 26, 1896.

wire, but the uncertain nature of this material has forced me to abandon its use. The suggestion for the employment of a bare wire immersed in oil is due to Mr. E. H. Griffiths, and it is with the developments of this suggestion that the present paper deals. There can be no doubt that with a bare wire immersed in oil the heating by the testing current is much less than with covered wire, and further, the temperature of the wire can be accurately measured; but perhaps one of the most important advantages in the use of a bare wire lies in the fact that the wire can be annealed in a most perfect manner by the passage of a sufficiently large current to heat it to a red-heat for a few seconds. It has long been known that well-annealed coils are much less liable to change than coils which have not been so treated. Dr. Lindeck, of the Reichsanstalt, has, in his various papers, laid great stress upon the necessity for annealing standard coils, which is effected by heating the coil in an air-bath to about 140° , the highest temperature a silk-covered coil can bear without material injury. The heating of the wire to a red-heat is, of course, much more efficient than the method adopted by Dr. Lindeck for freeing the wire from undue stress.

The wire which I have employed for the whole of the resistance-box was drawn from one cast of platinum-silver, a material which experience has shown to be the most perfect for resistance-measurement.

The two forms of resistance-box usually employed are the series form and the dial form. In the former a number of coils are placed in series, and any required number can be short-circuited. This form, though it requires comparatively few coils, has the disadvantage that any fluctuation in one plug causes an error in the final measurement, and if any considerable number be used, it is almost impossible to prevent variation in the sum of the contact-resistances. For these reasons it seemed advisable to adopt the second form of box, the dial pattern, in which the number of plugs employed is a minimum, and remains the same whatever resistance is being measured.

It is not essential to make standard resistances in order to calibrate a resistance-box, as each dial can be measured in terms of the preceding dial. This method, however, perpetuates any errors that may have been made in the measure-

ment of any of the smaller coils, and it is not possible to measure the coils in the box with the same accuracy that can be attained in the determination of the standard coils. I have therefore constructed standard coils for every dial, and their values have been obtained from a 1 ohm coil independently of the resistance-box.

II. *Construction of the Resistance-box and Standards.*

The resistance-box has five dials, each consisting of nine coils, ranging from $\frac{1}{10}$ of an ohm to 1000 ohms, and four pairs of proportional arms ranging from 1 to 1000 ohms.

The $\frac{1}{10}$ ohm coils have a length of 96 millim., the wire having a diameter of .63 millim. Being so short, there is no difficulty in supporting the coils.

The wire of the 1 ohm coils has the same diameter and a length of 960 millim. These coils were at first wound into short spirals, and were allowed to hang freely from their terminals. It soon, however, became apparent that the convolutions of the spiral required to be fixed rigidly to the ebonite top. The method that has been finally adopted is to screw the spiral into holes pierced in a mica plate; this plate being screwed top and bottom to a brass bar which is attached to the ebonite top.

In the coils of from 10 to 1000 ohms it is not easy to find a form of bobbin on which to wind the great lengths of bare wire required in such a manner as to avoid short circuits, and to bring the wire in contact as little as possible with the insulator. The form I have ultimately adopted is as follows (fig. 1):—To a central brass rod are attached, for the 10 ohm and 100 ohm coils, two small brass crosses at a distance of about 150 millim. apart; to the ends of the arms of these crosses are screwed serrated strips of mica, the pitch of the serrations being about 1 millim. The resistance-wire is then bifilarly wound into these serrations. The 1000 ohm coils are of the same form, but from their greater length, 250 millim., it is necessary to employ a third cross in order to strengthen the mica, and even then difficulty has been experienced owing to the mica bending under the coil-tension, culminating in several cases in the breakage of the wire.

For the 10 ohm coils wire having a diameter of .30 millim. and a length of 2400 millim. has been used. For the 100

ohm coils, a diameter of .15 millim. and a length of 6200 millim. For the 1000 ohm coils, a diameter of .075 millim. and a length of 14500 millim.

The top is of ebonite, 86 centimetres long, 28 centim. wide, and 19 millim. thick.

Instead of using divided blocks for the dials, conical plugs, having a maximum diameter of 9 millim., and a taper of 1 in 12, fitting into conical sockets have been employed*. The plugs are connected with the zero socket of the previous dial by means of heavy flexible leads, having a copper core, 5 millim. in diameter, composed of copper wires of diameter .2 of a millim. It was found necessary to use comparatively large wire, as the first set of connecting leads, built up of wire .07 millim. in diameter, parted with the constant turning and gave rise to a variable plug-resistance. The use of the conical plug in a solid conical socket is completely free from errors caused by the change of shape of the insulating material; further, such forms can be easily and accurately machined, a matter of no small importance to those who make their own apparatus.

The terminals are of large diameter (25 millim.), and are arranged with the screw in the moveable part, so that the top of the fixed part can be readily cleaned.

The proportional arms are fitted with the usual divided blocks.

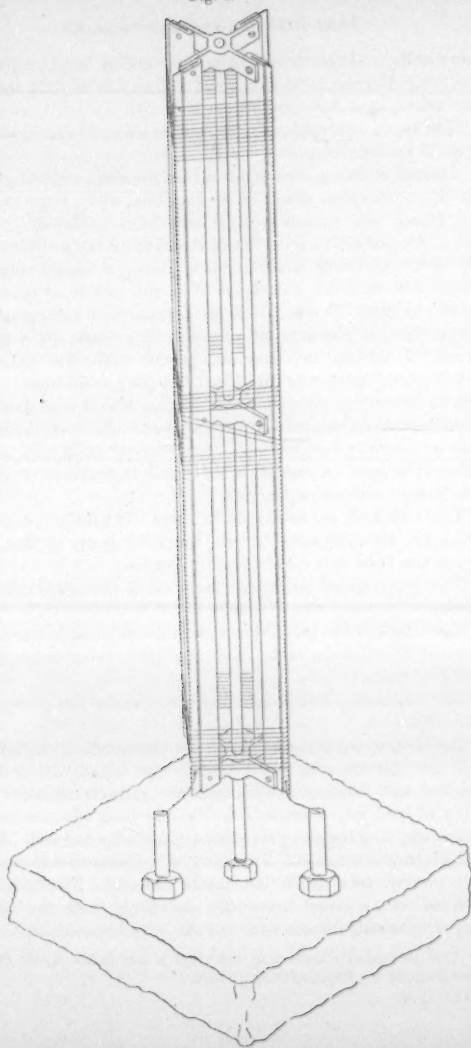
The whole of the metallic part is made of phosphor-bronze, a material which is harder and less liable to oxidation than ordinary brass.

The containing box is lined with copper for the reception of the oil.

The form of bobbin and the wire for the standard resistance-coils are the same in the 1 and 10 ohm standards; but in the 100 and 1000 ohm standards the mica is stiffened by being slipped into a brass back, like the back of an ordinary tenon saw, which quite prevents any tendency to bend. The temperature is measured by means of a thermometer placed in a pierced tube which lies inside the coil. The coils are inclosed in an outer brass case and fitted with the usual copper terminals for use with mercury-cup connexions.

* This method of constructing the plugs is due to Mr. R. W. Paul, Proceedings of the Physical Society, 1892.

Fig. 1.



The $\frac{1}{10}$ ohm standard is of manganin, of the usual British-Association pattern.

The oil employed is a pure heavy hydrocarbon oil, obtained from Messrs. Price's Patent Candle Company; it is free from any trace of alkali or acid, and experiments conducted by Messrs. Price prove that it has no action on the wire.

III. *Construction of Bridge and Galvanometer.*

The bridge employed was of the usual equal-armed pattern, having a straight wire of platino-iridium 1.5 millim. in diameter and 500 millim. long. The wire is stretched immediately over the scale by means of two brass split chucks, these chucks terminating in circular brass rods, having a diameter of 6 millim. The rods slide in heavy brass blocks, and the wire can be tightened by nuts on the ends of the rods. Thus the wire is not soldered to the terminals, and can be removed for repair. The scale is divided into 1000 parts, and the divisions are then divided into 10 by estimation; this is easily done with the aid of a good magnifier. The slider moves on the scale itself, and is provided with a fine adjustment. The connexion to the galvanometer is made by a platinum-tipped spring, resting on a platinum wire which is laid parallel to the main wire. The equal coils are made of manganin, and have a resistance of about 10 ohms. They have several times been adjusted to exact equality, but have never preserved that equality for a long period. This inequality, however, is unimportant, since it is eliminated by double balancing, which has been done at least once during each set of determinations. The actual amount of inequality at the present time is shown by the facts that, with 1 ohm in each of the outer arms, the bridge centre (*i. e.* the mean of the readings when the coils are interchanged) is 500.25, whilst with 1 ohm in each arm it is 500.4, and with 10 ohms in each arm it is 500.5.

The wire is mounted on a stout marble base. To protect the wire as much as possible from heat radiated from the observer's body two blocks of wood have been hollowed in such a manner that they can be slid over the wire; the slider itself being wrapped in cotton-wool. These precautions reduce

the effect of the thermo-currents to a few scale-divisions on the galvanometer. The effect of thermo-currents on the galvanometer was eliminated by the galvanometer-circuit being kept closed, the battery-circuit being made and broken as required. In order to connect the box-coils and the standards to the bridge, three pairs of heavy flexible copper leads have been constructed; one pair has mercury-cups, intended for the reception of the standard coils, at one extremity; a second pair has conical plugs, so that any particular coil in the box can be measured alone; the third pair and the other six extremities of the leads are provided with heavy copper spades, by means of which connexion can be made with the terminals of the box and the bridge. The three pairs of leads are adjusted to exact equality one with another, and each has a resistance of $\cdot 00966$ ohms at a temperature of $11^{\circ} \cdot 8$. The values of these resistances do not enter into any determination as the leads were always in pairs, one in each arm of the bridge.

The galvanometer is a four-coil astatic instrument, the resistance of each coil being 60 ohms. The magnet system is supported by a fine quartz fibre 260 millim. long. In order to diminish as far as possible the effects of the vibration caused by London traffic, thick rubber blocks are placed on the pier supporting the galvanometer; this precaution has made it possible to work at any time except when large masses of iron are being moved in the neighbouring goods station. The scale is placed 3 metres from the mirror. The motion of the mirror is observed by means of a small electric lamp which projects the image of a fine wire on to the mirror, thence it is reflected on to a ground-glass scale. The observer is at a considerable distance from the galvanometer, a matter of some convenience since the zero is thus not so liable to be changed. The galvanometer gives a distinct movement for an alteration of $\cdot 00001$ of an ohm, with 10 ohms in every arm, the battery being one Hellen cell, together with an added resistance in series with it of 10 ohms. The galvanometer is connected to the bridge by 20 gauge rubber-covered high insulation wires; the rubber covering is necessary as I have found serious errors introduced unless the insulation of the galvanometer connexion is extremely good.

IV. *Determination of Standards.*

The temperature difficulties which are most liable to cause inaccuracy in resistance measurements have been greatly simplified by the remarkable uniformity of temperature of the room in which the measurements have been made. The room is almost entirely underground, and is protected on all sides by other portions of the building from the heat of the sun. It is lighted by one small electric lamp, which gives rise to no appreciable change of temperature. The presence of one experimenter causes the temperature of the air to rise about $\cdot 2$ of a degree during the first half hour, and then it becomes constant; but the temperature of the resistance-box and standards, the latter being placed in earthenware vessels and surrounded by sawdust, takes several hours to rise as much as $\cdot 03$ of a degree.

The bridge-wire was calibrated by a method due, I believe, to Mr. Griffiths. In one of the outer arms is placed, by means of mercury-cups, a coil of low resistance, called the gauge-coil; on the other hand, the gap can be short-circuited by means of a thick copper bar. In the opposite outer arms are placed three resistance-boxes in parallel. The object of this arrangement is to secure the necessary fine adjustment. Commencing with the slider at one end of the wire, the gauge-coil being in place, a balance is obtained by adjusting the resistance in the boxes, the slider remaining fixed. The gauge-coil is then removed and the copper bar substituted; the slider is then moved until a new balance is obtained. Proceeding thus, the whole wire is divided into a number of parts the resistances of which are equal. From these observations a chart can be prepared showing the deviation in resistance of the wire from a wire of uniform resistance per unit of length. One of the great advantages of this method is that, when an abnormal value has been obtained, it is a simple matter to come back to that particular portion of the wire for a check measurement. The gauge-coil was equal to the resistance of 22.293 mean scale-divisions (each 0.5 millim.), and the greatest error at any point of the wire amounted to $\cdot 4$ of a scale-division. Throughout the whole of the comparison of the coils this calibration of the wire has been

employed, and several apparently discrepant observations have been thus brought into line.

The temperature-coefficient of the bridge-wire has been found by raising the temperature of the room from $10^{\circ}\cdot4$ to $18^{\circ}\cdot1$, the bridge having in one arm a coil of known temperature-coefficient, and was found to be $\cdot071$ per cent. per degree.

The thermometer employed, which is of hard Jena glass, is divided into tenths of a degree, each tenth being approximately $\cdot5$ millim. in length. By means of a small reading telescope it can with ease be read to $\cdot01$ of a degree. It was tested at the Reichsanstalt and, over the range employed, was accurate to at least $\cdot05$ of a degree.

The temperature-coefficient of the platinum-silver wire, over a range from 10° to 25° , was determined by immersing a coil of about 1 ohm in paraffin-oil at 25° , and allowing it to cool very slowly. It was found to be $\cdot000274$ per ohm per degree.

The 1 ohm coil from which the standards have been built up is of manganin, and when tested in June 1894 by Mr. Glazebrook had a resistance of $1\cdot00026$ ohms at $13^{\circ}\cdot4$. I was, however, so doubtful of the constancy of this standard that in December 1895 I constructed a bare platinum-silver 1 ohm coil. This was compared with the manganin standard, and the mean of eleven determinations gave its value as

$$1\cdot00000 + \cdot000274 (t - 14\cdot96).$$

No doubt the manganin coil had changed its value in the eighteen months that had elapsed since it was tested, so that the above figures were not true ohms.

The 10 ohm standard was obtained by the use of three approximate 3 ohm coils*, first placed in parallel and balanced against a 1 ohm coil, and then in series with the 1 ohm coil and balanced against the 10 ohm coil. Instead of connecting the legs of the coils by mercury-cups, as is usually done, the fusible metal made by Mr. F. Thomas, of Cambridge, was used. This metal, which melts at 60° , gives very constant contact resistances, and does not seem to injure

* This method is described in the B. A. Reports, 1883.

in any way the amalgamated terminals of the coils. I first determined the value of a manganin 10 ohm coil, which had been made for nearly a year. The value obtained on December 20, 1895, was 9.9987 at $9^{\circ}3$. This coil was used as a standard to which to adjust the platinum-silver 10 ohm coil. The value thus obtained for the latter was 9.9895 at $10^{\circ}3$. On stepping up to the platinum-silver 10 ohm coil, the value found on December 30 was 9.9905 at $10^{\circ}6$. Taking into account the known temperature-coefficient, the latter at $10^{\circ}3$ would be 9.9897, which differs by only two parts in 100,000 from its former value.

To obtain the value of the $\frac{1}{10}$ ohm coil, which was of manganin, a bridge was formed of two 1 ohm coils, a 10 ohm coil, and the $\frac{1}{10}$ ohm coil, and a balance obtained by shunting one of the 1 ohm coils*.

On January 12th, a value of .099934 at $11^{\circ}2$ was obtained.

" " 13th, " " .099932 " $10^{\circ}2$ " "

The temperature-coefficient of the wire was known to be .000004 per degree. Between these two determinations the bridge had been dismantled. This coil was again tested on April 8th, and had then a value of .099944 at $12^{\circ}1$, whereas, if it had remained unaltered, its value at that temperature should have been .099938. This change is, however, too small to cause any error in the box-coils.

For testing coils which are very nearly in the proportion of 1 to any power of 10 this method is extremely sensitive, and certainly more accurate than making the comparison by means of the wire bridge, provided that the resistances of three out of the four coils forming the bridge are accurately known.

In obtaining the values of the 1 ohm and 10 ohm standard coils, not more than 50 scale-divisions of the wire had been employed, hence a small error in the total resistance of the bridge-wire would not cause any inaccuracy in the values so obtained. The resistance of the bridge-wire was determined in the first instance by placing the standard 1 ohm coil in parallel with a coil of slightly smaller resistance than the

* This method is described by Mr. Glazebrook, B. A. Reports, 1894.

bridge-wire. This gave the resistance per scale-division of the wire as $\cdot 00087$ at $10^{\circ}1$. After the value of the $\frac{1}{10}$ ohm coil had been found, the resistance of the bridge-wire was determined by balancing the $\frac{1}{10}$ ohm coil against a coil of slightly smaller resistance than the bridge-wire; the $\frac{1}{10}$ ohm coil was then removed, and the gap short-circuited. The actual figures are of interest as showing the value of a calibrated wire.

$\cdot 1$ ohm in A. Gauge in D. Temperature $10^{\circ}8$.

Reading 394.0.

Correction for calibration $-.3$.

Short circuit in A. Gauge in D. Reading 967.9.

Correction for calibration, $-.2$.

Hence, if ρ be the resistance per scale-division,

$$\cdot 1 \text{ ohm coil} = 2 \times 574 \rho = 1148 \rho.$$

On interchanging the coils,—

$\cdot 1$ ohm in D. Gauge in A.

Reading 32.9. Correction for calibration, 0.

Short circuit in D. Gauge in A.

Reading 607.0. Correction for calibration, $-.1$.

This also gives

$$\cdot 1 \text{ ohm coil} = 1148 \rho.$$

Whence $\rho = \cdot 00008705$ at $10^{\circ}8$.

A and D being the two outer gaps of the bridge.

The 100 and 1000 ohm standard coils were determined on similar bridges. The 100 ohm coil,—

On April 14, had a value 99.913, at $10^{\circ}8$.

„ „ 17, „ „ 99.964, „ $12^{\circ}5$.

Whereas it should have been 99.960, according to the observation on the earlier date. This discrepancy is, I believe, due to the fact that the shunt required on one of the 10 ohm coils forming the bridge was greater than 12,000 ohms, which was the largest resistance at my disposal: in

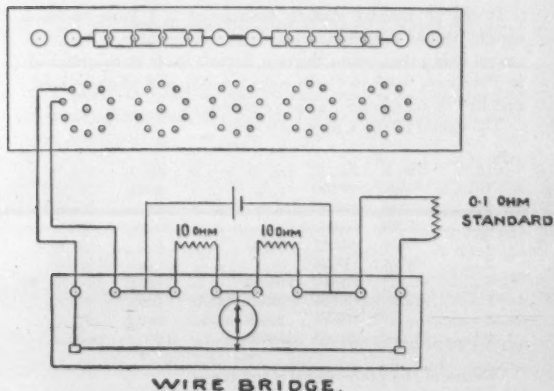
order to obtain a balance one of the 10 ohm coils had to be warmed, by means of a water-bath, about 2° above the temperature of the room, which led to some uncertainty as to the actual temperature.

The 1000 ohm standard coil was obtained by forming a bridge with the 1, 10, 100, and 1000 ohm coils, and had a value, on April 28, of 1000.24 ohms at $12^{\circ}.8$. No other determination has, as yet, been made. The coils were adjusted so as to be right at 15° .

V. Calibration of the Resistance-box.

For the first three of the dials the coils were balanced against standard coils (fig. 2), and the differences obtained

Fig. 2.



by means of the wire bridge. In order to connect the conical plugs to the arms of the wire bridge, the flexible leads with conical ends were used; the standard in the opposite arm being connected by means of the leads with mercury-cup ends.

For the 1 and 100 ohms in the proportional arms spade-

ended leads were used, and attached to the terminals at the ends of the proportional arms (see diagram).

For the sake of convenience of comparison, I have reduced the resistances to their values at 11°C .

The values of the $\frac{1}{10}$ ohm dial are as follows:—

No. of Coil.	Jan. 12.	Jan. 20.	Jan. 22.	Jan. 24.	Jan. 25.	Jan. 26.	Mar. 11.	Mar. 14.	Mean.
1 ...	·10067	·10071			·10065	·10065	·10073	·10073	·10073
2 ...	·10012	·10012	·10009	·10008	·10008		·10012		·10010
3 ...	·10067	·10069	·10063	·10061	·10063		·10050		·10050
4 ...	·10068	·10014				·10011	·10010		·10011
5 ...	·09976	·09974	·09975		·09973	·09972	·09975		·09974
6 ...	·09982	·09987			·09985	·09985			·09985
7 ...	·10046	·10046			·10042				·10045
8 ...	·10003	·10004				·10003			·10003
9 ...	·10055	·10061				·10066			·10061

It will be noticed that, in coils 1 and 3, I have rejected certain values determined on dates before March 11; the reason being that, when the new flexible leads were attached in February, these two coils were touched and slightly bent, and had in consequence to be re-annealed.

The values of the 1 ohm coils are—

No. of Coil.	Jan. 8.	Jan. 11.	Jan. 26.	Feb. 12.	Feb. 16.	Mean.
1	·99929	·99929			·99931	·99930
2	·99956	·99956				·99956
3	·99569	·99568				·99568
4	·99964	·99969	·99970			·99968
5	·99776	·99774				·99975
6		1·00007	1·00007		1·00008	1·00007
7		·99976	·99979	·99978	·99980	·99978
8		·99878	·99875	·99875	·99878	·99877
9		1·00053	1·00051		1·00053	1·00054

Values of the proportional arms are—

	April 3.	Mar. 8.	Mar. 16.	Mean.
1 in X	·99863	·99860	·99860	·99861
1 in Y	·99645	·99641	·99642	·99643

Considerable trouble was experienced in the determination of the 1 ohm coils in the proportional arms, owing mainly to variations in the resistance of the plugs, which were fitted into the usual divided blocks. The amount of this error is of

little moment when compared with the 10 ohm coil. The 1 ohm coils have not been employed in the determination of the larger resistances.

The values of the 10 ohm coils are:—

No. of Coil.	March 8.	April 3.	April 4.	Mean.
1	9.9765	9.9764		9.9764
2	9.9926	9.9925		9.9925
3		10.0033	10.0033	10.0033
4	9.9976		9.9973	9.9974
5	9.9999	10.0000	10.0001	10.0000
6	9.9574	9.9573		9.9573
7	9.9558	9.9555		9.9556
8	9.9863	9.9858	9.9864	9.9862
9	9.9748	9.9753	9.9758	9.9753

The values of the proportional arms are:—

	Mar. 8.	Mar. 16.	Mar. 17.	April 3.	April 4.	Mean.
10 in X	9.9513	9.9506	9.9513	9.9507	9.9509	9.9509
10 in Y	10.0106	10.0099	10.0103	10.0098	10.0101	10.0101

Desiring to obtain a check measurement, I balanced the nine 1 ohm coils against the 1 and 10 ohm standards in parallel, and obtained the value .90224 at 13° , while the first five coils of the dial, when balanced against the two 1 ohm coils in parallel, gave .50125 at 13° . The actual values for the sums of the nine and five respectively, as found by the separate determinations, were .90259 and .50147. Of course any contact-error due to the plugs would appear when the values of the coils are added up. When the separate values of the nine coils are added up, the contact-error will have entered 18 times into the result, and for five coils 10 times; on the other hand, when the coils are measured in series, the contact-error will only enter twice in each case, and the differences between the two sets of values in both cases are explained by the ascertained result that the contact-resistance of each plug is .00002.

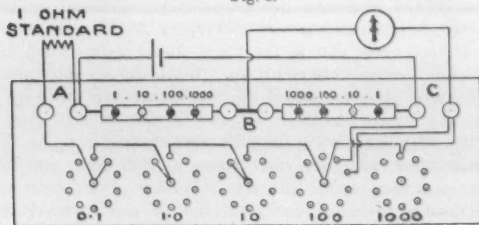
It is somewhat bold to assert that an error of .00004 ohms has not entered into the determination of each of the $\frac{1}{10}$ ohm coils. The standard itself is certainly correct, as will be seen by reference to its values on the various dates; the only possible explanation would be that the connecting leads of

the two arms had not equal resistances. A subsequent test, however, proved that they were equal. I was, therefore, driven to the conclusion that the contact-resistance as above found was real, and not due to an experimental error.

The resistance of the box from terminal to terminal with each plug in zero was obtained in three ways. Firstly, by short-circuiting the mercury-cups in one of the arms of the bridge, and measuring the resistance of the leads and plugs on the bridge-wire. This gave a value of $\cdot 00967$ ohm at $12^{\circ}8$. Secondly, the first coil of the $\frac{1}{10}$ ohm dial, together with the leads, was balanced against the $\frac{1}{10}$ ohm standard; subtracting from this resistance the known resistance of the box-coil, we get a value of $\cdot 00964$ ohm for the resistance of the leads, at $12^{\circ}4$. Thirdly, a similar method was employed to measure the first coil in the 1 ohm dial, and this determination gave the resistance of the leads as $\cdot 00954$ ohm at $10^{\circ}3$. In order that these three observations may agree it is necessary to assume that the temperature-coefficient of the copper leads is about $\cdot 00005$ ohm per degree. This is rather high, but the temperature range is too small for an accurate determination.

The values of the coils in the 100 dial were obtained on the box itself. In the gap A (fig. 3) the 1 ohm standard coil

Fig. 3.



was placed; this, together with the first three dials, gave a resistance of about 100.9 ohms; and constituted one arm of a Wheatstone bridge, the two 10 ohm coils in the proportional arms formed two other arms, the fourth arm was any one of

the 100 ohm coils, which was connected to the gap C by means of the two flexible leads with conical ends. The plug from the centre of the 100 dial was in the zero of the 1000 dial so as to complete the circuit from the gap A to the gap C. The smallest resistance in the box was $\frac{1}{10}$ of an ohm, so that the last two figures of the resistance had to be obtained by interpolation from the swings of the galvanometer. The battery employed was one storage-cell, and the current was commuted for each observation. With the coils of the galvanometer placed in series an alteration of $\frac{1}{10}$ of an ohm in the box caused a total galvanometer swing of about 116 scale-divisions.

The values of the 100 ohm coils are: —

No. of Coil.	April 5.	April 6.
1	100·511	100·510
2	100·967	100·971
3		100·266
4	100·300	100·305
5	100·362	100·365
6	100·671	100·676
7	100·603	100·607
8	100·434	100·435
9	100·845	100·850

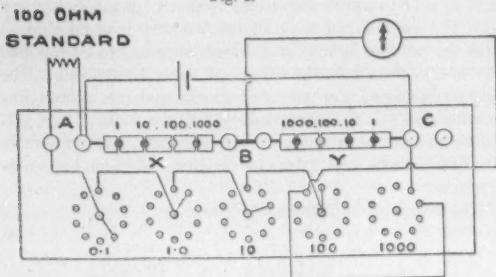
The 100 ohm proportional arms were determined by the same method as was used for the 100 ohm standard coils. Their values are:—

	April 17.	April 20.	April 21.	Mean.
100 in X .	100·224	100·219	100·220	100·221
100 in Y .	100·312	100·314	100·309	100·312

The 1000 ohm coils were also measured on the box itself. In the gap A (fig. 4) was placed the 100 ohm standard coil; this, together with the first four dials, formed one arm of a Wheatstone bridge; the two other arms were the 100 ohm coils in the proportional arms. A single flexible lead connected the last coil of the 100 dial with any required coil on the 1000 dial. Another lead connected the other terminal of the

coil to be measured to the terminal of the proportional arm Y. The current was provided by two storage-cells.

Fig. 4.



The values of the 1000 ohm coils are :—

	April 25.
1	1023·35
2	1027·30
3	942·05
4	1022·04
5	1017·21
6	1032·04
7	1007·44
8	1003·45
9	1026·42

The values of the proportional arms are :—

	April 27.
1000 in X	947·92
1000 in Y	994·21

My object in giving the whole of the figures relating to the resistance-coils is to show to what an extent bare wire coils are to be trusted. Whether the coils retain their values as well as the resistance standards of the British Association, time alone can decide. There is no reason to suppose that, if the coils are kept continuously in an oil-bath, and the oil kept free from water, any considerable alteration can take place.

XXVI. *On the Theory of Moving Electrons and Electric Charges.* By J. LARMOR*.

IN an interesting paper by Mr. W. B. Morton, communicated by the Physical Society† to the Philosophical Magazine for June, there is a criticism of a portion of my paper on "A Dynamical Theory of the Electric and Luminiferous Medium"‡, which if valid would affect its whole tenor. As, however, the formulæ of that paper were to a considerable extent obtained by two independent trains of reasoning, it would have to be shown that both were wrong before an error could be fully substantiated. As a matter of fact, Mr. Morton's criticism arises from his reading into the analysis assumptions which are not there, but which had been used, with the proper limitations to secure accuracy, in another place in the previous part of the paper. As the point is really fundamental, and as the analytical statement in the memoir is no doubt too brief to convey at once a grasp of the procedure employed, without somewhat detailed consideration on the reader's part, I beg leave to offer the following general explanation.

The facts of chemical physics point to electrification being distributed in an atomic manner, so that an atom of electricity, say an electron, has the same claims to separate and permanent existence as an atom of matter. The fundamental question then is, how far the conception of separate isolated electrons, pervading the æther of free space, can provide an explanation of electrodynamic and optical phenomena. In the paper referred to I have gone further back, and have considered the question how far such a simple underlying scheme is able by itself to provide an explanation of physical phenomena in general; for it will obviously not be permissible to import into our dynamical notion of an atom of matter more than simple electric properties, unless these latter prove to be insufficient to include all actual knowledge of its relations. The conclusion arrived at in the memoir is that there is

* Read June 26, 1896.

† The present volume, *ante*, p. 180.

‡ Phil. Trans. 1894 (A), pp. 719-822, and 1895 (A), pp. 695-743.

nothing in the ascertained laws of general physics which points to insufficiency in that scheme; while there are some experimental results which somewhat militate against the existence of interatomic forces of any kind other than those included in it.

The main feature of the theory referred to is that the æther is not matter, as ordinarily assumed, nor in any way like matter; it is the uniform substratum (analytical basis, if one is disposed to use that term, for it can never be the direct object of perception) in which the atoms of matter consist as permanent configurations of strain and motion. As was to be expected, the relations of inertia and elasticity of this uniform medium are simpler than those of matter, which is merely a molecular aggregate involved in its constitution. In fact, the only way to arrive at a scheme of the relations between æther and matter which shall be a complete dynamical theory and not merely descriptive, is to abolish the apparent duality in the phenomena, either by taking as here the molecules of matter to constitute singularities (in the mathematical sense of the theory of functions) in the uniform æther, or else by trying to make out the æther to be ordinary matter, and so giving up any attempt to explain why matter is molecularly constituted. This molecular constitution of matter is essential to the former theory, just as it is to all other theories or illustrations, like the vortex theories, which hypothecate a uniform underlying medium; it is quite unintelligible—or rather quite unexplained—on the latter type of theory.

But however these things may be, the point criticised by Mr. Morton does not involve any considerations so refined, or—as possibly may be said—so ambiguous. The sections to which he objects claim to be a reconstruction of ordinary electrodynamic theory on the basis of permanent electrons associated with the atoms of matter. Whatever view one may entertain as to the presence of qualities other than electric in the atom, all are I think now-a-days agreed that the electron is there. And whatever view one may have as to the validity and sufficiency of an æther with simple rotational elasticity, the formal equations to which that theory leads *for free space* are just those equations of Maxwell which Hertz's experi-

mental work has fully verified. The problem of electrodynamics is then that of the free æther, whose properties are represented analytically by these acknowledged equations, disturbed by the action of the electrons of material atoms moving about in it. The original Amperean electrodynamics, proceeding by consideration of elements of current, has not proved valid or sufficient in matters involving electric radiation, or even ordinary electrodynamic force. A most successful modification of it was that proposed by Weber, in which elements of current were replaced, as the fundamental object of consideration, by moving electric particles which acted on each other *at a distance* according to a law of force involving their velocities. This theory was, however, shown long ago by Lord Kelvin and Professor von Helmholtz to be untenable, on account of its violating the principles of the modern theory of energy ; now, of course, direct action at a distance is altogether out of court. The present question is whether a theory of electrons which act on each other, not directly according to a law of force, but mediately by propagation of the effect across the intervening æther, suffices to avoid the discrepancies of earlier theories and give a consistent account of electrical and optical phenomena ; and it is maintained that the answer is altogether in the affirmative. This question is, presumably, sufficiently important and fundamental to justify the present detailed explanation.

At the end of the first of the two papers referred to, building chiefly on the analytical results of previous theorists, the steady æthereal disturbance carried along by a moving electron had been investigated, and also the law of the force exerted on each other by two moving electrons through the intervention of the æther between them. This was on the hypothesis that each electron carried along with it a steady trail of æthereal disturbance, but that no sensible derangement of this steady motion ever occurred such as would lead to loss of the energy of the system by the starting of waves. If the velocities of the electrons remain always small compared with that of radiation, then, however their mutual influences alter their motions, this steady trail will instantaneously adjust itself to the new conditions without sensible excitation of radiation, and the theory will apply. But if any of the

electrons are moving with velocities comparable with that of radiation, a change in velocity will involve derangement of this steady trail of æthereal strain and motion, giving rise to wave-motion which will carry off some of the energy by radiation. Accordingly in such a case it is altogether nugatory to speak of laws of action between electrons: the complete theory must then take account not only of the positions and velocities of each of the electrons at each instant, but also of the state of each volume-element of the surrounding æther. And the theory of mutual actions of electrons as expressed in the memoir was in fact thus restricted to cases in which their velocities were small compared with that of radiation: unless that condition is satisfied there is no such theory at all.

In the second paper (§ 15, *seqq.*) the general problem is attacked: it is now not a question of a set of electrons by themselves, each with a definite steady trail, but of the æthereal medium in general, *including* such electrons as exist in it. The analysis there given determines from foundations which all who adopt Maxwell's electrical scheme for free æther must allow, expressions for the force (P', Q', R') which acts on an element of volume of the free æther, and for the force e (P, Q, R), ordinarily called electric force, which acts on an electron e ; and it uses these forces for further development of the theory. What Mr. Morton's computation virtually does is to assume that the trail of each electron is steady, and then to transfer to the electron itself the force due to (P', Q', R') acting on this æthereal trail. In the special case of no radiation, and of velocities small compared with that of radiation, this force can, as above explained, be transmitted through the æther to the electron itself, and be supposed there applied. But to so transmit it in general is to miss the point of the theory, and, as Mr. Morton himself remarks, to reach the absurdity that the force on a moving charge depends not only on the state of the surrounding æther but on the state of the æther at a distance.

As regards the main subject of Mr. Morton's paper, it may be of interest to state the following general theorem. Suppose a system of charged conductors is in steady translatory motion through the quiescent æther with velocity u , and let v represent the velocity of radiation in free æther: consider a cor-

relative system of conductors obtained by uniform geometrical elongation of the actual system along the direction of motion in the ratio of $(1-u^2/v^2)^{-\frac{1}{2}}$ to unity, and find the electrostatic distribution of the same charges on this system supposed at rest: then the actual distribution of the charges on the moving system will be exactly correlative, viz., equal charges will exist on all corresponding elements of the two systems. This proposition is, however, limited to the case in which none of the bodies of the moving system are dielectrics, but all are conductors.

Finally, I take advantage of the present opportunity to draw attention to some special points in which the analysis of the second part of my memoir is incomplete. In §§ 28, 29, on the mechanical pressure of radiation, a statical force has been overlooked; when this is included the result practically agrees with that given by Maxwell. In §§ 34-6, on the material forces in polarized media, the tractions on interfaces of transition remain to be developed, and the theory may be otherwise improved. I hope presently to treat this subject at length.

Cambridge, June 4, 1896.

XXVII. *On the Resistance of the Electric Arc.* By JULIUS FRITH, 1851 Exhibition Scholar, the Owens College, Manchester, and CHARLES RODGERS, B.Sc., 1851 Exhibition Scholar, Firth College, Sheffield*.

[Plates I. to III.]

THERE seems to be some uncertainty as to what is meant by the resistance of the arc. Any given arc is a phenomenon which exists at a definite P.D. and current, and any attempt to measure its resistance must alter the state of the arc as little as possible or else we are no longer dealing with the same phenomenon. Hence it seems to us that the only way in which the resistance of the arc can be measured is by the ratio of a very small increment of P.D. applied, to the small increment of current produced. It must also be

* Read May 8, 1896.

borne in mind that the change in the current must be of so short a duration that the form of the carbons is not in any way altered.

Apparently the only method which fulfils these conditions is one in which a small alternating current is used, superimposed on the main continuous current. This has the effect of rapidly increasing and decreasing by a small amount the current passing through the arc. It seems probable that the effect on the arc of each small increase of current is annulled by the decrease of current immediately following. In fact a comparatively large alternating current may be superimposed on the main continuous current without producing any visible effect on the arc.

We are thus led to define the resistance of the arc as the ratio of a small increment of P.D. applied, to the small increment of current produced. This may be briefly written $\frac{dV}{dA}$. It is most important to distinguish this quantity, which we call the "instantaneous" $\frac{dV}{dA}$, from the tangent of the inclination of the tangent line of the curve representing the steady values of V and A , which we will call, for the sake of clearness, the "steady" $\frac{dV}{dA}$.

We have performed some experiments to exemplify the difference that exists between these two quantities; and also to show that in cases analogous with the arc, where, however, the result can be verified, the instantaneous $\frac{dV}{dA}$, found by superimposing an alternating current, gives correct values for the resistance.

In one of these experiments a glow-lamp, taking 10 amperes at about 8 volts, was placed in series with three I.E.S. 50 ampere accumulators, and a current sent through against the E.M.F. of the cells. This arrangement is just what is wanted to test the method, namely a resistance in series with a back E.M.F., both of which are functions of the current; and, further, the resistance can be separated from the back E.M.F. and measured, and the result compared with the value obtained for the resistance by the instantaneous $\frac{dV}{dA}$.

The result of this experiment is represented on Pl. I. Here are plotted the curves connecting the current and P.D. between the outside terminals of the arrangement and also between the terminals of the lamp and of the battery. This last gives the back E.M.F., since the resistance of the cells was small enough to be neglected. The P.D. at the lamp divided by the current gives the resistance; this is plotted on the same diagram; to make it clearer, however, the scale of ohms is multiplied by 10. We have measured* the instantaneous $\frac{dV}{dA}$ at various current strengths, and the values of this are plotted in a dotted line. It is seen that there is a very close agreement between these two measurements of resistance. On the same sheet are plotted values for the steady $\frac{dV}{dA}$. This differs considerably from both the resistance-curves. This leads us to see that the rapid excursions caused by the alternating current are not along the curve joining the steady values of V and A , but along a line which is everywhere more vertical than the tangent to the curve. This line is formed by joining the point on the curve to the instantaneous origin, which is distant from the origin of the diagram by an amount equal to the back E.M.F. at that particular current. Were the electrical excursions to travel along any intermediate path, the value obtained for the instantaneous $\frac{dV}{dA}$ would be dependent on the frequency. As will be seen later, this is not the case between the experimental limits of 250 and 7 complete alternations per second. If, therefore, the arc, as has been affirmed by various authorities, consists of a back E.M.F. and a resistance, we feel justified in applying this method for the measurement of its resistance, which has been found correct in closely analogous cases.

Now at very low frequencies indeed the electrical oscillations would travel along the curve connecting the steady values of V and A ; and this is clearly the meaning of the critical frequency which we have observed with cored carbons (see p. 323), namely, that under the critical frequency the

* By Method I. below.

superimposed alternations travel on the steady value curve and become identical with the "steady" $\frac{dV}{dA}$.

Several experimenters have obtained values for the resistance of the arc which agree fairly well amongst themselves, and which seem to show that the arc has a positive resistance*.

It was pointed out, however, by Prof. Ayrton at the Ipswich meeting of the British Association, that although there was a marked agreement between the values obtained by these experimenters, they were not at all in accordance with the conclusions drawn by himself from consideration of the curves obtained by Mrs. Ayrton. These curves connect the P.D. between the carbons with the current passing through the arc, for various fixed lengths of arc, and from them it is seen that for a given arc-length an *increase* of P.D. is always accompanied by a *decrease* of current. From this fact Prof. Ayrton concluded that if an attempt were made to measure the resistance of the arc by altering the P.D. between the carbons and finding the corresponding alteration of current produced, the resistance found by taking this ratio must be *negative*.

This conclusion was strengthened by some experiments made by Mr. Mather at Prof. Ayrton's suggestion. In one of these experiments two points of equal potential were found in a circuit consisting of an arc, a battery, and a resistance. Another battery, consisting of a few cells, of known E.M.F. and resistance was applied between these two equipotential points and the current flowing through the battery was noted. The resistances of the two parallel halves of the circuit, excluding the arc, were known, so that the current which, taking the arc resistance as zero, should flow through this battery could be calculated. Now the value of this calculated current was found to be less than the observed value, no matter in which direction the P.D. was applied, and this result was also obtained when an alternating P.D. was used. Hence the resistance of the arc was apparently less than zero.

* An abstract of papers bearing on this subject was given by Mrs. Ayrton in the 'Electrician,' Sept. 13, 1895.

The other experiment consisted in running the arc at a steady P.D. and current, suddenly altering the resistance in circuit by a small amount, and noting the changes in the ammeter and voltmeter-readings so produced. The new conditions were maintained only long enough to allow of these readings being taken. The arc was then brought back to its former condition before taking another reading. It was found that a change of P.D. in *one* direction was always accompanied by a change of current in the *opposite* direction. The results of both experiments were, however, only qualitative.

All these experiments, together with the consideration of the curves found by Mrs. Ayrton, lead to the conclusion that the arc has a *negative* resistance, while former experimenters had all obtained a *positive* resistance.

It was in order to throw some light on this discrepancy that we undertook a series of experiments to determine with some degree of accuracy the resistance of the arc under various conditions.

Methods.

A number of methods were tried using alternating currents, of which the following were most successful :—

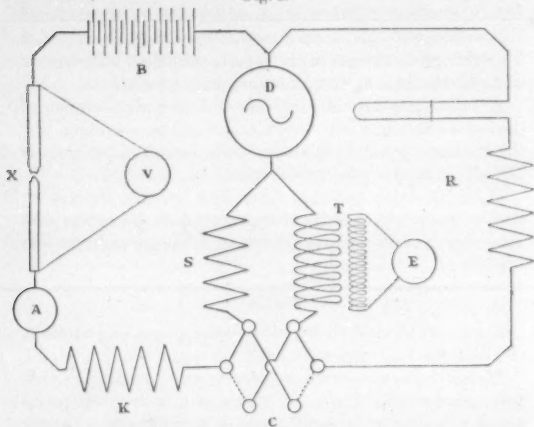
Method I. is represented diagrammatically in fig. 1. D is the armature of an alternator, the current from which passes round two circuits in parallel, one of which contains the arc X, and the other an adjustable resistance R. By adjusting R the alternating currents in the two halves can be made equal. When this is the case the impedances of the two halves to alternating currents must be equal.

In the diagram the continuous-current circuit is shown to the left. It consists of a battery of accumulators B, the hand-adjusted arc-lamp X, the resistance K, the ammeter A, and (with the commutator C as shown) the resistance S and the alternator D. It will be noticed that the alternator D carries the continuous current, but this of course does not prevent its acting as an alternator.

In order to measure the small alternating current independently of the continuous current flowing we used the air-transformer T, the thick wire coil of which was in series with

the alternator D, the thin wire coil being connected with an electrostatic voltmeter E. The reading of the electrostatic voltmeter is thus unaffected by the continuous current, while it is, at a given frequency, a measure of the alternating current flowing.

Fig. 1.



By means of the commutator C, the air-transformer T can be thrown into either circuit, the resistance S being by the same operation thrown into the other circuit. The resistance S is equal to that of the thick wire coil of T, so that when S replaces T the continuous current is unaffected by the change.

Method of Experimenting.

The arc was run at the required current and P.D. by altering the number of cells in B, K being always kept the same. The current was kept constant by adjusting the arc by hand. Under normal conditions the current could be kept constant to within 1 per cent. R was now adjusted till the deflexion of E was the same when T was in either circuit. When

balanced, a change of 0.01 ohm in R caused an appreciable difference in the deflexions of E . It was found to be useless to adjust more accurately than this, since the small variations in quality always found in carbons produced differences of this order in the resistance of the arc.

Let the value of R when a balance is obtained be R_1 . This is equal to the resistance, to alternating currents, of the battery (b_1), the resistance (k), the arc lamp and connexions (l), and the arc (x).

$$\therefore R_1 = k + b_1 + l + x. \quad (i.)$$

The carbons are now firmly screwed together and the number of cells in B reduced till the continuous current is the same as before. R is again adjusted till the deflexions of E are equal; and if R_2 is the new value thus obtained,

$$R_2 = k + b_2 + l; \quad (ii.)$$

b_2 being the resistance of the portion of the battery now used. The cells are next cut out, the mains leading to them short-circuited, and a third value R_3 obtained,

$$R_3 = k + l. \quad (iii.)$$

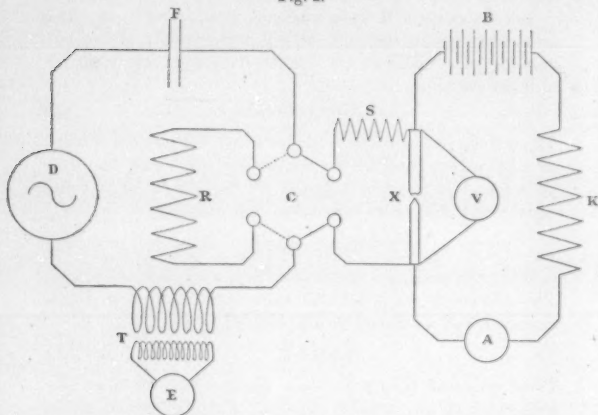
From (ii.) and (iii.) we obtain the resistance of b_2 , and by proportion of any number of cells. Putting these values in (i.) we obtain the value of x in ohms.

The advantages of this method are that no calibration of the electrometer is required, the speed of the alternator need not be constant for long periods together, various alternators giving different frequencies and wave-forms can be used without materially altering the circuit, and especially that the resistance of the arc can be obtained directly in ohms as the difference of two readings of the box R .

Method II. is shown diagrammatically in fig. 2; the arc circuit in this case being shown to the right. X is the arc, B the battery of accumulators, K the resistance, A the ammeter, and V the voltmeter across the arc. B and K were made large, K being about 11 ohms. The circuit shown on the left consists of the alternator D , the transformer T , which together with E is now used simply as a delicate alternate-current ammeter, a condenser F , and a commutator C . By

means of C we can put in circuit either the resistance R, or the arc-lamp X in series with the resistance S. The alternating current flowing through K is negligible compared with

Fig. 2.



that flowing through X on account of the high resistance of K. The condenser F prevents any continuous current due to the P.D. at the arc from passing through the alternator.

Method of Experimenting.

If L is the self-induction of the circuit and F its capacity, the impedance I of the circuit is given by $\sqrt{R^2 + \left(\frac{1}{F\omega} - L\omega\right)^2}$, where $\omega = 2\pi \times \text{frequency}$. This is a minimum when $LF\omega^2 = 1$. The alternator was run at a speed corresponding to this frequency, at which the arrangement is most sensitive to changes of R.

An experiment consists in adjusting R till the deflexions of E are the same whether R or $S+x$ is in circuit. Then

$$R = S + x.$$

The experiment was first tried without a resistance at S, but it was found impossible when using solid carbons to obtain a balance even when R was zero. In order to make it possible to obtain a balance with R greater than nothing, a resistance S numerically greater than the negative resistance of the arc had to be put in series with the arc. In our experiments we kept S constant and adjusted R. The experiment might have been performed by making $R=0$, and adjusting S till the readings of E were equal. Then $S+x$ must be equal to zero, and therefore $x=-S$, or *the resistance of the arc is equal to the resistance in the box S with its sign changed.*

This method gives results which agree well with those obtained by Method I.; but on account of the necessity of keeping the speed of the alternator constant the arrangement is more troublesome to manage, and therefore was used only as a check method.

Description of the Apparatus.

The *arc-lamp* had a hand adjustment for each carbon with centering arrangements for the positive. The arc-length was measured by projecting an image of the arc on to a screen by means of a lens in the usual way.

The resistance K consisted of broad platinoid strips bent back on themselves so as to avoid self-induction*. It did not heat appreciably with the largest currents used.

The battery consisted of 50 10-ampere E.P.S. cells arranged in 4 groups of 10 cells each, 4 of 2, and 2 groups of 1. These groups could be connected up by means of mercury cups.

The *ammeter* and *voltmeter* were Weston instruments of the horizontal type.

The *air-transformer* T was made in two sections which were usually used in series. The constants were as follows:—

Section No. 1:

Fine wire coil.

Diameter of wire (uncovered) = 4.3 mils.

“(covered) = 6.8”

Number of turns = 9730.

Resistance = 4707 Ω

* Described in Phil. Mag. February 1892.

* Described in Phil. Mag. February 1892.

Thick wire coil.

Size of wire No. 12 S.W.G.

Number of turns = 128.

Resistance = 0.143 ohm.

Section No. 2:

Fine wire coil.

Diameter of wire (uncovered) = 3 mils.

" " (covered) = 5 "

Number of turns = 29,500.

Resistance = 32,120 ohms.

Thick wire coil.

Size of wire No. 12 S.W.G.

Number of turns = 96.

Resistance = 0.112 ohm.

The thick wire was double-cotton-covered. The thin wire was silk-covered.

The *voltmeter* was an Ayrton-Mather reflecting electrostatic voltmeter giving a deflexion of 700 scale-divisions at a distance of 2500 scale-divisions for 100 volts. This instrument was admirably adapted to our purpose on account of its very quick swing and excellent damping.

The *resistance R* consisted of a box wound with thick german-silver wire plugging to 0.1 ohm, in series with which was a german-silver wire (No. 10 S.W.G.) doubled back on itself and carrying a slider. Points were marked out at distances corresponding to 0.01 ohm. These spaces were about 10.7 centims. long.

The *Alternator*. Several different alternators were used. The one used while obtaining the curves given was a Pyke and Harris machine, which was especially suitable, since it had no sliding contacts, both armature and field being fixed. The other machines were—for high frequencies a Ferranti, and for low frequencies a Gramme alternator. The fields in all cases were excited by storage-cells.

The condenser had usually a capacity of about 60 microfarads.

Variation of Conditions.

The conditions of the experiments were varied as much as possible. We have studied the effect on the resistance of the

arc of variations in the amount, frequency, and wave-form of the alternating current; the effect of different kinds of carbons and different P.Ds. and currents; the effect of using different combinations of cored and solid carbons, of carbons cored with substances other than carbon; and the effect of the relative size of the carbons.

Results.

The amount of alternating current between the limits we have used does not appear to have any influence on the value obtained for the resistance of the arc. The largest alternating current used had a R.M.S. (root mean square) value equal to about 10 per cent. of the continuous current. The smallest had about one-tenth of this value. In obtaining the curves shown, the R.M.S. of the alternating current was usually about 0.5 ampere.

The frequency also did not have any effect on the resistance of the arc between the limits 250 and 7 complete alternations per second. Frequencies above 150 were obtained from a Ferranti alternator; those between 150 and 45 from a Pyke and Harris; and those between 24 and 7 from a Gramme machine.

Besides these three machines a Mordey transformer of ratio 1:1 was sometimes used, with its primary in circuit with the Pyke and Harris alternator. On account of its having a closed iron circuit it was not so suitable, but it gave the same results as the alternators.

We have thus obtained considerable variety in the wave form as well as in the frequency. Hence the wave form does not affect our measurements of the resistance of the arc.

The values thus obtained for the resistance of the arc under various conditions are plotted in Pls. II. & III. Pl. II. shows the relation between the resistance of the arc and the current, the P.D. between the carbons being kept constant. Pl. III. fig. 1 shows the relation between the resistance of the arc and the P.D. between the carbons, the current being kept constant. Pl. III. fig. 2 shows the relation between the resistance of the arc and the arc length, the current being kept constant. In all these cases the resistance is measured from the central

line marked 0, positive values above the line and negative values below the line.

Certain makes of carbon have been selected for experiment, namely, Apostle, Brush, Thomson-Houston, and Carré carbons, as representative of the various qualities of carbon now in use commercially.

The diameter of the positive carbon was in all cases 11 millim., the diameter of the negative was 9 millim. in the case of the Apostle and Brush carbons, 8 millim. in the case of the Carré carbons, and 11 millim. in the case of the Thomson-Houston carbons. These sizes were selected as being most suitable for the currents we were able to take from our cells. The sizes of the Apostle carbons were those used by Mrs. Ayrton in her experiments.

That the diameter of the carbons used does not have any great effect on the resistance of the arc for the same current and voltage is shown by the curves D and E, Pl. II. and fig. 1, Pl. III. These are the curves for two solid Apostle carbons—one set D having carbons 11 millim. and 9 millim. and the other set E having carbons 18 millim. and 5 millim. in diameter respectively, and it will be seen that they lie very close together.

On the curves each make of carbons is represented by a particular kind of line as explained in the figure, the Apostle carbons, for instance, are represented by a dotted line. Curves bearing the same letter refer to the same combination of carbons in the different makes. For each make there are four combinations :—+ cored, — cored ; + cored, — solid ; + solid, — cored ; + solid, — solid ; represented respectively by the letters A, B, C, and D.

The general characteristics of these curves are that the ordinates of those for +solid —solid are always negative, and those for +cored —cored always positive, while the other curves all lie between these two extremes, those which have the positive carbon solid always being more negative than those which have the positive cored. The greatest uniformity is met with in the case of the +solid —solid combination, in which all the curves lie close together. For this reason the curves for the Thomson-Houston carbons have been omitted, as they lie completely on the other curves, and would tend to cause confusion. In the case of cored carbons, however, the

uniformity is not so marked, as the material of the core varies largely with the make. In fact, in a single carbon the core may vary considerably, and for this reason the curves for cored carbons are more troublesome to take.

Pl. II. shows the relation between the resistance of the arc and the current that the arc is taking, the voltage at the carbons being kept constant. It will be noticed that in the case of those combinations which have a solid positive, the number expressing the negative resistance of the arc increases as the current decreases. With the other two combinations—those having a cored positive—the curves for Brush and Carré carbons have the same general characteristics, while those for the Apostle carbons cut these latter. This is probably due to peculiarities in the coring of the Apostle carbons.

Pl. III. fig. 1 shows the relation between the resistance of the arc and the P.D. between the carbons, the current being kept constant at 10 amperes in all cases. These curves are more instructive than those in Pl. II., as the character of the arc alters more with change of voltage than with change of current. As before, the curves for the solid carbons are all very close together. In this case they all show a minimum (maximum negative) value at about 55 volts. With combinations having a cored positive, this minimum becomes more strongly marked and occurs at a lower voltage.

It has been our practice to examine the image of the arc on the screen and note when any change in the appearance of the arc takes place. It was first noticed by Mrs. Ayrton that with cored carbons under certain conditions a dark space appeared somewhere near the centre of the arc, dividing the purple glow into two parts. This was never noticed in the case of solid carbons. Our observations have fully confirmed this, and, moreover, we find that the dark space, when only one carbon is cored, is nearer this carbon. We have noted the appearance or disappearance of the dark space, and we find a remarkable coincidence between this point and the minimum of the curves. In the curves for cored carbons in Pl. III., to the right of the minimum point the arc always shows a dark space which becomes less marked on approaching the minimum, and at the minimum finally disappears. To

the left of this point, although the dark space has quite disappeared, yet the difference in colour between the two parts of the glow is still observable, that near the negative carbon being of a redder tint than that near the positive.

Pl. III. fig. 2 shows the relation between the length of the arc and its resistance at constant current. The general characteristics of these curves are the same as those of fig. 1, since the length of the arc is roughly proportional to the voltage.

Effect of the Core.

Since the presence of a core in either carbon has such a marked influence on the resistance of the arc, we tried the effect of boring out the core of a Carré carbon. The result of using this hollow carbon as the positive is shown by curve F in Pl. III. It will be seen that this curve follows very closely that obtained with both carbons solid. The effect was now tried of filling this hollow carbon first with plaster of Paris, and then with kaolin. In both cases the resistance of the arc so produced was positive. Another effect of these cores was to cause the arc to burn at a much lower voltage for a given current and arc length. The arc was, however, too unsteady to allow of a curve being taken.

Particulars of Cores.

The diameters of the various cores are as follows :—

Carbon.	Diam. of carbon. mm.	Diam. of core. mm.
Apostle	11	2·7
	9	2·33
Brush	11	2·73
	9	2·18
Carré	11	2·4
	8	1·9

Inverted Arcs.

In the experiments hitherto described the positive carbon has always been uppermost. Experiments were now made to ascertain if the resistance of the arc is dependent upon whether the positive carbon is the upper or the lower.

Using solid carbons, the resistance is not appreciably altered by inverting the arc. With cored carbons, however, the resistance of the arc is altered, and moreover the physical character of the arc is changed. As we have pointed out above, the distinguishing feature of the arc when using cored carbons is the appearance of a dark space situated near the centre of the purple glow, which appears to considerably increase the resistance of the arc. On inverting such an arc, however, we find that this dark space completely disappears, and, as we should have expected, the resistance is considerably diminished. At low currents or P.Ds., the difference between the resistances of ordinary and inverted arcs diminishes till the two values become nearly equal. These are the conditions under which the dark space ordinarily disappears. Hence when the dark space has disappeared, inverting the arc makes but little difference in its resistance, which is also the case with arcs using solid carbons, in which there is no dark space.

By placing the carbons horizontal, a value for the resistance of the arc can be obtained which is intermediate between the two values obtained by placing the positive carbon respectively above and beneath the negative carbon, the current and P.D. being the same in the three cases.

From consideration of the part played by the dark space, it would appear that the resistance of the arc is greatly affected by the state of the contact between the purple glow and the negative carbon. When the inside purple glow is in good contact with the negative carbon, the resistance of the arc is most negative. When, however, there is a dark space (caused by the presence of foreign matter or of carbon in a different physical state), and hence a lack of contact between this glow and the negative carbon, the resistance becomes increased.

Hissing Arcs.

An attempt was made, using Method I., to find the resistance of a direct-current hissing arc produced with accumulators, but it was found that even with the alternator at rest there was a large deflexion of the electrometer, showing that *the current through a hissing arc is oscillatory*. In order to determine the frequency of the oscillation, a condenser and

telephone were put across the arc. The sound heard in the telephone was, however, of no definite pitch.

A method suggested by Mr. Campbell was used to measure the amount of this oscillatory current. The current taken by the hissing arc was passed through the primary of a transformer, the secondary of which was put in series with a reflecting twisted-strip ammeter, the deflexion being noted. The arc lamp was then disconnected, and a known alternating current was sent through the primary, of such magnitude that the deflexion was the same as before. This known current has the same R.M.S. as the oscillatory current passing through the arc, for if the impedance of the secondary is large compared with its resistance, the deflexion of the ammeter is the same for the same R.M.S. current whatever the frequency. The arc was run at 14.5 amperes and 35 volts, Apostle carbons +9 mm. -9 mm. being used. While the arc was burning the deflexion of the ammeter varied between 200 and 250 scale-divisions. Primary currents of 0.43 ampere and 0.53 ampere, as measured by another twisted-strip ammeter, produced these scale-readings respectively. Hence in this case the R.M.S. of the oscillatory current amounted to about 3 per cent. of the continuous current.

Low Frequencies.

On comparing our results with the steady $\frac{dV}{dA}$ taken from the slope of the curves published by Mrs. Ayrton, the agreement as to sign, which held in the case of solid carbons, was found to break down in the case of cored carbons; for while the general shape of the curves obtained by Mrs. Ayrton was the same for cored as for solid carbons, that is both curves except for short arc-lengths indicated a negative resistance*, we find that with both carbons cored the resistance is *always* positive. With a view to elucidating this point, we repeated Mr. Mather's experiments using both carbons cored, and found the $\frac{\delta V}{\delta A}$ negative as shown by the following table:—

* Mrs. Ayrton does indeed find that for cored carbons at short lengths the curves slope slightly upwards, thus indicating a positive resistance.

Brush Carbons.	Both cored.
+11 mm.	-9 mm.

Arc taking 9 amperes at 43 volts.

Resistance in circuit about 5 ohms.

Resistance introduced.	Volts.	δV .	Amperes.	δA .
+·5	44·8	+1·8	8·10	-·90
+·4	44·2	+1·2	8·30	-·70
+·3	44	+1·0	8·45	-·55
+·2	43·7	+·7	8·62	-·38
+·1	43·3	+·3	8·75	-·25
-·1	42·8	-·2	9·20	+·20
-·2	42·4	-·6	9·38	+·38
-·3	42·0	-1·0	9·60	+·60
-·4	41·6	-1·4	9·85	+·85
-·5	41·1	-1·9	10·08	+1·08

In the curves connecting the P.D. between the carbons and the current, each point is obtained by taking the readings of ammeter and voltmeter after the arc has been made to burn for a *very considerable time* at constant current and arc-length, so that one may say that the "frequency" is zero. In this case, the resistance, if deduced from the slope of these curves, would be negative both for cored and for solid carbons. In Prof. Ayrton's original method, although the readings are taken as quickly as possible, still the "frequency" is small compared with the lowest frequency we have hitherto used, and both the carbons and the arc may have time to alter before the ammeter and voltmeter can be read. This method would lead to the same conclusion as before, viz. that the resistance of the arc is always negative. Our measurements, however, with frequencies as low as 7·5 gave a positive resistance for arcs using cored carbons. Hence it seems that there must be a critical frequency, above which the resistance has a positive value which is independent of the frequency, and below which it appears to have a negative value, and further, that this frequency must lie between 7·5 and 0.

Our former methods were not suitable for working with a frequency lower than 7, for at this frequency the needles of the voltmeter and ammeter began to vibrate with the alter-

nating current. This vibration of the needles was made use of in the following way, to indicate the sign of the $\frac{dV}{dA}$ at frequencies lower than 7. The Gramme alternator used in the previous method was run as a rotatory transformer. By adjusting the continuous current passing through this transformer, and hence its speed, any desired frequency could be obtained down to as low as one complete alternation in 2 seconds. By an arrangement of mirrors the needles and scales of both ammeter and voltmeter could be observed simultaneously. In this way it could be seen whether the two needles were at any instant vibrating in the same or in opposite directions. It is evident that if the needles are vibrating in phase, that is if an increase of P.D. is accompanied by an increase of current, then $\frac{dV}{dA}$ must be positive; while if they are vibrating out of phase, that is if an increase of P.D. is accompanied by a decrease of current, then it is negative. With this arrangement, *solid carbons at any frequency gave a negative resistance. With cored carbons, however, we found as we expected, that above a certain frequency the resistance was positive, while below this frequency the method gave a negative value for $\frac{dV}{dA}$.* The critical frequency was found by observing the point at which the swing of the needles was uncertain, being sometimes in phase and sometimes out of phase. At this point the frequency was 1.8, at 1.9 the sign was certainly positive, and at 1.7 was negative. This result was obtained with all arcs whose resistance as given by our curves was positive (with one exception mentioned below), and was not obtained with any whose resistance was negative. In the case of arcs whose curves cross the zero line, the transition from positive to negative, as shown by the disappearance of the critical frequency, was well shown and corresponded with the crossing of the zero line. As an example, we ran an arc with Brush carbons +11 mm. cored -9 mm. solid at 10 amperes, the curve for which is marked B in fig. 1, Pl. III. It will be noticed that this curve crosses the zero line twice, at 55 volts and again at 41.3 volts: at 60 volts we found the sign of the $\frac{dA}{dV}$, as shown

by the swing of the needles, to be positive at frequencies above 1·8 and negative below this frequency, at 45 volts it was negative at all frequencies; at 35 volts, however, it was positive at frequencies above 1·8, and also at frequencies below 1·8. That the resistance would be positive at all frequencies seemed probable, since the curves connecting the P.D. and current at constant arc-length for such carbons show that with small arc-lengths an increase of P.D. is accompanied by an increase of current.

The experiments have been carried out in the Physical Laboratory of the Central Technical College, and our best thanks are due to Professor Ayrton and Mr. Mather for much kind advice during the course of the work.

NOTE.—Fluctuation of the Current given by a Continuous-Current Dynamo.

An attempt was made to replace the cells in Method I. by a dynamo, but it was found that even with the alternator at rest the electrostatic voltmeter showed a large deflexion. This was evidently due to the oscillation of the current owing to the commutator of the dynamo having a finite number of segments. The oscillation was found to increase as the brushes were moved out of the sparkless position. The dynamo was a 5 kilowatt 2-pole machine, and was giving 10 amperes at 70 volts. By adjusting the brushes to the sparkless position this oscillatory current, measured in the same way as in the case of the hissing arc above, could not be reduced below 2·5 per cent. of the continuous current, while by rocking the brushes out of this position it could be increased to as much as 9 per cent. without the sparking at the brushes being excessive. The commutator was in good condition. This oscillation may introduce errors in the measurement by polarized instruments of the currents given by dynamos owing to the differences between the mean and the R.M.S. of such a current.

DISCUSSION.

Prof. A. GRAY doubted whether it was right to give the name "true resistance" of the arc to the slope of the curve connecting the potential difference (V) and the current (A).

The authors' method of deducing $\frac{\delta V}{\delta A}$ was only true if the curve was a straight line; while in the case of the arc, E and a may both vary with the current.

Mrs. AYRTON said that with reference to the question of the existence of a back E.M.F., the evidence tended to show that it did not exist. By using an exploring carbon no constant back E.M.F. could be found.

Prof. AYRTON said that, considering the arc as consisting of a back E.M.F. and a resistance, it was necessary to separate these two. Simply obtaining one value of the P.D. and the current was of no assistance in solving this question, but a series of values had to be taken. By taking the changes in the P.D. and current sufficiently small, the curve over the range considered was practically straight. It was curious to note that as long as observers obtained a positive value for the resistance of the arc, no fault was found with the method, but that now a negative value was found, the accuracy of this method was questioned. If a back E.M.F. does really exist, then it follows that the arc must have a negative resistance. Mr. Frith has shown why some people have got positive and some negative values for the resistance of the arc, and also that with an alternating current you may get either one or the other.

Mr. CARTER asked if the fact that the arc had a negative resistance did not imply a back E.M.F. in order that the arc might be stable. If so, was a negative resistance such an absurdity?

Mr. CAMPBELL said that he was very pleased to see that the authors had applied a method which he (Mr. Campbell) had suggested for measuring pulsating currents. If a pulsating current, such as could be obtained by means of a make and break, were passed through a thermopile you would get a back E.M.F., while if an alternating current were employed, you would not.

Mr. FRITH, in his reply, said that he had defined the "true resistance" as $\frac{dV}{dA}$. Mrs. Ayrton has shown that an arc will not run unless a certain resistance is placed in series with it;

this resistance must be numerically equal to the negative resistance of the arc itself.

Prof. AYRTON said Mr. Frith's remarks as to the cause of the want of stability of an arc without outside resistance were most suggestive.

The CHAIRMAN said he did not like the expression P.D. He suggested the employment of photography to facilitate the accurate registration of the instrument readings.

XXVIII. *Admittance and Impedance Loci.*

By FREDERICK BEDELL*.

THE quantities chiefly considered in the discussion of alternating currents are electromotive forces and currents, the values of these being determined for different conditions. Electromotive forces and currents are commonly represented by vector diagrams; and the change in these diagrams, as some one quantity is varied, is shown by the loci of the vectors which are altered thereby. What may be termed electromotive force and current loci are thus determined. The numerical values for which these are constructed necessarily depend upon some condition involving an assumed value either of the current or of the electromotive force: thus we may assume a certain current to be constant (as the primary current of a transformer), and construct an electromotive-force diagram with loci showing the changes in the various electromotive forces as some part of the circuit is changed; or we may assume the impressed electromotive force constant, and ascertain current loci for the same variation. Let us limit ourselves to the transformer. In the first case above, it will be found that the constant assumed primary current I_1 is a factor in the value of every line representing the components of the primary electromotive force E_1 . By factoring out I_1 we have an impedance diagram similar to the electromotive-force diagram, and without any assumption as to the value of the current or electromotive force. Similarly each

* Read June 26, 1896.

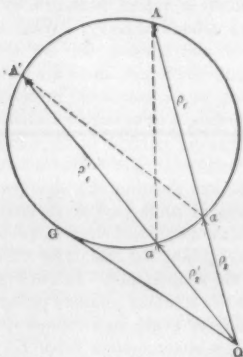
line in a current diagram, constructed for a constant impressed electromotive force E_1 , represents a current which is a multiple of an admittance (the reciprocal of an impedance) and the factor E_1 . By factoring out E_1 an admittance diagram is consequently obtained, similar to the current diagram but with no assumption as to the current or electromotive force. Admittance and impedance diagrams accordingly correspond to current and electromotive-force diagrams respectively, differing from them only by a factor.

Impedance and admittance loci, or electromotive force and current loci, for the primary of a transformer will in general be arcs of circles for changes in any one of the constants of the primary or the secondary circuit.

Some interesting relations arise from the reciprocal nature of admittance and impedance. Let us note the following relations between reciprocal vectors:—

If any vector has an arc of a circle for its locus, a vector proportional to its reciprocal will have an arc of a circle for

Fig. 1.—Reciprocal Vectors, ρ_1 and ρ_2 .



a locus. In fig. 1 let ρ_1 be any vector from the origin O, having its locus as shown upon the arc of a circle. The vector ρ_2 , drawn in the direction of ρ_1 and proportional to its reciprocal, will have its locus upon an arc of a circle, which

may be shown as follows. Let ρ_1 and ρ_1' represent the vector in any two positions, OA and OA' . The intercepts Oa and Oa' will represent the reciprocal vectors ρ_2 and ρ_2' ; for in the similar triangles $OA'a$ and OAA' ,

$$\rho_1 : \rho_1' :: \rho_2' : \rho_2.$$

Hence

$$\rho_1' \rho_2' = \rho_1 \rho_2 = \text{a constant.}$$

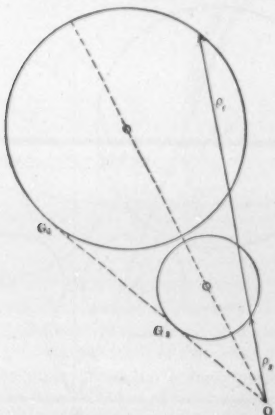
The value of this constant product of ρ_1 and ρ_2 is $\overline{OG}^2$.

By a suitable selection of scale, the loci of ρ_1 and its reciprocal ρ_2 may be represented as arcs of the same circle, as in fig. 1; or they may be represented by different circles, as in fig. 2. In the latter case,

$$\rho_1 \rho_2 = OG_1 \cdot OG_2 = \text{a constant.}$$

As the origin O approaches the circle which represents the locus of ρ_1 , the centre of the reciprocal circle becomes more

Fig. 2.—Reciprocal Vectors.



distant and its radius becomes greater. When the origin O is a point in the circumference of the first circle, the centre

force on open circuit, and includes the effects of primary resistance and the losses due to hysteresis and eddy currents; HJ is the electromotive force to overcome the primary self-induction. These are in the direction of the primary current, and at right angles to it, respectively. A line from J to K, at right angles to the secondary current, would show the reaction of the secondary upon the primary. It is to be noted that the line NH represents the effects due to magnetic leakage. It is desired to find the locus for the primary current when the primary electromotive force has a constant value, and is drawn in the direction OA. The angle of lag θ_1 , between the primary electromotive force and current on open circuit, is JOH. Accordingly, with a constant electromotive force in the direction OA, the open circuit current I_0 is laid off lagging behind the electromotive force at an angle of $AOj' = \theta_1 = JOH$. The open circuit current I_0 may be laid off to any convenient scale. To construct the locus for the primary current proceed as follows:—Lay off the line OC_2 so that the angles AOC_1 and AOC_2 are equal. The point C_2 is located so that $OC_2 : OC_1 :: Oj' : Oj$. The primary current locus is then drawn as the arc of a circle with C_2 as a centre, passing through j' .

The limits of the primary electromotive force locus are the points J and N. The corresponding limits of the primary current locus are the points j' and n' . It will be noted that these points correspond to the points j and n on the circle C_1 , which are reciprocal to the points J and N.

In the absence of magnetic leakage the points N and H coincide. The point n' would then lie in the line OA. The deviation of the primary current locus from the line OA is produced by magnetic leakage.

An experimental curve showing the primary current locus for a constant potential transformer, as affected by magnetic leakage, is shown in fig. 5.

The reciprocal relation between admittance and impedance vectors gives a simple method for determining the conditions for consonance and resonance in transformer circuits*.

* "Resonance in Transformer Circuits," by Bedell and Crehore, 'The Physical Review,' vol. ii. p. 442.

Figure 6 is given as a particular instance in illustration of the statement given above that loci produced by the variation of any one constant are usually arcs of circles. The primary

Fig. 5.—Primary Current Locus for Constant Potential Transformer; Determined Experimentally.

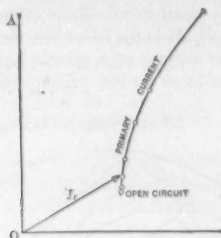
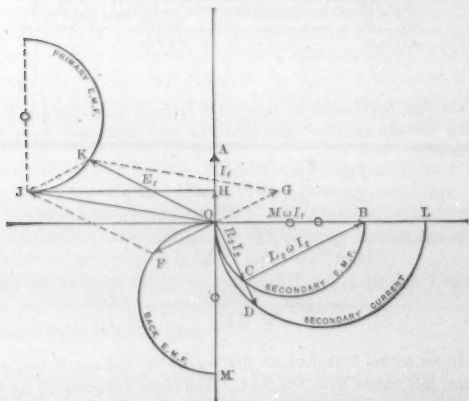


Fig. 6.—Effect of the Variation of the External Secondary Self-Induction in a Constant Current Transformer.

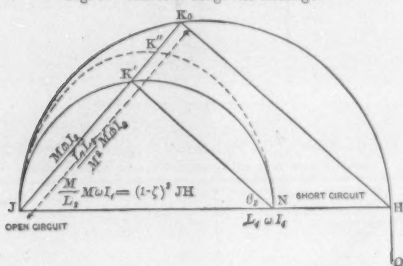


loci are always arcs of circles. The diagram shows the changes due to a variation in the secondary self-induction.

Fig. 7 shows the effect of magnetic leakage. The curves shown are loci for primary electromotive force, when the primary current is I_1 . The primary electromotive force is composed of the components OH, to overcome ohmic resistance and supply open-circuit losses, HJ to overcome self-induction, and (with absence of magnetic leakage) JK_0 to overcome the back electromotive force of mutual induction.

The semicircle JK_0H is the locus for the primary electromotive force in the absence of magnetic leakage. The semicircle $JK'N$ is the locus for the primary electromotive force

Fig. 7.—Effect of Magnetic Leakage.



when the coefficient of magnetic leakage ζ is constant from open circuit to short circuit. In this case we have the relations

$$\frac{JK'}{JK_0} = \frac{JN}{JH} = \frac{M^2}{L_1 L_2} = (1-\zeta)^2;$$

$$1-\zeta = \sqrt{\frac{JK'}{JK_0}} = \sqrt{\frac{JN}{JH}};$$

$$\zeta = 1 - \sqrt{\frac{JK'}{JK_0}} = 1 - \sqrt{\frac{JN}{JH}}.$$

In an actual transformer the magnetic leakage is not constant, but varies with the load. The locus represented by the dotted curve $JK''N$ is for such a case in which the magnetic leakage is zero on open circuit and increases to the maximum at short circuit. Where the magnetic leakage is variable, it

is determined for any point as K'' by the ratio of JK'' to JK_0 .

$$\zeta = 1 - \sqrt{\frac{JK''}{JK_0}}$$

Thus let us suppose that the back electromotive force JK'' , actually induced in the primary by the secondary current I_2 , is eighty-one volts; and that JK_0 , which would be the back electromotive force in the absence of magnetic leakage, is 100 volts. We then have the equation

$$1 - \zeta = \sqrt{\frac{81}{100}} = \frac{9}{10} = 0.9;$$

which indicates that the mutual induction is nine-tenths of the value it would have in the case of no magnetic leakage. The coefficient of magnetic leakage is accordingly found to be 10 per cent.; thus:—

$$\zeta = 1 - 0.9 = 0.10.$$

Fig. 7 is drawn to represent the values of the various electromotive forces in the primary circuit of a transformer, for a given value of the primary current and for different values of the secondary resistance. If the magnitude of each line is divided by the primary current I_1 , fig. 7 represents the values for the primary impedance (without any assumption as to constant current or electromotive force) for different values of the secondary resistance. The effect of magnetic leakage upon the primary impedance of any transformer is thus shown for different values of the secondary resistance.

The above construction affords a simple method of studying the conditions for a decrease of the primary impedance of a transformer when the secondary circuit is closed*.

Curves corresponding to the dotted curve in fig. 7 have been experimentally determined† by the writer.

Cornell University, May 1896.

* Discussed at length before the Physical Society by Mr. E. C. Rimington in his paper on "On Air-core Transformers," Proc. Physical Soc. vol. xiii. p. 1.

† 'Proceedings of the International Electrical Congress,' Chicago. 1893, p. 234.

DISCUSSION.

Mr. BLAKESLEY asked the author what was his test of resonance? Was it that the primary current and E.M.F. were exactly in the same, or in opposite phase? The term "resonance" was an acoustical one, and he did not see why it should be applied to one particular case in the electrical problem. The statement made by the author, that "impedance and admittance loci, or electromotive force and current loci, for the primary of a transformer will in general be arcs of circles for changes in any one of the constants of the primary or the secondary circuit," is only true under greater limitations than those cited by the author.

Mr. INWARDS asked what degree of accuracy the author had obtained.

The AUTHOR, in reply, said that if the applied E.M.F. and the current were brought into phase by means of a condenser in the secondary, then he called that a case of resonance. The agreement between the experimental and theoretical results was generally within from one to three per cent.

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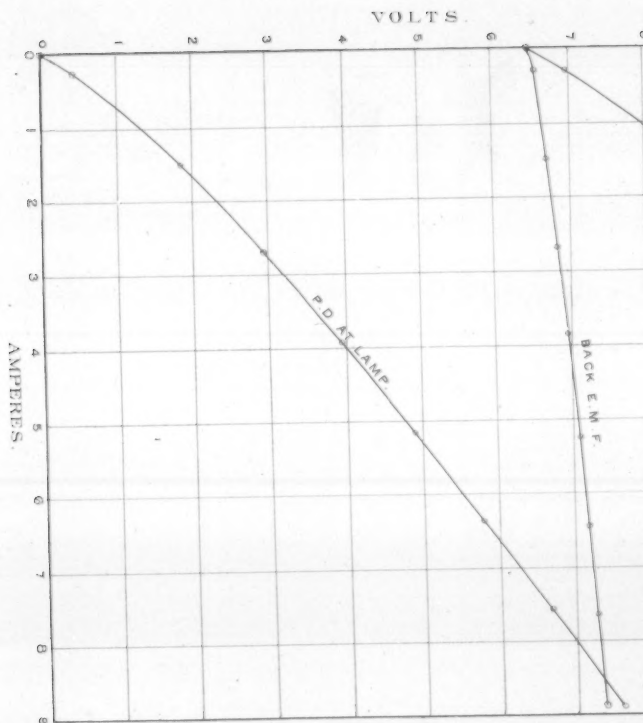
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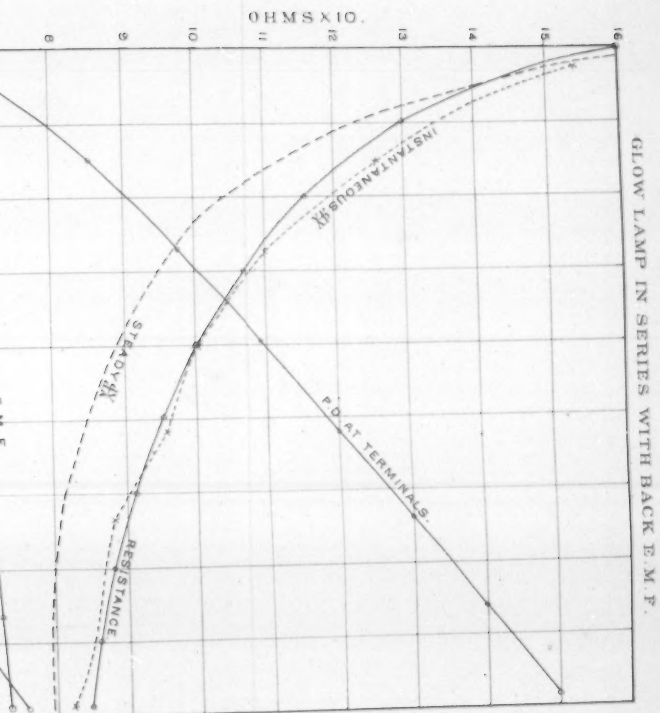
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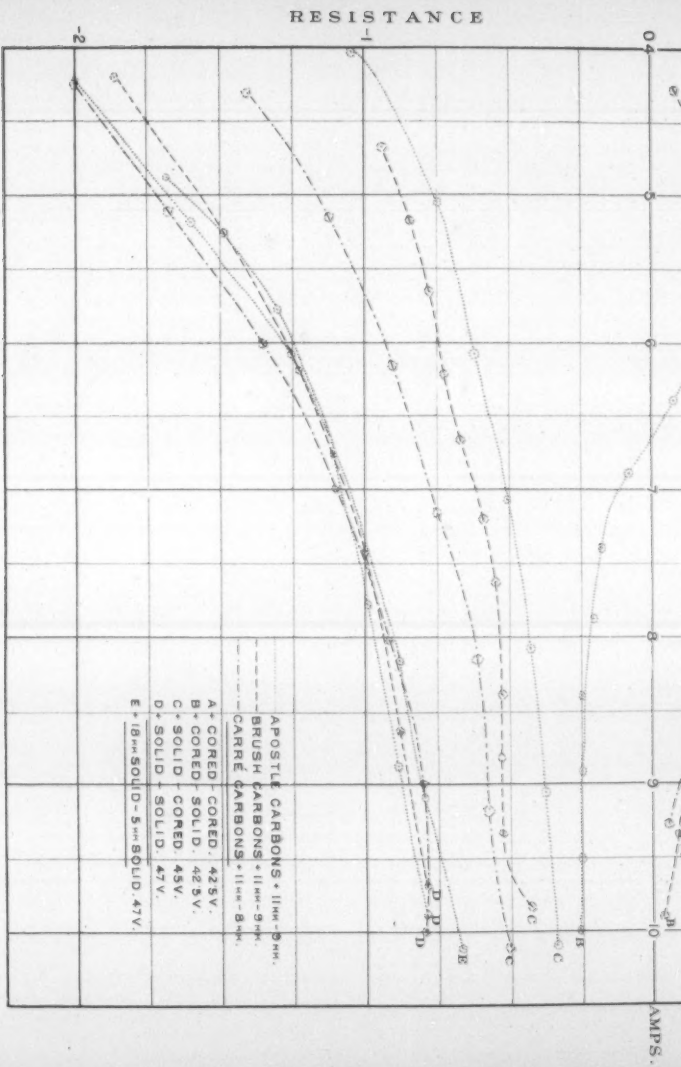
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Maxwell's Dyn. Ill.

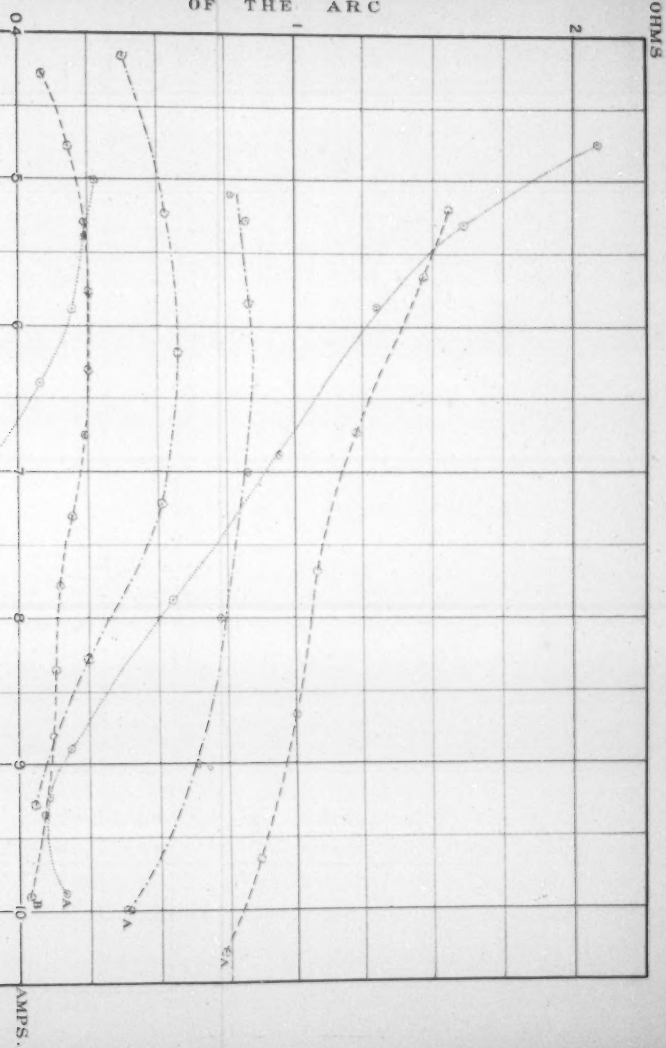




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RESISTANCE OF THE ARC

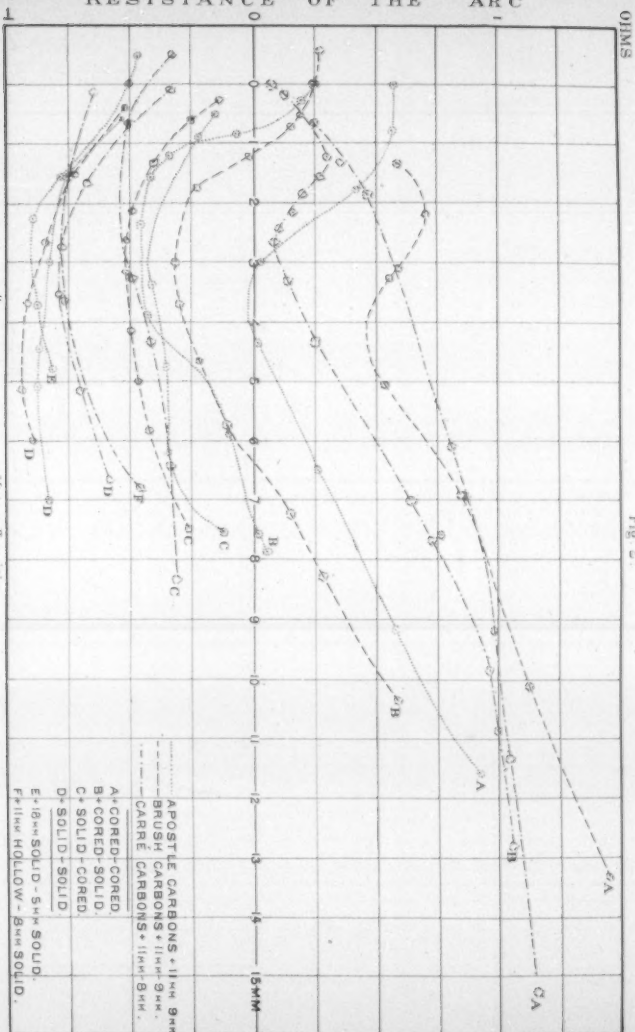
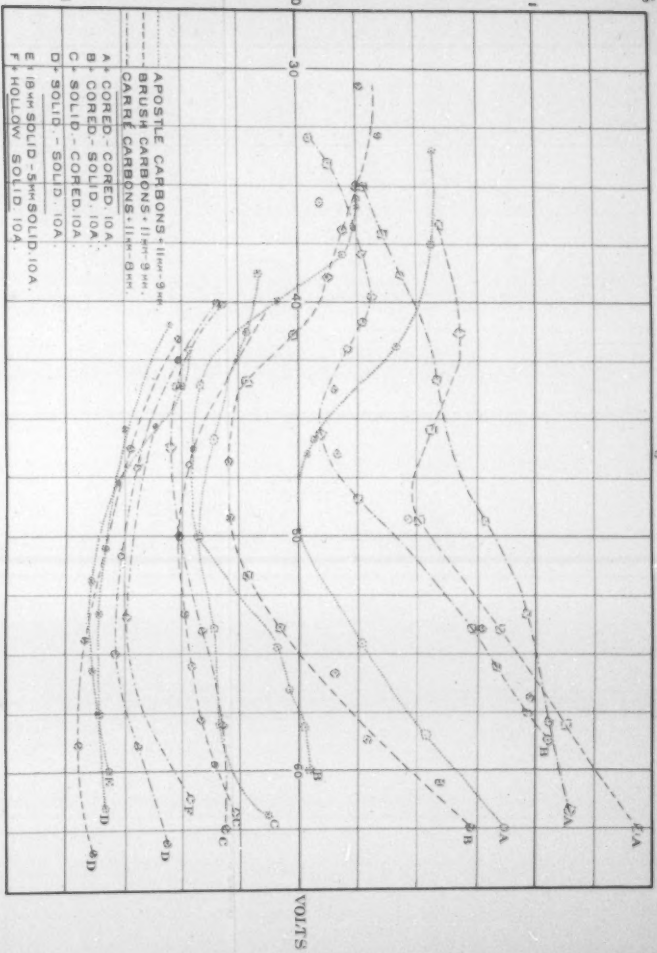


Fig. 1

RESISTANCE OF THE ARC





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	* Baynes, R. E., M.A. <i>Christ Church, and 2 Norham Gardens, Oxford.</i>
June 14, 1895.	Beale, S. <i>Banbury Technical School, Banbury.</i>
Feb. 28, 1896.	Bedell, F. <i>Physical Laboratory, Cornell University, Ithaca, New York, U.S.A.</i>
May 22, 1886.	Bell, Chichester A., M.B. 3 <i>Mansfield Place, Richmond, Surrey.</i>
Dec. 13, 1896.	Bell, H. G., B.A. <i>Bancroft's School, Woodford, Essex.</i>
June 28, 1895.	Berridge, Miss C. E. 17 <i>Rotunda Terrace, Cheltenham.</i>
Nov. 25, 1892.	Bessemer, A. G., Jun. 27 <i>Killieser Avenue, Streatham Hill, London, S.W.</i>
Mar. 30, 1877.	Bidwell, Shelford, M.A., LL.B., F.R.S. <i>Riverstone Lodge, Wandsworth, London, S.W.</i>
Mar. 23, 1889.	Biggs, C. H. W.
May 28, 1881.	Blaikley, D. J. 13 <i>Elsworthy Terrace, Primrose Hill, London, N.W.</i>
Nov. 28, 1885.	Blakesley, T. H., M.A., M.Inst.C.E. <i>Royal Naval College, Greenwich, S.E.</i>
May 26, 1877.	Blytheswood, Baron. 2 <i>Seamore Place, Curzon Street, W.</i>
Mar. 8, 1895.	Borns, H., Ph.D. 19 <i>Alexandra Road, Wimbledon.</i>
May 13, 1876.	Bosanquet, R. H. M., M.A., F.R.S.
	* Bottomley, J. T., F.R.S. <i>University Physical Laboratory, Glasgow.</i>
Jan. 16, 1875.	Bottomley, W. <i>The University, Glasgow.</i>
	* Bourne, C. W., M.A. <i>King's College, Strand, London, W.C.</i>
Feb. 12, 1892.	Bower, William Richard. <i>Municipal School of Science, Brighton.</i>
June 12, 1880.	Boys, C. Vernon, F.R.S. <i>Physical Laboratory, Royal College of Science, South Kensington, London, S.W.</i>
May 24, 1895.	Bramwell, Sir F. J., Bart., D.C.L., F.R.S., M.I.C.E. 5 <i>Great George Street, Westminster.</i>
Feb. 8, 1895.	Briscoe, Albert Edward. <i>University College, Nottingham.</i>
June 8, 1889.	Brittle, J. R. <i>Faral Villa, Vanbrugh Hill, Blackheath, London, S.E.</i>
Nov. 21, 1874.	Brown, Prof. F. D., M.A. <i>University College, Auckland, New Zealand.</i>
Feb. 12, 1887.	Brown, J. <i>Edenderry House, Shaw's Bridge, Belfast.</i>
Mar. 12, 1881.	Browne, Colvile. <i>Plantation Cottages, Hextable, Kent.</i>
Dec. 14, 1894.	Bryan, G. H., M.A., F.R.S. <i>Thorulea, Trumpington Road, Cambridge.</i>
Feb. 25, 1882.	Buchanan, J., B.Sc. <i>Gordon's College, Aberdeen.</i>
May 11, 1895.	Buckmaster, C. A., M.A., F.L.S. 16 <i>Heathfield Road, Mill Hill Park, Acton, W.</i>
Mar. 8, 1895.	Burbury, S. H., M.A., F.R.S. 1 <i>New Square, Lincoln's Inn, W.C.</i>

Date of Election.

Nov. 23, 1894.	Burke, John, B.A. <i>Owens College, Manchester.</i>
Feb. 8, 1895.	Burstall, Henry F. W., M.A., Assoc. M.I.C.E. 76 <i>King's Road, Camden Road, N.W.</i>
Jan. 24, 1896.	Burstall, H. R. J. 76 <i>King's Road, Camden Road, N.W.</i>
Mar. 10, 1888.	Burton, Chas. V., D.Sc. 24 <i>Wimpole Street, London, W.</i>
Nov. 27, 1875.	Busk, C.
June 14, 1884.	Butler, Prof. A. S., M.A. <i>St. Andrew's University, N.B.</i>
Apr. 26, 1895.	Campbell, A., B.A. 15 <i>Rothwell Street, Primrose Hill, N.W.</i>
Dec. 14, 1894.	Capstick, J. W., D.Sc., B.A. <i>Trinity College, Cambridge.</i>
June 22, 1889.	Cardew, Major P., R.E. <i>Beechcroft, Woking.</i>
Dec. 10, 1887.	Carus-Wilson, C. A. <i>McGill College, Montreal; and 2 Holford Road, Hampstead, N.W.</i>
Nov. 28, 1885.	Casella, C. F. 17 <i>Campden-Hill Square, London, W.</i>
Jan. 24, 1896.	Cassie, W. 70 <i>Regent Street, Cambridge.</i>
Mar. 8, 1895.	Catchpool, E., B.Sc.
Feb. 28, 1885.	** Cecil, Lord Sackville. <i>Hayes Common, Beckenham.</i>
Apr. 26, 1884.	Chadwick, O., A.Inst.C.E.
May 12, 1877.	Chattock, A. P. <i>University College, Bristol.</i>
Nov. 22, 1895.	Christie, W. H. M., M.A., F.R.S. <i>Royal Observatory, Greenwich, S.E.</i>
Dec. 10, 1881.	Clark, G. M., B.A. 9 <i>Alma Road, Canonbury, N.</i>
Nov. 27, 1875.	* Clark, Latimer, C.E., F.R.S. 11 <i>Victoria Street, London, S.W.</i>
Dec. 13, 1895.	Clayden, A. W., M.A. <i>St. John's, Polsloe Road, Exeter.</i>
Mar. 23, 1889.	Clifton, Prof. R. B., M.A., F.R.S. <i>The Museum, Oxford.</i>
Mar. 11, 1876.	Cobham, G. W. 1 & 3 <i>Edwin Street, Gravesend.</i>
Nov. 10, 1893.	Cockburn, A., F.C.S. 22 <i>Streetley Road, Brondesbury, N.W.</i>
May 16, 1890.	Coffin, W. H. 94 <i>Cornwall Gardens, South Kensington, London, S.W.</i>
Jan. 29, 1876.	Cole, R. S., M.A. 26 <i>Victoria Place, Devonport.</i>
Mar. 13, 1875.	Coleman, Professor W. M., A.B. <i>Sam Houston Normal Institute, Huntsville, Texas, U.S.A.</i>
Feb. 22, 1895.	Conroy, Sir John, Bart., M.A., F.R.S. <i>Balliol College, Oxford.</i>
Feb. 2, 1878.	Cooke, Conrad W. 2 <i>Victoria Mansions, Victoria Street, London, S.W.; Rothsay, Macaulay Road, Clapham Common, S.W.</i>
June 23, 1893.	Cooper, W. R., M.A. <i>Woodlands, Clapham Park, London, S.W.</i>
June 12, 1875.	**Core, Prof. T. H. <i>Owens College, Manchester.</i>
Mar. 8, 1895.	Cormack, M. T., M.A. 25 <i>Grafton Square, Clapham, London, S.W.</i>
	**Cotterill, Prof. J. H., M.A., F.R.S. <i>Royal Naval College, Greenwich, S.E.</i>
	*Cownley, A. J. 13 <i>Fenchurch Avenue, London, E.C.</i>
	Cox, John, M.A. <i>McGill University, Montreal, Canada.</i>
	Crawford and Balcarres, Earl of, F.R.S., F.R.A.S. 2 <i>Cavendish Square, London, W.</i>
	Crawley, C. W. S. 44 <i>Castletown Road, West Kensington, London, W.</i>
	* Crawshaw, Edward. 25 <i>Tollington Park, London, N.</i>
	* Crisp, Frank, B.A., LL.B., J.P. 5 <i>Lansdowne Road, London, W.</i>

Date of Election.	
Apr. 18, 1890.	Croft, William Bleaden, M.A. <i>The College, Winchester.</i>
Mar. 23, 1889.	Crompton, R. E. <i>Thriplands, Kensington Court, London, W.</i>
May 12, 1888.	** Crookes, W., F.R.S. <i>7 Kensington Park Gardens, London, W.</i> Cunynghame, H. H. <i>Hurlingham Lodge, Fulham, London, S.W.</i>
Mar. 13, 1886.	Curtis-Hayward, A. C., M.A. <i>27 Chancery Lane, London, W.C.</i>
Nov. 25, 1892.	D'Albe, Edmund Edward Fournier, B.Sc. <i>3 Old-Court Terrace, Bray, Ireland.</i>
May 12, 1877.	Darwin, Capt. L., R.E. <i>18 Wetherby Place, London, S.W.</i>
Dec. 4, 1891.	Davey, Henry, M.Inst.C.E. <i>3 Princes Street, Westminster, S.W.</i>
Feb. 13, 1875.	Day, R. Evan. <i>135 Fellows Road, London, N.W.</i>
May 26, 1877.	Debus, Dr. H., F.R.S. <i>1 Obere Sophienstrasse, Cassel, Germany.</i>
June 24, 1876.	Dewar, Prof. James, F.R.S. <i>Cambridge.</i>
June 12, 1891.	Dines, William H., B.A.
Feb. 25, 1882.	Dockrell, Staff-Commander H. J., R.N.
Mar. 23, 1889.	* Donnelly, Maj.-Gen. Sir J. F. D., R.E., K.C.B. <i>Science and Art Department, South Kensington, London, S.W.</i>
Apr. 13, 1889.	Douglass, Sir J., F.R.S. <i>Trinity House, London, E.C.</i>
May 26, 1877.	Dowson, J. E., M.Inst.C.E. <i>39 Old Queen Street, Westminster, London, S.W.</i>
	* Durham, Henry. <i>32 Fitzroy Street, London, W.</i> Dyer, W. T. Thiselton, C.M.G., M.A., B.Sc., F.R.S. <i>Kew Gardens, Surrey.</i>
Feb. 12, 1892.	Edser, Edwin. <i>62 Ringford Road, West Hill, Wandsworth.</i>
Apr. 26, 1895.	Edwards, Miss Eleanor M. Wynn. <i>Ladies' College, Cheltenham.</i>
Dec. 11, 1886.	Elder, H. M., M.A. <i>50 City Road, London, E.C.</i>
Jan. 25, 1894.	Emtage, W. T. A., M.A. <i>49 Schubert Road, Putney, S.W.</i>
June 26, 1891.	Enright, James, B.Sc. <i>202 Holland Road, London, W.</i>
Nov. 27, 1875.	Esson, W., M.A., F.R.S. <i>Merton College, Oxford.</i>
Mar. 8, 1895.	Eumorfopoulos, N., B.Sc. <i>1 Kensington Park Gardens, London, S.W.</i>
Mar. 12, 1875.	* Eve, H. W., M.A. <i>University College School, Gower Street, London, W.C.</i>
Nov. 22, 1895.	Everett, Prof. J. D., D.C.L., F.R.S. <i>Queen's College and Derryfolgie Avenue, Belfast.</i>
Feb. 21, 1890.	Everett, W. H., B.A. <i>University College, Nottingham.</i>
	Evershed, Sydney. <i>1 Woodfield Road, Harrow Road, London, W.</i>
	** Festing, Maj.-Gen. E. R., R.E., F.R.S. <i>The Museum, South Kensington, London, S.W.</i>
Feb. 8, 1879.	Finch, G. B., M.A. <i>1 St. Peter's Terrace, Cambridge.</i>
May 23, 1885.	Fison, A. H., D.Sc. <i>14 Dean Road, Willesden Green, N.W.</i>
Mar. 23, 1889.	Fitzgerald, D. G. <i>46 Loughborough Road, Brixton, London, S.W.</i>
Feb. 25, 1882.	Fitzgerald, Prof. G. F., M.A., F.R.S. <i>Trinity College, Dublin.</i>

Date of Election.

- Dec. 14, 1894. || Fitzpatrick, Rev. T. C., M.A. *Christ's College, Cambridge.*
 * Fleming, Prof. J. A., M.A., D.Sc., F.R.S. *University College, Gower Street, W.C.*
- Mar. 25, 1882. || Fletcher, L., M.A., F.R.S. 36 Woodville Road, Ealing, W.
- Apr. 14, 1883. || Forbes, Prof. G., M.A., F.R.S. 34 Great George Street, London, S.W.
 ** || Foster, Prof. G. C., F.R.S. *University College, Gower Street, London, W.C.*
- June 14, 1879. || Fox, St. George Lane. *Travellers' Club, Pall Mall, London, S.W.*
- Nov. 18, 1876. || Francis, Dr. W., F.R.A.S. *Red Lion Court, Fleet Street, London, E.C.*
- May 12, 1877. || Frankland, Prof. E., D.C.L., F.R.S. *The Fews, Reigate Hill, Reigate.*
 * || Freeman, Rev. A., M.A. *Murston Rectory, Sittingbourne, Kent.*
- Dec. 13, 1895. || Fridlander, E. D. *Holyhead Road, Coventry.*
- Nov. 22, 1895. || Frith, J. 1 Torrington Square, W.C.
- Nov. 25, 1892. || Fry, Francis James. *Leigh Woods, Clifton, Bristol.*
- Dec. 4, 1891. || Fulcher, L. W., B.Sc. *South Kensington Museum, S.W.; and 56 Burkleigh Road, Streatham Common, S.W.*
- Apr. 29, 1876. || Fuller, Prof. F., M.A. 9 Palace Road, Surbiton, *Kingston-on-Thames.*
- Feb. 14, 1885. || Fuller, G., M. Inst. C.E. 71 Lesham Gardens, *Kensington, London, W.*
- Mar. 23, 1889. || Galton, Sir D., K.C.B., D.C.L., F.R.S. 12 *Chester Street, Grosvenor Place, London, S.W.*
- Feb. 22, 1895. || Gardiner, J. H. 59 Wroughton Road, *Bulham, London, S.W.*
 * Garnett, Prof. William, M.A., D.C.L.
- Nov. 1, 1889. || Garrett, T. A., B.A. 30 *The Avenue, Ealing, London, W.*
- May 13, 1892. || Gerrans, H. G.
 ** || Gladstone, Dr. J. H., F.R.S. 17 *Pembridge Square, Bayswater, London, W.*
- Dec. 13, 1879. || Glazebrook, R. T., M.A., F.R.S. *Trinity College, Cambridge.*
- Dec. 14, 1878. || Gleed, W., M.A. *Royal Naval College, Greenwich, S.E.*
 * || Goolden, Walter T., M.A. *Woodfield Road, Harrow Road, London, W.*
- Mar. 23, 1889. || Gorham, J. M. 2 *Princes Mansion, Victoria Street, London, S.W.*
- Mar. 30, 1877. || Grant, W. *University College, Gower Street, London, W.C.*
- Feb. 27, 1891. || Gray, Professor Andrew, M.A., F.R.S.E. *University College of North Wales, Bangor.*
- Dec. 13, 1879. || Gray, J., B.Sc. *H.M. Patent Office, Chancery Lane, London, W.C.*
- Mar. 13, 1880. || Gray, J. Macfarlane. 79 *Mark Lane, London, E.C., and 4 Ladbrooke Crescent, Notting Hill, W.*
 ** || Gray, M. 106 *Cannon Street, London, E.C.*
- Dec. 4, 1891. || Gray, Philip Leman, B.Sc. 15 *Francis Road, Edgbaston, Birmingham.*
- Feb. 13, 1875. || Gray, R. K. *Telegraph Works Company, Silvertown, S.E.*
- June 22, 1878. || Grierson, F. W. *Holmwood Cottage, Stafford Road, Wallington, Surrey.*

Date of Election.

- Nov. 21, 1874. Griffith, George, M.A. *Harrow.*
 Feb. 22, 1896. Griffiths, A., B.Sc. *Owens College, Manchester.*
 Feb. 8, 1895. Griffiths, E. H., M.A., F.R.S. *12 Parkside, Cambridge.*
 Feb. 10, 1893. Grove, C. E. *Thames Iron Works, Orchard Road, Blackwall.*
 Feb. 23, 1884. ||Grundy, J. *39 Victoria Terrace, Manchester Road, Bolton.*
 Mar. 23, 1889. Gumpel, C. G. *22 Christchurch Road, Streatham Hill, London, S.W.*
 **||Gurney, Rev. H. P., M.A. *Rosworth, Gosforth, Northumberland.*
 **Haddon, A. *Royal Naval College, Greenwich, S.E.*
 *||Hall, Captain Marshall. *Easterton Lodge, Parkstone, R.S.O., Dorset.*
 Mar. 3, 1877. Hall, Samuel. *19 Aberdeen Park, Highbury, London, N.*
 Feb. 14, 1896. Halliday, George. *13 North Crescent, Bedford Square, W.C.*
 *||Harding, Alfred B. *Belmont, Bromley Road, Catford, London, S.E.*
 Feb. 14, 1880. Hare, A. T. *Neston Lodge, East Twickenham.*
 Apr. 28, 1893. ||Harrison, Fred., M.A. *Dulwich College, London, S.E.*
 Dec. 9, 1882. ||Harrison, H. E., B.Sc. *2 Park Place, Regent's Park.*
 Jan. 24, 1896. Harvey, H. S. S. *8 Maryon Road Charlton.*
 Jan. 13, 1893. ||Haselfoot, C. E., M.A. *Hertford College, Oxford.*
 Mar. 28, 1885. Hawes, F. B. *1 Wellington Chambers, Buckingham Gate, S.W.*
 Apr. 26, 1896. Hay, A., B.Sc. *University College, Liverpool.*
 May 6, 1882. ||Heaton, W. H., M.A. *University College, Nottingham.*
 Mar. 13, 1896. Henkel, F. W. Jr., F.R.A.S. *73 Pembroke Road, Walthamstow.*
 **Henrici, Prof. O., Ph.D., F.R.S. *City and Guilds' of London Central Institute, South Kensington, London, S.W.; 34 Clarendon Road, Notting Hill, W.*
 May 9, 1885. ||Herroun, E. F. *Riverdale, Humber Road, Westcombe Park, Blackheath, S.E.*
 Mar. 9, 1888. Herschel, Prof. A. S., M.A., F.R.S. *Observatory House, Slough.*
 Mar. 16, 1878. Hicks, W. M., M.A., F.R.S. *Endcliffe Crescent, Sheffield.*
 Dec. 11, 1875. ||Higgins, Clement, M.A. *5 Paper Buildings, Temple, E.C.*
 Jan. 22, 1892. Highfield, E. G. *23 Hartwood Road, Southport.*
 June 12, 1880. Hilger, A. *204 Stanhope Street, Hampstead Road, London, N.W.*
 Feb. 12, 1876. Hodgkinson, W. R., Ph.D., F.G.S., F.R.S.E. *8 Park Villas, Blackheath, S.E.*
 Dec. 4, 1891. Hoffert, Hermann H., D.Sc. *4 Harewood Grove, Darlington.*
 Feb. 7, 1890. ||Holland, Charles E., B.A. *4 Redington Road, Hampstead, London, N.W.*
 Apr. 22, 1882. ||Hopkinson, Edward, M.A., D.Sc. *Oaksleigh, Timperley, Cheshire.*
 Mar. 16, 1878. ||Hopkinson, J., M.A., D.Sc., F.R.S., M.Inst.C.E. *Holmwood, Wimbledon.*
 **Hopps, J. *Coopers Hill, Staines.*

Date of Election.

- Dec. 8, 1893. ||Hovenden, Fredk., F.L.S., F.G.S., F.R.Met.S. *Glenlea, Thurlow-Park Road, West Dulwich, S.E.*
- Jan. 24, 1884. ||Howard, Alfred, A.R.S.M., A.N.S.S. 186 Stanstead Road, Forest Hill, S.E.
- May 13, 1876. ||Howard, David. *Devon House, Buckhurst Hill, Essex.*
- June 9, 1888. ||Howard, James Livsey, D.Sc. 86 St. John's Road, Waterloo, Liverpool.
- Nov. 21, 1874. ||Huggins, W., D.C.L., LL.D., F.R.S. *Upper Tulse Hill, London, S.W.*
- Mar. 13, 1880. ||Hughes, Prof. D. E., F.R.S. 40 Langham Street Portland Place, W.
- **||Huntington, Prof. A. K. *King's College, Strand, London, W.C.*
- **||Huntington, J. B., M.Inst.C.E. 19 Bedford Row, London, W.C.
- *||Hurst, John Thomas. *Raymond Villa, Geraldine Road, Wandsworth, London, S.W.*
- Apr. 26, 1884. Inwards, R., F.R.A.S. 20 Bartholomew Villas, Kentish Town, London, N.W.
- May 26, 1877. ||Jack, Prof. W., M.A., LL.D. *University, Glasgow.*
- Mar. 25, 1882. ||Jackson, M. J., B.A., D.Sc. *The College, Kurrachee, India.*
- Mar. 8, 1879. ||Jacob, F. *Messrs. Siemens' Works, Woolwich, London, S.E.*
- May 22, 1886. ||Johnson, W. Claude. *Westcombe Park, Blackheath, London, S.E.*
- Feb. 17, 1886. ||Jones, D. E., B.Sc. 4 Conway Road, Cardiff.
- Mar. 11, 1882. ||Jones, D. Rhys. 19 Paget Street, Cardiff.
- Nov. 27, 1880. ||Jones, H. C. *Royal College of Science, South Kensington, London, S.W.*
- June 24, 1882. ||Jones, Prof. J. V., M.A., F.R.S. *University College, Cardiff.*
- Feb. 13, 1891. ||Joyce, Samuel. *Latchford House, Denmark Road, Manchester.*
- Apr. 10, 1880. ||Judd, Prof. J. W., F.R.S. *Royal College of Science, South Kensington, London, S.W.*
- Dec. 15, 1877. ||Judson, J. E., B.A. *Charterhouse, Godalming.*
- Apr. 26, 1895. Kaye, S. R. *Boys' High School, Allahabad, India.*
- Feb. 23, 1893. Kearton, J. W. 44 Tyrwhitt Road, St. John's, S.E.
- June 12, 1875. ||Kelvin, The Right Hon. Lord, LL.D., D.C.L., F.R.S. *University, Glasgow.*
- Mar. 13, 1875. Kennedy, Prof. A. B. W., C.E., F.R.S. 2 Gloucester Place, Portman Square, W.
- *||Kesselmeyer, C. A. *Villa Mon Repos, Altrincham, Cheshire.*
- *||Kesselmeyer, W. J. *Rose Villa, Vale Road, Bowdon, Cheshire.*
- Feb. 23, 1889. Kilgour, Martin H. 3 Clarence Parade, Cheltenham.
- May 28, 1881. Kilner, Dr. Walter, B.A., M.B. 104 Ladbroke Grove Road, London, W.
- Dec. 5, 1874. ||Kinch, Edward. *Royal Agricultural College, Cirencester.*
- Mar. 2, 1878. ||Kirkman, J. Parke, M.A. 4 St. Augustin's Road, Bedford.
- Apr. 8, 1876. Klaassen, H. M., F.G.S. *Aberfeldy, Campden Road, Croydon.*
- Mar. 13, 1896. Korda, Desiré. 64 Rue Caumartin, Paris.

Date of Election.

- *||Lambert, Prof. C. J., M.A. *Royal Naval College, Greenwich, S.E.*
- Jan. 25, 1879. ||Lankester, Prof. E. Ray, M.A., F.R.S. *2 Bradmore Road, Oxford.*
- Nov. 22, 1895. Larden, W. *37 Haddington Road, Devonport.*
- June 22, 1894. ||Larmor, Prof. Joseph, M.A., D.Sc., F.R.S. *St. John's College, Cambridge.*
- May 27, 1876. ||Law, Channell. *Ilsham Dene, Torquay.*
- May 25, 1889. ||Lawson, Dr. T. A. *114 Alexandra Road, St. John's Wood, London, N.W.*
- Nov. 6, 1891. Lee, Miss Alice, B.Sc., B.A. *Bedford College, Baker Street, W.*
- Nov. 22, 1895. ||Lehfeldt, R. A. *Beethoven House, Sheffield.*
- June 28, 1895. Leonard, Miss Agatha. *Ladies' College, Cheltenham.*
- May 11, 1889. Lewes, Prof. V. B. *Royal Naval College, Greenwich, S.E.*
- Feb. 28, 1896. Lewis, D. Morgan. *University College, Aberystwyth.*
- Feb. 14, 1880. ||Lewis, T. C., M.A. *Director of Public Instruction, N. W. P. and Oudh, Allahabad, India.*
- Jan. 17, 1882. ||Lewis, Prof. W. J., M.A. *Mineralogical Museum, Cambridge.*
- June 14, 1895. Littlewood, T. H. *Bancroft's School, Woodford Wells, Essex.*
- Feb. 22, 1879. ||Liveing, Prof. G. D., M.A., F.R.S. *Newnham, Cambridge.*
- Mar. 25, 1876. ||Liversidge, Prof. A., M.A., F.R.S. *University of Sydney, New South Wales.*
- Feb. 3, 1877. Lockyer, J. Norman, C.B., F.R.S. *South Kensington Museum, London, S.W.*
- Feb. 13, 1875. ||Lodge, Prof. O. J., D.Sc., F.R.S. *2 Grove Park, Liverpool.*
- Feb. 23, 1884. ||Love, E. F. J., M.A. *The University, Melbourne, Australia.*
- June 10, 1892. Lovibond, Joseph W. *26 St. Ann Street, Salisbury.*
- Mar. 6, 1891. Lucas, William, M.A. *53 Thornhill Square, Barnsbury, N.*
- *||Lupton, Sydney, M.A. *Grove Cottage, Roundhay, Leeds.*
- Jan. 25, 1879. Maedonell, Prof. A., B.A. *C. E. College, Sibpur, Bengal.*
- Dec. 14, 1894. MacMahon, Major P., R.A., F.R.S. *52 Shaftesbury Mansions, W.C.*
- Nov. 6, 1891. ||McClean, Frank, M.A., F.R.S. *Rusthall House, Tunbridge Wells.*
- Dec. 14, 1878. McGregor, Prof. J. Gordon, M.A., D.Sc. *Dalhousie College, Halifax, Nova Scotia.*
- **M'Leod, Prof. Herbert, F.R.S. *Coopers Hill, Staines.*
- May 11, 1878. ||Magnus, Sir Philip, B.Sc.
- Nov. 24, 1893. Maitland, Col. E., C.B. *12 Hill Top Road, West Hampstead, N.W.*
- Nov. 18, 1876. ||Malcolm, Col. E. D., R.E., C.B.
- Mar. 7, 1890. ||Mance, Sir Henry. *32 Earl's Court Square, London, S.W.*
- June 28, 1895. Marshall, Miss D., B.Sc. *13 Belsize Avenue, London, N.W.*
- June 10, 1882. Martin, Major-General C. N., R.E.
- Feb. 14, 1880. Martineau, Miss Caroline A. *1 Clifton Place, Sussex Square, London, W.*
- Mar. 23, 1889. ||Massey, W. H., M.Inst.C.E. *Twyford, R.S.O., Berks.*
- May 14, 1887. ||Mather, T. *1 Solent Crescent, West Hampstead, N.W.*
- **||Meldola, Prof. R., F.R.S. *6 Brunswick Square, London, W.C.*

Date of Election.	
Mar. 6, 1891.	Miers, Prof. H. A., F.G.S. <i>Magdalen College, Oxford.</i>
Feb. 13, 1886.	Mills, A. E., M.A., M.I.E.E. 72 <i>Upper Tollington Park, London, N.</i>
	** Mills, Dr. E. J., F.R.S. 60 <i>John Street, Glasgow.</i>
Nov. 15, 1889.	Milne, Prof. J., F.R.S. <i>Shide Hill House, Isle of Wight.</i>
Mar. 13, 1880.	Minchin, Prof. G. M., M.A., F.R.S. <i>Coopers Hill, Staines; and Malahide, Englefield Green, Surrey.</i>
Apr. 8, 1892.	Mitchell, C. T., M.A., Barrister-at-Law. 41 <i>Upper Addison Gardens, Kensington, W.</i>
June 8, 1889.	Mond, Ludwig, F.R.S. <i>The Poplars, 20 Avenue Road, London, N.W.</i>
Dec. 4, 1891.	Mond, Robert Ludwig, B.A. (Camb.), F.R.S.E. 20 <i>Avenue Road, London, N.W.</i>
Apr. 13, 1889.	Mordey, W. M. <i>Redholm, Loughborough, Leicestershire.</i>
May 12, 1877.	Morley, H. F., M.A., B.Sc. 47 <i>Broadhurst Gardens, South Hampstead, London, N.W.</i>
Mar. 22, 1895.	Morris, D. K. 13 <i>Somers Place, Hyde Park Square, London, W.</i>
Jan. 27, 1893.	Morton, Thomas William. 18 <i>Hogarth Road, Earls Court, W.</i>
Mar. 22, 1895.	Morton, W. B., M.A. 9 <i>Chilworth Buildings, Strandmillis Road, Belfast.</i>
Mar. 8, 1895.	Mostyn, S. G., M.A. 49 <i>Grosvenor Road, Canonbury, N.</i>
June 28, 1879.	Moulton, J. F., M.A., Q.C., F.R.S. 57 <i>Onslow Square, London, S.W.</i>
Mar. 23, 1889.	Mudford, F. J. <i>Arrow Electrical Works, Jackson Road, Holloway, London, N.</i>
	** Muirhead, Alexander, D.Sc. 3 <i>Elm Court, Temple, London, E.C.</i>
Nov. 26, 1887.	Mukhopādhyāy, Prof. Asutosh, M.A., F.R.A.S., F.R.S.E. 77 <i>Russa Road North, Bhowanipore, Calcutta.</i>
	* Müller, Dr. Hugo, F.R.S. 13 <i>Park Square East, London, N.W.</i>
Dec. 11, 1886.	Natanson, Ladislas. <i>Professor, Cracow University; 32 Wolska, Cracow (Austria).</i>
	Neville, Francis Henry, M.A. <i>Sidney College, Cambridge.</i>
Feb. 25, 1888.	Newall, H. F. <i>Madingly Rise, Cambridge.</i>
Nov. 12, 1881.	Niven, W. D., M.A., F.R.S. <i>Royal Naval College, Greenwich, S.E.</i>
Oct. 25, 1895.	Noble, Sir Andrew, K.C.B., F.R.S. <i>Jesmond Dene House, Newcastle-on-Tyne.</i>
June 9, 1877.	Northcott, W. H. 7 <i>St. Mary's Road, Peckham, London, S.E.</i>
Apr. 26, 1895.	Notcutt, S. A., LL.M., B.A., B.Sc. 98 <i>Anglesca Road, Ipswich.</i>
May 8, 1880.	Obach, Eugene, Ph.D., F.I.C., F.C.S. <i>Messrs. Siemens Brothers & Co., Woolwich.</i>
Apr. 10, 1875.	Odling, Prof. W., M.A., F.R.S. <i>The Museum, Oxford.</i>
Apr. 13, 1889.	O'Keefe, E. A., B.E. <i>City and Guilds' Technical Institute, Leonard Street, London, E.C., and 6 Staple Inn, Holborn, London, E.C.</i>

Date of Election.

- Feb. 24, 1883. || Page, F. J. M. 54 *Sutherland Street, Pimlico.*
 Mar. 23, 1889. Paul, R. W. 44 *Hatton Garden, E.C.*
 Jan. 22, 1892. Peace, James Bennet, M.A. *Emmanuel College, Cambridge.*
 *Perigal, Henry, F.R.A.S. 9 *North Crescent, Bedford Square, London, W.C.*
 Jan. 16, 1875. || Perkin, W. H., F.R.S. *The Chestnuts, Sudbury, Harrow.*
 ** || Perry, Prof. J., F.R.S. 31 *Brunswick Square, London, W.C.*
 May 25, 1878. || Phelps, W. R. *Montacute House, Ilminster.*
 Feb. 17, 1877. || Philips, T. W.
 Mar. 13, 1886. || Pickering, Prof. S. U., M.A., F.R.S. 48 *Bryanston Square, London, W.*
 Jan. 16, 1891. || Pidgeon, Winter Randall, M.A. 42 *Porchester Square, W.*
 Jan. 16, 1891. Porter, Alfred William, B.Sc. *University College, Gower Street, W.C.*
 Nov. 11, 1892. || Porter, Rev. T. C., M.A. *Eton College, Windsor.*
 Dec. 13, 1879. || Poynting, Prof. J. H., M.A., F.R.S. *Foxhill, Alvechurch, Worcestershire.*
 Mar. 16, 1878. || Prance, Miss Edith. *Froggnall, Hampstead, London, N.W.*
 Nov. 4, 1876. || Preece, W. H., C.B., C.E., F.R.S. *Gothic Lodge, Wimbledon.*
 Mar. 25, 1876. || Preston, Dr. S. Tolver. *Seiler Strasse 29, Hamburg.*
 June 24, 1882. || Price, Rev. Prof. Bartholomew, M.A., F.R.S. 11 *St. Giles', Oxford.*
 May 8, 1886. || Price, W. A., M.A. 5 *Hardy Road, Westcombe Park, Blackheath.*
 Nov. 18, 1876. || Purser, Prof. J., M.A. *Queen's College, Belfast.*
 June 27, 1885. || Ramsay, Prof. W., Ph.D., F.R.S. *University College, Gower Street, London, W.C.; and 12 Arundel Gardens, W.*
 Dec. 9, 1892. Rawson, Major Herbert E., R.E. *Cavalry Barracks, Canterbury.*
 Dec. 15, 1877. || Rayleigh, The Right Hon. Lord, M.A., D.C.L., Sec.R.S. *Terling Place, Witham, Essex; and 4 Carlton Gardens, London, S.W.*
 Nov. 22, 1879. Reilly, Prof. Calcott, M.Inst.C.E. *Coopers Hill, Staines.*
 ** || Reinold, Prof. A. W., M.A., F.R.S. *Royal Naval College, Greenwich, S.E.*
 Nov. 27, 1875. || Reynolds, Prof. Osborne, M.A., F.R.S. *Owens College, Manchester.*
 Apr. 26, 1895. Rheam, W., B.Sc. *Freshfield, near Liverpool.*
 Apr. 13, 1894. Rhodes, W. G., M.Sc. 1 *Rothsay Villas, Hamilton Road, Quick's Road, S. Wimbledon.*
 June 12, 1886. || Rideal, S., D.Sc. 28 *Victoria Street, Westminster, S.W.*
 May 10, 1895. Rigby, Rev. E. *Lowe House, St. Helens, Lancashire.*
 June 12, 1886. Rimington, E. C. *Lynton House, The Park, Church Road, Norwood, S.E.*
 Nov. 9, 1894. Rintoul, David, M.A. 4 *Northcote Road, Clifton, Bristol.*
 May 26, 1888. || Roberts, Samuel Oliver, M.A. 3 *Furnival's Inn, Holborn, London, E.C.*
 ** || Roberts-Austen, Prof. W. Chandler, C.B., F.R.S. *Royal Mint, London, E.*
 May 11, 1889. Robinson, W., M.E., A.M.Inst.C.E. *University College, Nottingham.*
 Feb. 22, 1895. || Rodger, J. W. 80 *Anerley Park, London, S.E.*

Date of Election.

- Feb. 28, 1895. Rodgers, C., B.Sc. 179 *Finborough Road, Earls Court, S.W.*
 May 8, 1880. ||Roscoe, Prof. Sir Henry E., Ph.D., F.R.S. 10 *Bramham Gardens, Wetherby Road, London, S.W.*
 Jan. 24, 1885. ||Rose-Innes, J., B.Sc. 5 *Gayton Crescent, Hampstead, N.W.*
 June 26, 1875. ||Rosse, The Earl of, D.C.L., F.R.S. *Parsonstown, Ireland.*
 * ||Rücker, Prof. A. W., M.A., F.R.S. 19 *Gledhow Gardens, London, S.W.*
 Jan. 19, 1878. ||Russell, J. W., M.A. 16 *Bardwell Road, Oxford.*
 June 24, 1876. ||Russell, The Hon. F. A. R., M.A. *Pembroke Lodge, Richmond Park, Surrey.*
 Mar. 2, 1878. ||Russell, W. J., Ph.D., F.R.S. 34 *Upper Hamilton Terrace, St. John's Wood, London, W.*
 Dec. 13, 1895. Rylands, L. G., B.A. 7 *Holtham Road, N.W.*
 May 26, 1877. ||Sale, Capt. M. T., R.E., C.M.G. *Ugia Bank, Abbey Wood, Kent.*
 Mar. 25, 1876. ||Salisbury, The Marquis of, D.C.L., F.R.S. *Hatfield House, Herts.*
 Jan. 15, 1876. ||Salomons, Sir D. L., Bart. *Broom Hill, Tunbridge Wells.*
 Jan. 30, 1875. ||Salter, M. J. *Northcote, Beaconsfield Road, New Southgate, London, N.*
 June 27, 1885. Sanderson, F. W., B.A. *The School House, Oundle.*
 Dec. 9, 1882. ||Saunders, Rev. S. T. H., M.A. *Merchant Taylors' School, London, E.C.*
 ** ||Schuster, Prof. A., Ph.D., F.R.S. *Owens College, Manchester.*
 Feb. 21, 1883. Scott, Prof. A. W. *St. David's College, Lampeter, South Wales.*
 Dec. 19, 1894. ||Searle, G. F. C., M.A.. 11 *Fitz-William Street, Cambridge.*
 Mar. 17, 1877. ||Scaton, W. S. *Montague House, Richmond, Surrey.*
 May 9, 1885. ||Selby, A. L., M.A. *University College, Cardiff.*
 Feb. 9, 1889. Sellon, Percy R. *Brush Electric Engineering Company, 49 Queen Victoria Street, E.C.*
 Mar. 8, 1895. Shaw, P. E., B.A., B.Sc. *University College, Nottingham.*
 Feb. 8, 1879. ||Shaw, W. N., M.A., F.R.S. *Emmanuel College, Cambridge.*
 Nov. 6, 1891. Shenstone, W. A. *Clifton College, Bristol.*
 Dec. 13, 1879. Shettle, R. C., M.D. 73 *London Street, Reading.*
 Mar. 7, 1890. ||Shorter, Lewis R. 21 *Campden Hill Road, Kensington, W.*
 Oct. 25, 1895. ||Shurlock, F. W., B.A., B.Sc., 67 *Waverley Road, Redland, Bristol.*
 Apr. 13, 1889. ||Siemens, A. 12 *Queen Anne's Gate, London, S.W.*
 Jan. 22, 1881. ||Simpson, G. P., B.Sc. 12 *Queen Anne's Gate, London, S.W.*
 Dec. 10, 1881. ||Sircom, Prof. the Rev. Sebastian, M.A. *Presbytery, Holy Name, Oxford Road, Manchester.*
 Dec. 14, 1894. ||Skinner, S., M.A. *Cromwell House, Trumpington, near Cambridge.*
 Jan. 16, 1891. Skirrow, Benjamin Beck, B.A. 9 *Alfred Street, Plymouth.*
 May 10, 1884. Slade, Lieut. E. J. W., R.N.
 Apr. 28, 1888. Slater, A. W.
 Dec. 13, 1895. ||Slater, R. C. *Charterhouse, Godalming, Surrey.*
 Nov. 23, 1878. ||Sleeman, Rev. P. R., F.R.A.S., 65 *Pembroke Road, Clifton, Bristol.*
 * ||Smith, Basil Woodd, F.R.A.S. *Branch Hill Lodge, Hampstead Heath, London, N.W.*

Date of Election.

- Feb. 7, 1890. ||Smith, Edward Wythe. 66 College Street, Chelsea, London, S.W.
- Feb. 23, 1884. Smith, Rev. F. J., M.A. 28 Norham Gardens, Oxford.
- Apr. 10, 1880. ||Smith, W. O.
- June 8, 1878. ||Solly, R. H., F.G.S. Gordon Villa, Cambridge.
- Apr. 29, 1895. Soper, H. E., M.A. 1 Christchurch Terrace, Cheltenham.
- May 12, 1883. Soward, A. W. 28 Therapia Road, Honor Oak, London, S.E.
- Mar. 23, 1889. Spagnoletti, C., M.Inst.C.E. The Poplars, Aberdeen Place, London, N.W.
- Mar. 8, 1895. ||Stansfield, H. Aldersyde, Hainault Road, Leytonstone.
- Feb. 25, 1882. ||Stanley, W. Ford, F.G.S. Cumberlow, South Norwood, London, S.E.
- June 11, 1881. ||Stead, J. E. Redcar.
- June 28, 1895. ||Stewart, J. J., B.A., B.Sc. 7 Mount View Road, Crouch Hill, N.
- *Stirling, Hon. Sir James, M.A. 51 Great Cumberland Place, London, W.
- Dec. 15, 1877. ||Stocker, Prof. W. N., M.A. Coopers Hill, Staines.
- Nov. 18, 1876. ||Stoney, G. Johnstone, M.A., F.R.S. 8 Upper Horsey Rise, London, N.
- Mar. 13, 1880. ||Stroh, Augustus. 98 Haverstock Hill, London, N.W.
- June 9, 1893. ||Stroud, H., D.Sc., M.A. Durham College of Science, Newcastle-on-Tyne.
- Feb. 26, 1887. ||Stroud, Prof. W., D.Sc. Yorkshire College, Leeds.
- Dec. 10, 1887. ||Sumpner, W. E., D.Sc. 19 Greenfield Crescent, Edgbaston, Birmingham.
- Dec. 6, 1889. ||Swan, J. W., F.C.S. 58 Holland Park, W.
- May 22, 1886. ||Swinburne, J. 66 Victoria Street, S.W.
- Feb. 8, 1895. Swinton, A. A. Campbell, A.M.I.C.E. 66 Victoria Street, S.W.
- May 27, 1876. Taylor, Herbert A. 25 Collingham Road, London, S.W.
- Apr. 29, 1895. Thomas, E., B.Sc. 81 Sisters' Avenue, Clapham Common, S.W.
- May 10, 1895. Thomas, G. L., B.Sc. 35 Richmond Terrace, Clifton, Bristol.
- Dec. 11, 1875. ||Thompson, Prof. S. P., D.Sc., F.R.S. Finsbury Technical College, Leonard Street, London, E.C.
- Nov. 27, 1875. Thomson, Prof. John Miller. 85 Addison Road, Kensington, W.
- Mar. 22, 1895. ||Thomson, Prof. J. J., M.A., F.R.S. 6 Scrope Terrace, Cambridge.
- Feb. 13, 1891. Thorp, Walter, B.Sc. Analytical Laboratory, Limerick.
- **||Thorp, William, B.Sc. 24 Crouch Hall Road, Crouch End, London, N.
- Nov. 28, 1885. ||Thorpe, Prof. T. E., Ph.D., F.R.S. Government Laboratory, Somerset House, W.C.
- Mar. 9, 1889. ||Threlfall, Prof. R., M.A. The University, Sydney.
- *Tomlinson, Charles, F.R.S. 7 North Road, Highgate, London, N.
- *Tomlinson, Herbert, B.A., F.R.S. South-West London Polytechnic, Manresa Road, Chelsea.
- Dec. 8, 1888. ||Trotter, Alexander Pelham, B.A. Treasury, Cape Town.

Date of Election.

- Mar. 8, 1895. Trouton, F. T., B.A., Sc.D. *Caerleon, Killiney, Co. Dublin.*
 Dec. 2, 1876. Tunzelmann, G. W. von. *65 Longridge Road, Earl's Court Road, London, S.W.*
- ** Unwin, Prof. W. C., B.Sc., F.R.S. *City and Guilds' of London Central Institute, Exhibition Road, London, S.W.*
- Nov. 22, 1895. Vincent, J. H., B.Sc. *Royal College of Science, South Kensington, S.W.*
- June 9, 1893. Wade, John, B.Sc., F.I.C. *Wyvenhoe, Purley, Surrey.*
 Nov. 27, 1875. Waghorn, J. W. W., R.N., D.Sc. *Royal Naval College, Greenwich, S.E.*
- ** Walenn, W. H. *9 Carleton Road, Tufnell Park, London, N.*
 Dec. 1, 1883. || Walker, J., M.A. *30 Norham Gardens, Oxford.*
 Dec. 5, 1874. || Walker, J. F., M.A. *45 Bootham, York.*
- ** || Walker, J. J., M.A., F.R.S. *12 Denning Road, Hampstead, London, N.W.*
- Mar. 7, 1890. || Waller, Dr. Augustus D., F.R.S. *Weston Lodge, 16 Grove End Road, N.W.*
- Nov. 24, 1888. || Walmsley, R. M., D.Sc. *Northampton Institute, London, E.C.*
- Mar. 25, 1892. Walter, Miss L. Edna, B.Sc. *38 Woodbury Grove, Finsbury Park, London, N.*
- Dec. 11, 1891. || Watson, William, B.Sc. *7 Upper Cheyne Row, S.W.*
 Mar. 8, 1895. Watt, J. *Bradfield College, Berks.*
- ** Watts, Dr. W. M. *Giggleswick Grammar School, and Goldielands, Settle.*
- Dec. 11, 1886. Wertheimer, Prof. Julius, B.A., B.Sc. *Merchant Venturers' Technical College, Bristol.*
- June 9, 1877. Whalley, L. J. *26 Park Place, Greenwich, S.E.*
- Dec. 15, 1894. || Whetham, W. C. D., M.A. *Trinity College, Cambridge.*
 May 10, 1895. || White, S. C. F., M.A. *72 Cambridge Street, Eccleston Sq., W.*
 Dec. 15, 1894. || Wilberforce, L. R., M.A. *6 Pemberton Terrace, Cambridge.*
 Mar. 10, 1893. Williams, W. *Physical Laboratory, Royal College of Science, South Kensington.*
- Feb. 8, 1895. || Wilson, W.E., J.P. *Daramona, Streets, Rathowen, Westmeath, Ireland.*
- Feb. 9, 1889. Wimbhurst, James. *4 The Grove, Clapham Common, London, S.W.*
- Mar. 13, 1880. Wingfield, Lieut. G. T., R.N.
- Jan. 22, 1887. Womack, Frederick, B.Sc., M.B. *9 Dennington Park Road, West Hampstead, London, N.W.*
- Dec. 15, 1877. || Wood, Sir H. T., B.A. *Society of Arts, Adelphi, London, W.C.*
- Feb. 28, 1896. Wood, S. *South Kensington Museum, S.W.*
 May 8, 1880. Woods, Edward, C.E. *45 Onslow Gardens, South Kensington, London, S.W.*
- * Woodward, C. J., B.Sc. *97 Harborne Road, Birmingham.*
 Dec. 10, 1881. || Woolcombe, W. G., M.A., B.Sc., F.R.A.S. *King Edward's High School, New Street, Birmingham.*

Date of Election.

Jan. 24, 1884.	* Wormell, Dr. R. <i>Cowper-Street Schools, London, E.C.</i> Worthington, Prof. A. M., M.A., F.R.S. <i>59 Oxford Terrace, Hyde Park, W.</i>
Mar. 27, 1886.	Wright, A. R. <i>46 Marmora Road, Honor Oak, London, S.E.</i>
Mar. 26, 1881.	Wright, Lewis. <i>52 Cromwell Avenue, Highgate, London, N.</i>
Dec. 13, 1880.	Wrightson, T., C.E. <i>Norton Hall, Stockton-on-Tees.</i>
Oct. 28, 1892.	Wyatt, G. H., B.Sc. <i>Emmanuel School, Wandsworth Common, S.W.</i>
Feb. 27, 1886.	Young, Sydney, D.Sc., F.R.S. <i>University College, Bristol.</i>
Feb. 13, 1891.	Yule, G. U. <i>4 Carlingford Road, Hampstead, N.W.</i>

PROCEEDINGS

OF THE

PHYSICAL SOCIETY OF LONDON.

From October 1896 to October 1897.

VOL. XV.

LONDON:

TAYLOR AND FRANCIS, RED LION COURT, FLEET STREET.

MDCCCXCVII.



PRINTED BY TAYLOR AND FRANCIS,
RED LION COURT, FLEET STREET.

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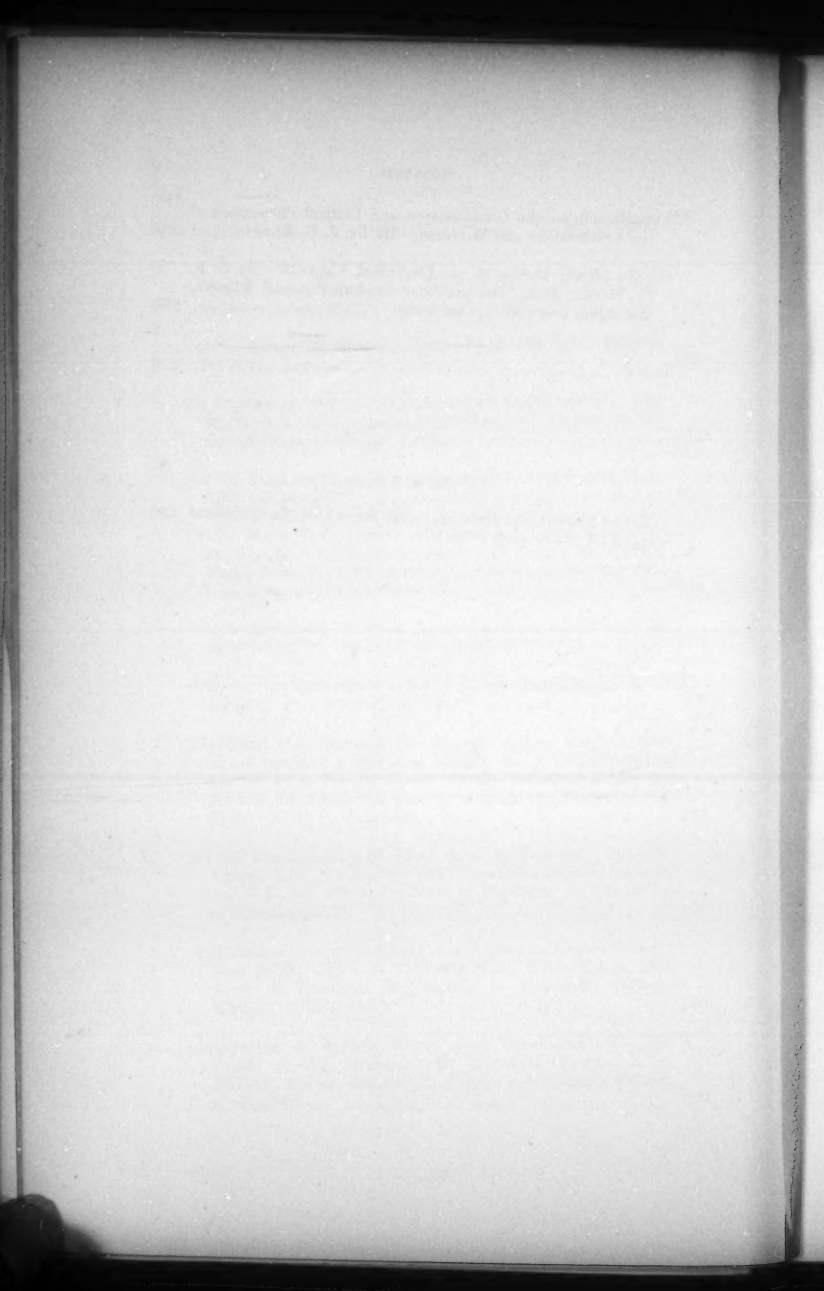
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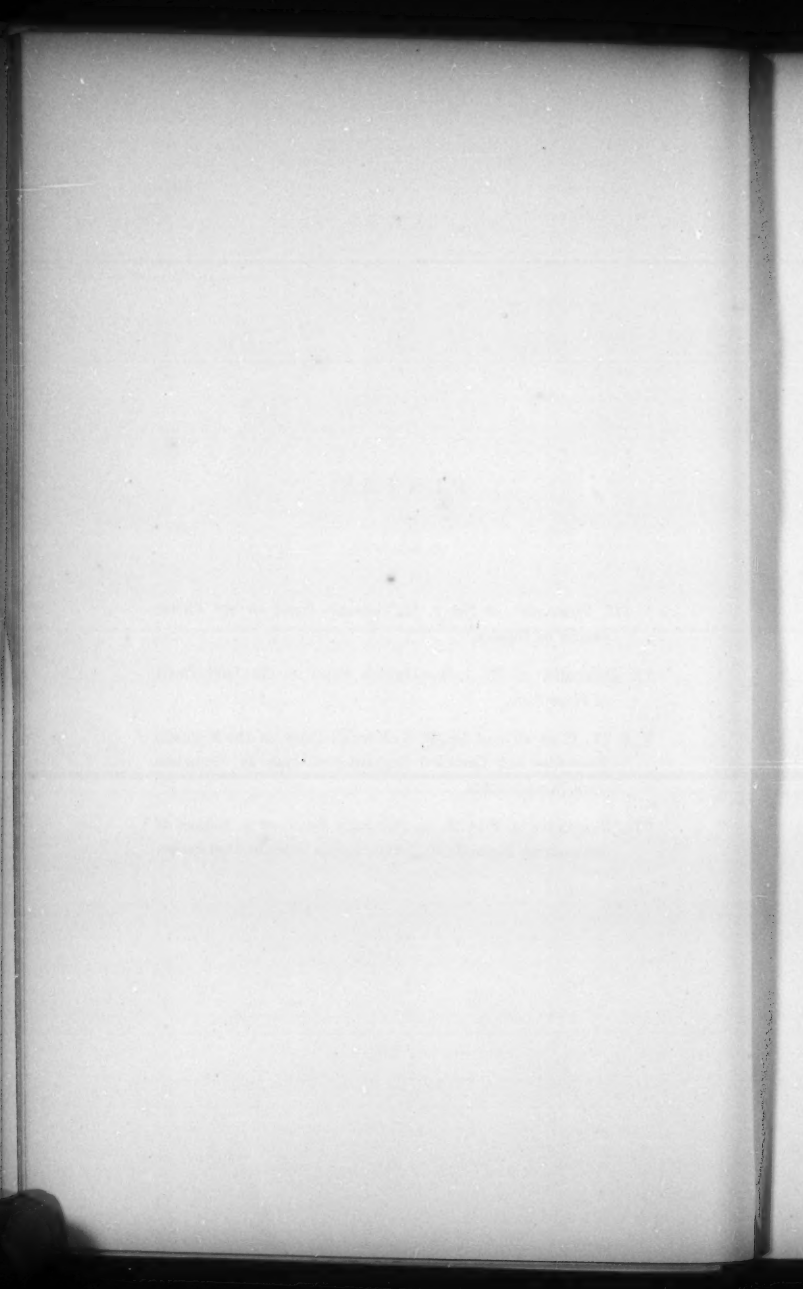
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- I.-III. Illustrative of Mr. J. H. Vincent's Paper on the Photography of Ripples.
- IV. Illustrative of Mr. J. Rose-Innes's Paper on the Isothermals of Isopentane.
- V. & VI. Illustrative of Dr. D. K. Morris's Paper on the Magnetic Properties and Electrical Resistance of Iron as dependent upon Temperature.
- VII. Illustrative of Prof. J. A. Fleming's Paper on a Method of determining Magnetic Hysteresis loss in Straight Iron Strips.



PROCEEDINGS
OF
THE PHYSICAL SOCIETY
OF LONDON.

NOVEMBER 1896.

- I. *On some Experiments with Röntgen's Radiation.* By RICHARD TRELFALL, M.A., *Professor of Physics*, and JAMES ARTHUR POLLACK, *Demonstrator of Physics in the University of Sydney, N.S.W.**

THE experiments to be described were performed during April and May of this year, and were made with the object of elucidating the nature of the radiation. It was thought that the following possible explanations should be tested:—

1. The radiation consists of a swarm of material particles projected through the glass of the generating tube. Electrical changes taking place at the glass surface are invoked to explain the differences between Röntgen's and Lenard's rays.
2. The radiation consists of an "æther wind." Æther is sucked through the glass towards the source of radiation and then blown outwards. The question as to whether the radiation observed by Röntgen is the expression of the motion of æther to or from the source remaining open.
3. The radiation consists of æther vortices moving to or from the source.

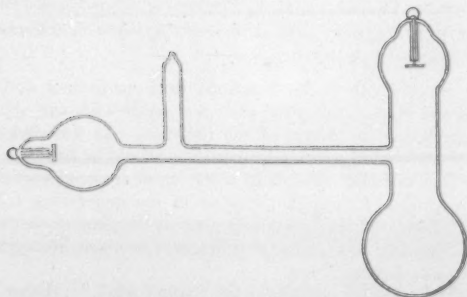
* Read November 13, 1896.

4. The radiation consists of æther waves: that is waves of regular or irregular æther motions.
5. The radiation consists of electromagnetic waves of either very small wave-length or having longitudinal components. This is probably a special case of (4)—at least if we are to look to the æther to explain electricity and magnetism dynamically.
6. The radiation is a phenomenon of a new order entirely unconnected in any way with anything in our past experience.

Source of Radiation.

Being thrown entirely on our own resources for means of production of the radiation—all the Crookes's tubes in our possession being almost useless—we arrived at the form of tube shown in fig. 1. These tubes are easily made; the surface

Fig. 1.



opposite to the kathode being spherical, can be made very thin, and the electrodes are kept well apart. The kathode is best made very nearly plane—if concave it will easily fuse the thin glass against which its rays are projected; we have lost many tubes from this cause. We have found that the bulb may be conveniently about three or four centimetres in diameter and the main tube as little as 1.5 to 2 cm. in diameter. The expansions round the electrodes are intended to obviate

local heating, for it is not always easy to prevent oscillatory discharges and consequent "kathoding" from the "anode." The chief merits from our point of view, however, were that the tubes were very easy to make out of comparatively small glass tubing. Their volume is small, so that they can be exhausted quickly, and they give very intense action. In fact one tube—the bulb of which ultimately fused under the influence of the kathode-discharge—gave quite as intense if not more intense radiation than a "focus"-tube made in our laboratory, which appeared to act perfectly, so far as we could judge.

Experiment to test Hypothesis 1.

When a vacuum-tube is prepared with electrodes of aluminium-wire whose ends are about 1 centimetre apart, and exhausted until the discharge will rather jump across three or four centimetres of air between balls of 1.5 cm. diameter than pass in the tube, it is generally noticed that the discharge, when forced to pass by the tube, goes rather more easily in one direction than the other. By placing a spark-gap with spherical electrodes in parallel with the exhausted tube and properly adjusting the distance of the balls from one another, it is easy to so arrange matters that the sparks pass mostly by the spark-gap when the current is in one direction and by the tube when it is reversed. An arrangement of this kind is exceedingly sensitive to small changes of pressure in the exhausted tube. In the experiment to be described, the spark-gap was generally so adjusted that when the coil-commutator was in one position the whole of the discharge passed over the gap—only the faintest glow being discernible in the tube in a dark room. When the current was reversed, however, the discharge was about equally divided between the gap and the tube. No very delicate adjustment of the gap seems to be necessary, at all events when the discharges follow each other rapidly.

Having thus obtained a means of testing the vacuum in a discharge-tube more rapidly and probably much more delicately than by any kind of gauge, we thought it worth while to try whether Röntgen's rays would project particles into the exhausted tube. If hypothesis (1) be correct,

then particles must be carried into the exhausted tube if it is thin enough to be transparent; and if in addition it contains a piece of platinum-foil which stops the radiation, the particles would also be stopped; also if the particles are not wholly entangled in the platinum, some change in the vacuous state of the tube is to be expected.

A tube about 12 cm. long and 1.5 cm. in diameter, and having a bulb about 4 cm. in diameter in the middle of its length, was prepared of German glass. It was provided with electrodes fused in from either end, and extending to within 1 cm. of each other in the centre of the bulb. A bit of platinum-foil lay in the bulb, and the tube was fused on to a Sprengel pump through about a metre of tubing some millimetres in internal diameter. A phosphorus-pentoxide tube was included just above the fall-tube. The whole apparatus, including the pentoxide tube, was repeatedly heated by a Bunsen burner and exhausted until the discharge preferred to pass through 3 cm. of air-gap rather than through the tube. So sensitive, however, was this means of testing the vacuum that for the first few days, despite frequent heatings and pumpings, the vacuum would not remain constant for more than some minutes. After about a fortnight of heating and pumping, however, the vacuum became so steady that the change in twelve hours, which was sufficient to entirely stop the discharge in the spark-gap, could be rectified by the fall of at most 50 drops of mercury in the fall-tube—*i. e.* by about 30 seconds' pumping. The change of vacuum occurring during an hour could still be easily detected by testing with a current in alternate directions.

Under these circumstances, experiments were made by urging Röntgen tubes to their utmost, almost in contact with the bulb of the exhausted tube, and directing their radiation on to the platinum-foil. Though many very active tubes were fused or otherwise destroyed, yet during an hour's action on several occasions no change of vacuum in the exhausted tube could be detected when due allowance for the slight progressive deterioration was made.

This experiment was repeated several times, and a tube which had successfully exhibited the fluorescence of a screen of tungstate of calcium through an aluminium plate .7 mm.

thick to an entire audience, was melted down in the operation ; but no effect whatever was observed.

Professor Wright, of Yale, has given some reasons for thinking that Röntgen rays when passing through gold-leaf carry particles (of gold ?) off with them ; so that if a positive effect had been obtained in the experiment described it would not have been quite conclusive ; neither is the negative effect observed conclusive against *any* particles being carried forward ; it only shows that the particles so carried (if any) either refuse to behave as gaseous particles, or are exceedingly few in number.

The experiments of Professor Minchin which are now available have, however, rendered the solid particle theory so unlikely that it is hardly worth while pushing the investigation further—at all events until much more powerful appliances are to hand. In case the experiment appears worthy of repetition, we would call attention to the fact that it appears desirable to use specially purified phosphorus pentoxide in the drying-tube ; for the presence of the vapour of phosphorus or its lower oxide fouls the mercury when impure pentoxide is heated.

Experiments made to test whether Röntgen's Radiation is associated with æther currents in any way.

With regard to hypothesis (2) Michelson has put into our hands a comparatively simple method of obtaining the interference of two beams of light which may be used for detecting the presence of æther currents by their influence on the velocity of light travelling through the moving æther.

The arrangement employed in fig. 2 was made use of for the purpose of determining whether Röntgen's radiation is associated with æther movements.

In this arrangement the light from L is divided into two beams at A which travel round from mirror to mirror in opposite directions, eventually reaching the telescope T. When the adjustment is correct, interference-bands are seen on looking into the telescope.

In this experiment it was estimated that a shift of the bands equal to one tenth of the width of a single band, or a widening of the bands by one fifth of the same amount,

No disturbance of the bands commencing when the coil started, and stopping when it stopped, was ever detected, though a great many excellent observations were made. Of course, effects due to the heating and electrification of the glass can be easily distinguished from those we are in search of in view of their persistence after the coil is put out of action. The tube employed gave quite visible fluorescence by barium platinocyanide at a metre's distance and through aluminium .7 mm. thick. We can get an idea of the order of the minimum æther velocity which could be detected by this means. The path BC being some 25 cm. long, we will suppose that only 10 cm. of it are influenced by the tube and that the radiation is along the path. The sensitiveness of the method was the same whether we employed air or benzene.

A shift of the bands amounting to a fifth of the distance from band to band would be produced by an æther velocity sufficient to change the time of passage of light over the path by one tenth of a period. As sodium light was employed, we may call this

$$5.9/3 \times 10^{-16} \text{ seconds.}$$

But the time required for light to travel over ten centimetres in air is about

$$1/3 \times 10^{-9} \text{ seconds ;}$$

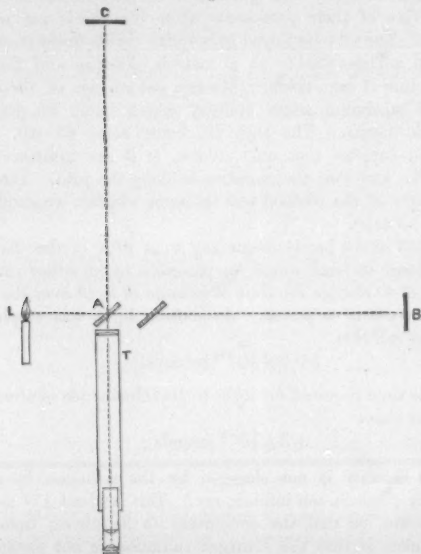
or the velocity is not changed by the radiation by more than six parts in ten million, say. This is about 177 metres per second, so that the conclusion to be drawn from the experiment is that the Röntgen radiation is not associated with æther velocities greater than, say, one fifth of a kilometre per second, or about a thousand times less than that of cathode rays as measured in a vacuum tube by Prof. J. J. Thomson (Phil. Mag. vol. xxxviii. p. 364).

Experiment to test whether Röntgen Rays produce any change in æther affecting the velocity of light.

An experiment (fig. 3) was arranged on Michelson's principles. In this case the sodium light from L is divided at the partially silvered mirror A, one beam travelling to C and thence to T, the other travelling to B and thence back

to T. The path AC is operated upon by the tube, and the path AB is screened by a heavy cast-iron screen. The active surface of the tube is brought up to the path CA, and the

Fig. 3.



Röntgen radiation allowed to traverse it in a variety of directions from parallelism to normality, and is even thrown on to the mirror at C. For experiments on benzene, troughs of that liquid as similar as possible are inserted in both paths, the one in the path AB acting merely as a compensator. This experiment is much more difficult than the one previously described, and the benzene requires to be well stirred if good definition is required. In the experiments in air a shift of the bands by $\frac{1}{50}$ of the width of a band could be observed, while in benzene a change of about half the distance from

band to band only could be seen. A widening of the bands to about half the above amounts could have been detected in each case. When the benzene was used the fringes were unsteady, and opportunities for observation had to be waited for. Of course, when the active surface of the Röntgen tube dipped into the benzene so as to appear in the field of view of the telescope great disturbances due to thermal and electric changes became visible. These, however, did not appear instantaneously on starting the coil, nor did they disappear on stopping it. In no case was any real effect observed.

The chief interest of this experiment lies in the fact that if the Röntgen radiation consisted of longitudinal æther waves, *i. e.*, waves of longitudinal æther displacement, some effect was to be expected. If the waves are long compared with the path AC (the path AB being screened) a widening of the fringes, or in the extreme case a total disappearance of the fringes, is to be anticipated. This can easily be realized by blowing an organ-pipe in the neighbourhood. If the waves are short compared with the dimensions AC then all will depend on the azimuth of the tube, or rather on the inclination of the Röntgen-ray path to the path of the light rays. This appears from the fact that whole waves would produce no effect in the case contemplated—everything would depend on the fractional parts of the waves included in or projected on AC.

The conclusion to be drawn from the experiment is that neither in air nor benzene are the light-transmitting properties of the æther interfered with. The limits of observational accuracy are of the same order as in the case of Experiment II.

Action of Röntgen's Rays on a Selenium Cell.

It so happened that one of us was engaged early in the year in experimenting with photo-resistance cells made of selenium which had been laboriously purified and which was probably as pure as any that has ever been obtained. Contrary to expectation, such cells showed a quite normal light-sensitiveness whether the electrodes were of platinum or of aluminium. The cells were made according to the directions given by Mr. Shelford Bidwell* in his paper read before the

* Phil. Mag. vol. xl. p. 233; Proc. Phys. Soc. vol. xiv. p. 552.

Society last year—the only difference being that purified selenium was employed.

The result of a good many observations was to show that a certain selenium cell with platinum electrodes was acted on to about the same extent and in the same direction whether it was exposed to the radiation of a “standard” candle at a distance of three metres, or to the Röntgen radiation at a distance of ten centimetres and passing through .7 mm. of aluminium and about 3 mm. of wood. The tube was working so as to cause visible fluorescence in a barium platino-cyanide screen—not of quite the best quality—at a distance of rather over a metre in a room nearly but not absolutely dark. This is of course a very rough way of stating the degree of activity of the tube, but when the experiments were made Professor Minchin’s work had not reached us, and consequently the simple scale of tube intensities which it implies was not available.

In order to test whether the action of the light differed in kind from that of the Röntgen radiation two experiments were made—in one the rate of resistance-recovery of the selenium cell was carefully studied and compared with the rate of recovery of the cell after exposure to candle-light; in the other tests were made in the hope of discovering that a permanent electromotive force was established by the radiation, and that it persisted after the radiation was cut off. Neither of these experiments led to positive results. The rate of change of resistance during the twenty seconds of exposure to Röntgen’s radiation was, so far as could be seen, exactly the same as when the candle-flame was substituted at the proper distance. The recovery curves, extending over about half an hour, were also very similar on the whole (several tests were made), though both curves themselves exhibited great irregularities.

In order to obtain effects as little complicated as possible by previous history the sensitive cell was kept in circuit on the bridge, and was traversed by the testing current for two or three days before the observations.

It was for the same reason that exposures were limited to 20 seconds, for the rate of recovery of resistance with the cell employed was very slow. On one occasion when an

accidental exposure for several minutes to Röntgen's radiation was made, the cell had by no means recovered four hours afterwards.

The resistance of the sensitive cell employed was reduced from 1209 ohms to 1185 ohms in 20 seconds by the radiation under the conditions mentioned. The testing battery consisted of two "Obach" cells, and the bridge was made up of two equal arms of 1000 ohms each—the selenium and the variable arm. We mention this in case it may ever turn out that the effect depends on the testing current.

With regard to the electromotive force which it was supposed might be set up. The cell was kept at rest and undisturbed for three days before the final trial; it was placed three centimetres from the active tube, which was, as before, in a metallic box, together with the coil. The tube was shut off from the cell by an aluminium plate 7 mm. thick.

A very sensitive high resistance galvanometer in our possession, which has been described in a paper read before the Royal Society but as yet unpublished, was employed to test for any electromotive force which might be set up. An exposure to the radiation was made while the cell was in series with the galvanometer, and it was found that the cell always exhibited a small electromotive force whether it was exposed to the rays or not. This prevented the test from being very sensitive, but in no case was any electromotive force attributable to the radiation discovered, though if a voltage of 10^{-7} volts had come into operation its effect could probably not have escaped observation.

At the time these experiments were made we were uninformed as to the discharging action of the rays, which has since been so copiously studied. As soon as we saw an account of some of this work we felt that the change of resistance of the selenium cell was no longer an isolated phenomenon to be worked out by itself, but must be studied in conjunction with the similar phenomena observable in other substances, and it is for this reason that the experimental work was not extended so as to include other cells.

These notes may, perhaps, be summed up as follows:—

(1) It is easy to make a Röntgen tube of great activity by the most elementary glass-blowing.

(2) The Röntgen radiation does not consist in the projection of gaseous matter, or if it does the amount of such matter involved is extraordinarily small.

(3) The Röntgen radiation does not consist in the projection of æther streams having a velocity above a couple of hundred metres per second: this is true whether the radiation takes place in air or in benzene.

(4) The properties of æther regarded as determining the velocity of electromagnetic waves are not greatly changed (*i. e.* not at all within our experimental limits) by the Röntgen radiation, and this applies alike together in air and in benzene.

(5) A selenium cell composed of platinum electrodes and highly purified selenium is affected by Röntgen radiation to an extent which is comparable with the effect produced by diffused daylight.

(6) No permanent or temporary electromotive force is set up in a selenium cell by the Röntgen radiation.

DISCUSSION.

Mr. SHELFORD BIDWELL said he had made some experiments on the effect of Röntgen rays on the resistance of selenium, but with a negative result, although he could have detected a much smaller change than that found by the authors. It might be that this difference was due to the tube, for in his experiment the radiation started from a platinum plate within the tube, while in the authors' arrangement the radiation starts from where the kathode rays strike the glass of the tube.

Prof. SILVANUS THOMPSON said there were a number of points with reference to the Röntgen radiation which required clearing up. For instance, the suggestion that they were vortices in the æther had not been tested. Again, Lafay says that if the rays are passed through a metal screen which is charged with electricity, then the rays can be deflected by a magnet. He (Prof. Thompson) had not been able to repeat this experiment, neither had he that of Galitzine on the polarization of the rays by tourmaline. The statement of Prof. J. J. Thomson, that under the influence of the radiation paraffin became a conductor, had not been satisfactorily

proved. As to the wave-length, while some observers obtained values about one-tenth that of the extreme violet, another had obtained a value greater than that of the extreme red. He (the speaker) did not understand the authors' device for detecting changes in the vacuum of a tube, since everyone who has worked with Crookes tubes has found that the resistance is always greater for a spark in one direction than in the other, and also varies with the battery-power employed. Lenard, adopting Hertz's arrangement, uses as anode a cylinder surrounding the kathode (a disk), the idea being that by using such a symmetrical arrangement the kathode radiation was more homogeneous. It might be advisable, when seeking to produce homogeneous Röntgen rays, to adopt such a symmetrical arrangement.

II. *A Satisfactory Method of Measuring Electrolytic Conductivity by means of Continuous Currents.* By Prof. W. STROUD, D.Sc., M.A., and J. B. HENDERSON, B.Sc.*

THE devices for eliminating or reducing the disturbing effects of polarization in the measurement of the conductivity of electrolytes are very numerous. Wheatstone, Horsford, Wiedemann, Beetz, Paalzow, Ewing and Macgregor, Bouty, and others have all experimented on this subject. It will, however, be universally conceded that Kohlrausch's method of measuring the conductivity of electrolytes by the use of alternating currents and a telephone is superior to any method at present in use in which continuous currents are employed. That there are difficulties in the use of this method, arising from self-induction, capacity, &c., is admitted by Kohlrausch himself. Ever since the introduction of this method in 1875 physicists have not ceased to try to improve the continuous current methods, with the desire unquestionably of employing a galvanometer instead of a telephone as the indicating apparatus. In the method used

* Read October 30, 1896.

by Fitzpatrick*, where the alternating currents are confined to the four arms of the bridge, we have an example of a successful attempt to utilize the advantages of alternating currents while dispensing with the telephone in favour of the more satisfactory galvanometer.

In the face of all the experimental work done in the past it may seem a bold thing to say that if the proper conditions are only satisfied, continuous currents are preferable in every way to alternating currents for the measurement of electrolytic conductivity, but such we believe to be the fact.

The idea underlying the method to be described struck one of us some six months ago. This idea was to obviate the detrimental effects of polarization in the electrolytic cell by inserting a second cell with the same size of plates &c., but with a very different length of electrolytic conductor in the corresponding arm of a Wheatstone bridge circuit.

The notion of employing such a balancing cell we subsequently found was suggested in 1877 by Kohlrausch and employed by Tollinger†.

The method used by Tollinger consisted in having an electrolytic cell in each arm of the bridge, but the method of finding the resistance was to obtain a balance for a particular position of the electrodes in one cell, then to diminish the distance between them by a measured amount and increase the resistance in that arm so as to restore the balance. A conceivable source of error arises in consequence of the motion of the electrode through the liquid producing a possible alteration in its polarization.

The results of Tollinger's work show that resistances measured by his continuous current method were 0.6 per cent. lower on the average than the same resistances measured by the alternating current method.

Elsas‡ has also employed the balancing cell method. He used a long trough of known section with two fixed non-polarizable electrodes at the ends (Cu electrodes in CuSO_4) and one movable electrode between them. A resistance in

* Brit. Assoc. Rep. 1886, p. 328.

† Wied. Ann. vol. i. 1877, p. 510.

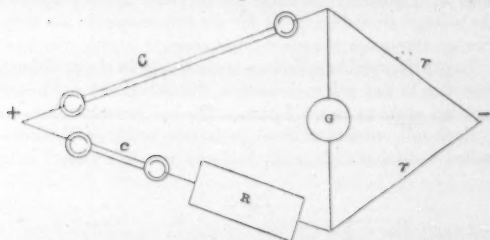
‡ Wied. Ann. xlv. p. 666.

series with the shorter column of liquid was adjusted to balance the bridge. The resistances so measured were of the order of 12 ohms or less. For many purposes the use of non-polarizable electrodes is impossible or inconvenient.

The method to be described is in many respects similar to those of Tollinger and Elsas, but it differs from them in the form of electrolytic cell employed, and especially in a very material point, viz.:—the employment of high voltages and high resistances so as to effectually drown any residual error arising from differential polarization.

Fig. 1 will make the arrangement clear, rr are equal resistances, Cc are two similar electrolytic cells equal in all

Fig. 1.



respects, except that in C the electrolytic conductor is very long, in c very short. The resistance-box R is adjusted till there is a balance, when, of course, the resistance in the box is equal to the difference in the resistances between the two cells C and c . Since the resistances rr are equal it will be clear that when approximate balance has been obtained equal currents will be traversing both electrolytic cells, and therefore there should be the same polarization in each cell, and these polarizations are clearly opposed and are in theory eliminated.

Instead of the arrangement shown in fig. 1, in which the cells are arranged in parallel, we have also tried the effect of interchanging the battery and galvanometer so as to arrange the cells in series. In the first case the polarization of one cell is opposed to that of the second, so that the polarization of each cell is much more persistent; in the other case the

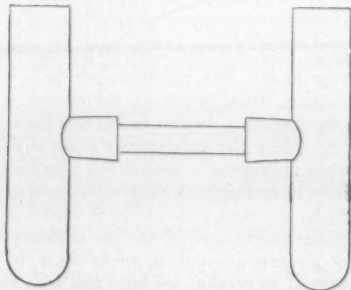
cells are joined in series, so that the polarization falls as soon as the battery circuit is broken. Experiment shows that the arrangement shown in the figure is much the better of the two.

In practice, however, it is found that the polarization in the two cells is not always exactly the same; generally speaking at least 99 per cent. of the polarization may be balanced in this manner, but ordinarily the balance is much more perfect than this. The next point then is to drown the residual differential polarization by working with high resistances and high voltages, and in this way the detrimental effects of polarization are reduced to insignificance. With electrolytic cells having a difference in resistance of about 20,000 ohms, the remaining arms being 1000 ohms each, and with a D'Arsonval galvanometer of about 300 ohms, when the voltage on the bridge is 30 the measurements are very like measurements of metallic resistances.

To get the specific resistance immediately in ohms, without reference to any other electrolyte, the cells C and c are constructed as shown in the figure.

Each cell consists of three parts, two being small, thick-walled test-tubes with necks half-way up their sides. Into

Fig. 2.



these necks fit the well-ground ends of a tube of nearly uniform bore. In the case of the cell C this tube is 30 centim. or more in length, while in the case of c it is only a few

centim. long. In every other respect the two cells are as nearly as possible alike.

The vertical limbs of the H cells were about 1.2 centim. in diameter, and about 6 centim. high, and the horizontal tubes about 0.6 centim. external diameter, the diameter of the bore being chosen to give a convenient resistance with the electrolyte used. The lengths of the tubes were about 30 centim. in the one case and 5 centim. in the other. The tubes were calibrated by measuring their lengths and weighing in a watch-glass the mercury required to fill them. In this way a constant can be obtained for a particular pair of tubes such that on multiplying by the observed resistance of the electrolyte its specific resistance is at once obtained.

In determining the weight of mercury required to fill the tubes we found that the results were most concordant when the finely ground ends of the tube were stopped by means of a small piece of thin cover-glass backed with cork about $\frac{1}{10}$ inch thick, the diameter of the cover-glass and cork being about the same as the external diameter of the tube, so that the pressure of one finger might suffice to keep the cover-glass firmly against the end of the tube. In this way any tilting of the cover-glass relatively to the end of the tube was avoided.

The following weighings of mercury and watch-glass for one pair of tubes were made: 29.836, 29.837, and 29.835 grms. for the long tube, and 11.482, 11.481, and 11.481 for the short one. The lengths of the tubes were 29.70 and 4.89 centims. From these data, assuming the density of mercury to be 13.558 at the temperature of the experiment, the constant for this particular pair of tubes was $\frac{1}{454.6}$.

If we were making fresh cells we should make them with the cross-tube near the foot instead of in the middle of the tubes. This would much facilitate mixing when a portion of the electrolyte is removed and replaced by water so as to alter the concentration. All that would then be necessary would be to tilt the cell two or three times to ensure effective mixing. The electrodes were pieces of platinum foil bent into cylindrical form to fit the vertical tubes of the cells. A platinum wire was welded on to each electrode and electrical connexion established with the arms of the bridge by the

intervention of a mercury cup. By this means the electrodes were easily removable from the cells.

The cells and mercury cups were mounted on a wooden stand placed in an oil bath, oil being used as it was found that with a water bath the apparent resistance of the electrolyte depended on the direction of the current. The explanation of this was traced to leakage over the glass surface and through the water, and was entirely obviated by the substitution of an insulating liquid for the fluid in the bath.

The following tables give the results of some experiments made with a solution of potassium chloride, the strength being $\frac{1}{2}$ molecular equivalent per litre. The arms rr of the bridge were each 1000 ohms, 30 volts were used, and the balancing resistance R was of the order of 20,000 ohms. Between each two measurements of resistance the electrodes were washed, heated to redness, replaced, and the current reversed. The method of making an experiment was as follows:—The current was made for about half a minute to polarize the plates, and then broken; two minutes were allowed to elapse to enable the electrolyte to assume the temperature of the bath, which was constantly stirred, and then the balance was obtained by momentarily making the current and adjusting the resistance.

Solution I.—The solution was made from KCl twice recrystallized from water and dried in the air between filter-paper. With Kohlrausch's notation we found

$$\frac{k}{\mu} = 1005.5 \times 10^{-8}.$$

We give no details of the experiment with this solution because the atomic weights used in making up the solution were only approximate, viz.:— $K=39$ and $Cl=35.5$. Correcting for this error the result would come about 0.1 per cent. higher. The salt, too, was dried in air, and it is probable that all the moisture was not removed, and any error resulting from this would cause the value obtained to be too low.

Solution II. (prepared for us with great care by Dr. Ewan).—The salt was prepared by fusing KClO_3 in a platinum crucible until no more gas was evolved. It was then recrystallized from dilute hydrochloric acid and dried by heating over a bunsen. The molecular weights used were $\text{K}=39.13$ and $\text{Cl}=35.45$. The strength was as before $\frac{1}{2}$ molecular equivalent per litre.

TABLE I.

Direction of current.	Temp.	Resistance.	Resistance reduced to 18° C.
+	18.26	21035	21149
-	18.19	21045	21129
+	18.17	21045	21119
-	18.11	21065	21113
+	18.08	21085	21120
-	18.06	21095	21121

The temperature coefficient used in the reduction was taken as 2.1 per cent. per 1° C.

The galvanometer gave a clear indication for a variation of 10 ohms.

$$\text{Specific Resistance} = \frac{21125}{454.6} = 46.47 \text{ ohm-centim. units.}$$

$$\text{Specific Conductivity in c.g.s. units} = 2151.9 \times 10^{-14}.$$

$$\text{Conductivity compared with mercury} = 2024.4 \times 10^{-9}.$$

$$\frac{k}{\mu} = 1012.2 \times 10^{-6}.$$

Solution III. (prepared by Dr. Ewan).—The details of preparation of this solution were similar to those employed in the previous case, except that the salt was not recrystallized from dilute hydrochloric acid, so that it is possible that a trace of chlorate or perchlorate might have been present, and this would tend to diminish the conductivity.

TABLE II.

Direction of current.	Temp.	Resistance.	Resistance reduced to 18° C.
+.....	18.11	21155	21204
-.....	17.98	21225	21214
+.....	17.84	21275	21204
-.....	17.70	21345	21211
			Mean 21208

whence as before $\frac{k}{\mu} = 1008.3$.

Kohlrausch's latest result for the same strength of solution of the same salt at the same temperature is 1009, Bouty's value is 1035, Krannhal's 1003 (obtained by graphical interpolation from the data given in Ostwald's *Lehrbuch der allgemeinen Chemie*, p. 732).

Let us now investigate the different errors which are present in the determination, and assign to each its relative importance. Taking the numbers given in Table I., the mean error of observation comes to be ± 4.8 , and the probable error ± 3.2 . The mean error of a single observation is 12, and the probable error of a single observation is 8, so that taking only one determination, R is probably correct to 1 in 2000 or $\frac{1}{20}$ per cent. The probable error of the mean is about 1 in 7000.

Another source of error would arise if the heat caused by the passage of the current through the electrolyte had not sufficient time to escape into the liquid in the bath. The current in our experiments would produce a rise of temperature of $\frac{1}{2}$ ° C. per minute, but the surface exposed is so enormous compared with the mass of electrolyte in the cross-tube that calculation shows that the difference in temperature between bath and electrolyte after vigorous stirring for two minutes is infinitesimal.

The temperature of the bath was read by means of a thermometer graduated in tenths of a degree, and standardized

by comparison with a thermometer certified by the Reichsanstalt. It was, however, difficult to read the thermometer with certainty to less than $\frac{1}{50}^{\circ}$ C. The error arising from this cause might amount to $\frac{1}{25}$ per cent.

We estimate the possible error in the calibration of the tubes to amount to not more than $\frac{1}{20}$ per cent., but this source of error arises largely from the fact that we had not in our possession apparatus for measuring the lengths of the tubes to less than 0.01 centim. With suitable apparatus there would be no difficulty in determining the constant for the tubes with much greater accuracy.

The greatest source of error appears to be a chemical one, viz.:—the preparation of the solutions, for while the one solution prepared in one way by Dr. Ewan gave 1008.3 for the conductivity, the second prepared by a different method gave 1012.2.

We have also made experiments to see whether the alternating current method of measuring electrolytic resistance would be improved by adopting the balancing-cell which we have described. To express the results very briefly, we find that for resistances not greater than 1000 ohms or thereabouts it is distinctly an improvement, enabling dead silence to be obtained in the telephone, while without the balancing-cell there was always a feeble buzz. For resistances much greater than 1000 ohms the introduction of the balancing-cell seemed of no avail whatever. We did not pursue these experiments with alternating currents very far, because it seemed clear that there was very little chance of getting as great accuracy as with the continuous current method described.

Conclusions.

1. The form of electrolytic cell described is very suitable for determining the specific conductivity of electrolytes directly without reference to the physical properties of any second electrolyte.

2. The method of measuring the conductivity of electrolytes described in this paper is, in our opinion, more convenient and more accurate than the method in which alternating currents are used.

DISCUSSION.

Prof. PERRY asked if the authors had tested whether the difference in resistance of the two cells was proportional to the difference in length of the liquid columns.

Mr. APLEYARD said he had found that the resistance of an electrolyte appeared to vary, because in the ordinary arrangement the cell was short-circuited through the arms of the bridge. He suggested as a remedy the making and breaking of the circuit by a special key, so arranged that except when taking a reading the cell is on open circuit.

Mr. BLAKESLEY asked if the authors had tried the method in which the resistances are adjusted till, when the battery-circuit is broken, there is no immediate change in the galvanometer deflection. It is possible by this method to measure a resistance of between 6,000 and 10,000 ohms to within 0.1 per cent.

Prof. AYRTON said the method referred to by Mr. Blakesley was the ordinary "false zero" method. In using this method you were working to a continuously altering zero; in Prof. Stroud's method, however, the zero was constant.

Mr. APLEYARD said he had found the "false zero" method troublesome to use.

Prof. STROUD, in reply, said they had not tested the proportionality between the resistance and length, and they had not tried the "false zero" method.

III. *Absorption of Electric Waves along Wires by a Terminal Bridge.* By EDWIN H. BARTON, D.Sc., F.R.S.E., Senior Lecturer in Physics at University College, Nottingham, and GEO. B. BRYAN, B.Sc., "1851 Exhibition" Research Scholar*.

IN the discussion on Mr. Yule's electrical paper last year†, one of us stated his intention of endeavouring to realise experimentally the extinction of electric waves along wires on their arrival at the end of the line, the urgent desirability of this having been felt by several investigators‡. The question is probably of interest also in connexion with Telephony. This task, small as it may seem, we had no opportunity to attack till this summer. The adoption of a resistance-bridge for the absorption of the electric waves was originally suggested by Mr. Oliver Heaviside's mathematical proof§ that, given a bridge of suitable resistance at the end of a line, then the waves arriving there would be immediately absorbed; "the electricity is all gobbled up at once, so to speak"||. And, a few preliminary difficulties being overcome, this important theoretical conclusion readily received experimental confirmation.

Experimental Arrangement.—The electric vibrations were generated by an oscillator of the type and size previously used by Mr. V. Bjerknes¶ and others. The waves, about $8\frac{1}{2}$ metres long, were propagated along a "line" consisting of a pair of parallel copper wires 1.5 mm. in diameter and 8 cm.

* Read November 13, 1896.

† G. U. Yule, "On the Passage of an Oscillator Wave-Train through a Plate of Conducting Dielectric," Proc. Phys. Soc. vol. xiii. pp. 358-392 (1895); Phil. Mag. [5] xxxix. p. 309.

‡ See e.g. J. von Geitler, pp. 7-8, 'Inaugural Dissertation,' Bonn, 1893; E. H. Barton, pp. 70-71, Proc. Roy. Soc. vol. lvii. 1894; G. U. Yule, Proc. Phys. Soc. t. c. pp. 384-387; Phil. Mag. t. c. pp. 334-337.

§ 'Electrical Papers,' vol. ii. pp. 127 and 132-133.

|| *Loc. cit.* p. 127.

¶ V. Bjerknes, Wied. Ann. vol. xlv. pp. 519-520 (1891); E. H. Barton, Proc. Roy. Soc. vol. liv. p. 86 (1893); G. U. Yule, Proc. Roy. Soc. vol. liv. p. 97 (1893).

apart. The waves were detected and relatively measured by an attraction-electrometer made on the principle of that devised by Herr von Geitler*. The length of the line to the electrometer was 116 m. This instrument has a needle of aluminium foil suspended by a quartz fibre between disks which may be connected to the respective wires of the line. On the passage of a wave-train the needle, initially uncharged, is charged inductively, and its ends are accordingly attracted to the disks whatever the sign of their potential-difference.

The electrometer therefore responds by a deflexion of the needle towards its disks. Let us call this a positive deflexion. In addition to the needle just described, and fixed to it in the same vertical plane, is a second similar one, but with its pair of attracting disks so situated that when they alone are connected to the line a negative deflexion ensues. When both needles have their respective disks connected to the line the electrometer is said to be used differentially. When so used, and with the instrument in adjustment, no deflexion is produced by the simple passage of a wave-train. Suppose now, owing to anything on the line which causes reflexion, stationary waves are produced at the electrometer. Then, provided one needle be connected to a point on the line which is a *node* for the waves, and the other be at a *loop*, the electrometer must give a deflexion and thus prove that *reflexion* of the waves has occurred. It is easily seen that when a single needle is used the first throw is proportional to the time-integral of the square of the potential-difference of the two leads of the line at the place to which the disks are connected.

The electrometer-needle carried a light plane mirror, and the throws were read by a distant telescope and scale.

Theory of Absorption.—Mr. Heaviside's † theory shows that in the case of plane electric waves proceeding along parallel leads to a bridge at the end of the line, the ratio of the potential-differences of the two leads due respectively to the reflected and incident waves is given by

$$\rho = \frac{R - Lv}{R + Lv}, \quad \dots \dots \dots (1)$$

* Wied. Ann. vol. xlix. p. 188 (1893).

† 'Electrical Papers,' vol. ii. pp. 132-133.

where ρ is the ratio in question, or *reflexion coefficient*, R is the resistance of the bridge, which must be of negligible inductance, L is the inductance per unit length of the line, and v is the velocity of light. Electromagnetic units are to be understood throughout.

Thus,

$$\rho = -1, \text{ for } R=0, \dots \dots \dots (2)$$

$$\rho = +1, \text{ for } R=\infty, \dots \dots \dots (3)$$

$$\text{and } \rho = 0, \text{ for } R=Lv. \dots \dots \dots (4)$$

Now $v=3 \times 10^{10}$ cm. per sec. nearly, and in the present case $L=4 \log \left(\frac{8}{0.075} \right) = 18.68$ nearly. Hence the critical value of R to give complete absorption is,

$$R=3 \times 10^{10} \times 18.68 \text{ absolute units of resistance} = 560 \text{ ohms. } (5)$$

Details of Bridge. At first an ordinary resistance-box was used as a bridge across the two wires of the line. But this was soon rejected lest the faces of the two bars at a plug opening should act as the plates of a condenser of appreciable capacity, which is inadmissible in the bridge desired. We next wound a small coil non-inductively on a bobbin in the usual manner and used it without any box or screw terminals. It was of well-covered wire and further insulated by thorough soaking in paraffin-wax both before and after winding. But this too proved unsatisfactory. For a time it stood, but then the insulation broke down and sparks were seen to pass between different turns of the wire. It was accordingly abandoned.

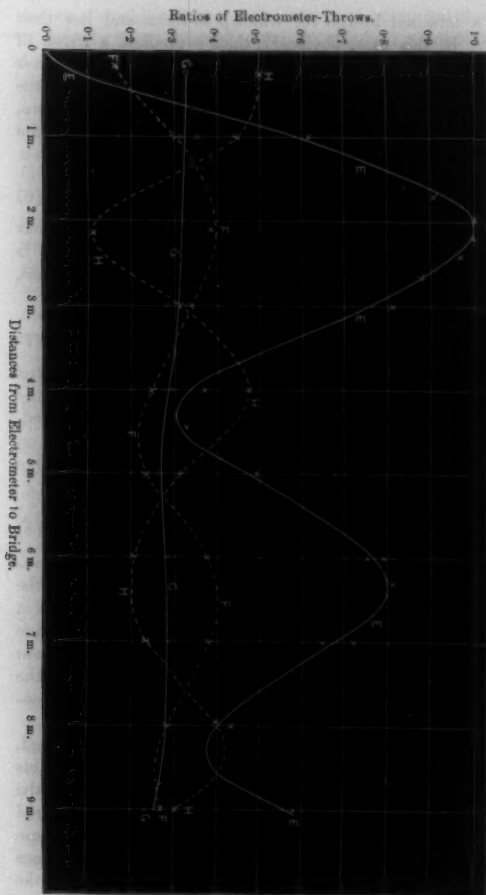
The simple expedient of making an electrical resistance by rubbing a common lead pencil on a disk of ground glass, small pieces of tin-foil gripped on by binding-screws forming the terminals, was then tried, and seemed in every way satisfactory. The electrostatic capacity of the terminals was proved to be negligible by testing such a disk and screws without any pencil. In this case no reflexion of the waves could be detected. Moreover, by making the track of the pencil-marks between the terminals very wide, the self-induction of the bridge was reduced to a minimum. It was also found easy to adjust the resistance to a few ohms

and to restore it nearly to its original value after the change which spontaneously occurred in it from day to day. Three bridges of this type were used, of resistances 261, 549 to 560, and 1336 to 1355 ohms. Let these be denoted respectively by the letters F, G, and H. Finally, since the pencil-markings on the glass form an extremely thin sheet, the resistance may be assumed to be practically the same to high-frequency waves as to the steady current by which they were measured on the Post-office box.

Experiments.—The oscillator emits rapidly-damped electric vibrations, only the first dozen, say, being appreciable. We thus have, advancing along the line with the speed of light, a damped wave-train, its large end, or head, leading and its tail, after about twelve waves, being negligibly small. Suppose now one needle of the electrometer to be connected to the line and a bridge of no resistance to be placed at the end of the line a little beyond the electrometer. We shall then have, at the electrometer, stationary waves due to interference between the incident waves and those reflected at the bridge. Hence, if a series of readings be taken with the electrometer at different distances from the end of the line the throws will be found to periodically wax and wane. But when the bridge is distant a few wave-lengths from the electrometer, we have the head of the wave-train interfering with the tail only. And with somewhat greater distances between electrometer and bridge the interferences of the waves and, consequently, the waxing and waning of the electrometer-throws cease to be appreciable. Thus, if the distances between electrometer and terminal bridge are plotted as abscissæ and electrometer throws as ordinates, we should expect the experiment to yield a damped wavy curve. And this is the case, as first shown by Mr. V. Bjerknes*, and utilized by him and his successors to determine the wave-length of the oscillations in use. The result of this experiment in our case is shown by the full-line curve E on the diagram.

In the case just considered the reflexion coefficient, ρ , of the bridge is -1 , as shown by equation (2). The waves are therefore unchanged in magnitude by the act of reflexion. Turn now to the general case of a precisely similar experiment

* Wied. Ann. vol. xliv. pp. 522-523 (1891).



Curves showing Interference due to Reflexion.
Line G G, being almost straight, indicates complete absorption by the corresponding bridge.

with a terminal bridge for which ρ is finite and less than unity. In this case, obviously, theory predicts a curve of similar form, wavy and damped, but lying between narrower limits, since the reflected wave is now always smaller than the incident one. Such an experimental result may, therefore, be accepted as proof that the bridge in use diminishes the wave in the act of reflexion.

Further,

$$\text{for } R < L_v, \rho \text{ has a negative value, } \dots (6)$$

and consequently the maxima and minima of the curves occur for the same values of the abscissæ as when $R=0$. This is seen to be the case with the dotted curve, F, on the diagram, obtained with the bridge whose resistance was 261 ohms.

Again,

$$\text{for } R > L_v, \rho \text{ has a positive value; } \dots (7)$$

hence this curve is, broadly speaking, an inversion of that just considered, the maxima occurring for those values of the abscissæ at which the minima of the curve for $R=0$ occurred, and *vice versâ*. This is exemplified in the dotted curve, H, on the diagram, resulting from the experiment with the bridge whose resistance was of the order 1350 ohms.

Finally, we have

$$\text{for } R = L_v, \rho = 0. \dots (8)$$

In this case, since no waves are reflected, no stationary waves can obtain, and consequently no waxing and waning of the electrometer-throws can occur. Thus theory predicts a straight line as the outcome of an experiment with a bridge of this critical resistance. This is approximately the case with the full-line curve, G, on the diagram, obtained with the bridge whose resistance was of the order 560 ohms, the theoretical value of the critical resistance for the line used.

Of course an absolutely straight line cannot be expected, since the sparks in the oscillator vary slightly. For this reason, throughout the experiments hitherto considered the throws with the bridge under examination at any desired distance beyond the electrometer were alternated with those obtained with a no-resistance bridge at the standard distance of a quarter-wave-length beyond. And the ordinates of the curves on the diagram represent, not the actual electrometer-

throws with the bridge and distance in question, but the ratio of these to those obtained under the standard conditions just described. Thus the maximum ordinate in the diagram is unity, all the curves are reduced to the same scale, and the errors due to variations of the sparking at the oscillator are in great measure eliminated. In the above experiments the electrometer was used as a single-needle instrument, and the throws obtained varied up to 100 scale-divisions.

As a further test of the non-reflexion of waves by the bridge G, the electrometer was afterwards used differentially, the upper and lower needles having their disks attached to points on the line a quarter-wave-length apart. The bridges were placed at the end of the line 0.25 m. beyond the second needle. With a no-resistance bridge the throws were positive and of the order 50 scale-divisions; with a simple disconnection ("infinity bridge") the throws were of the same order but negative. The bridges F and H gave smaller throws, positive and negative respectively. The bridge G also appeared to yield a small throw; but this was no larger than might be due to a creep of the zero or an imperfection in the adjustment of the electrometer-needles. The use of the differential electrometer is thus seen to confirm the results of the experiments conducted with a single needle.

It should be noted that this paper deals only with the absorption of waves by *terminal* bridges; no *intermediate* resistance-bridge will absorb all the waves arriving there. The reflexion and transmission coefficients ρ' and τ' for an intermediate bridge of resistance R are given by*

$$\rho' = \frac{-Lv}{2R + Lv} \quad \text{and} \quad \tau' = \frac{2R}{2R + Lv}, \quad \dots \quad (9)$$

the notation being the same as that previously used. Thus the condition for non-reflexion is $R = \infty$, in which case all is transmitted. Hence complete absorption is impossible.

If, therefore, we wish to pass electric waves along wires through any layer of dielectric, or through any other arrangement which affects them, and are desirous of avoiding the annoying disturbance due to return of the transmitted portion

* Oliver Heaviside's 'Electrical Papers,' vol. ii. pp. 141-142.

after reflexion at the end of the line, the following method will suffice.

1. Place at the end of the line a bridge of the form herein described, whose resistance has the value given by the theory for plane waves.

2. Test experimentally the absorbing-power of the bridge, and, if necessary, adjust its resistance until no reflexion can be detected.

University College, Nottingham,
September 1896.

DISCUSSION.

Mr. CAMPBELL asked if the resistances given were expressed in ohms or in electromagnetic units.

Mr. BIDWELL asked if the authors had found that the pencil-trace resistances obeyed Ohm's law. He found that if you balance with one cell in the battery circuit, then, on increasing the battery-power to two cells, the resistance alters.

Mr. APPLEYARD suggested that the variation was caused by the contacts at the ends not being good.

Mr. CAMPBELL said the same variation occurred in the case of mixtures of clay and plumbago where the contacts could be made quite good.

Mr. CARTER suggested electroplating the ends to give good contact.

Mr. BRYAN, in reply, said that in their case they did not require to know the resistance very accurately.

IV. *Applications of Physics and Mathematics to Seismology.**By C. CHREE, Sc.D.**

Sections

CONTENTS.

- 1-3. Introductory.
- 4. Influence of surface-load on observed level.
- 5. Direct pressure effect.
- 6-7. Relation between pressure and gravitational effects.
- 8-10. Pure pressure effects, general formulæ, application to rectangle.
- 11-13. Special cases of loaded rectangle.
- 14-18. Numerical results, and conclusions.
- 19-20. Pressure effects below the surface.
- 21-23. Luni-solar effects, explanatory.
- 24-26. Solution of problem.
- 27-30. Application to earth, working formulæ.
- 31. Numerical estimates.
- 32. Final conclusions.
- 33-34. Subsidiary remarks.

Introductory.

§ 1. THE existence of apparent movements in the earth's surface-strata appeals in the first instance to seismologists. Prof. J. Milne, however, and others have been of late attempting to bring it home to astronomers and meteorologists that they too may have a vital interest in the matter. The presentations of the subject which have come under my notice take little or no heed of the theoretical aspects of the case, in which, as an elastician, I have long been interested. As the neglect of theoretical results may be due not so much to their defects as to the slowness with which mathematical investigations become generally known, I have decided to group together and discuss in a more or less popular way the theoretical conclusions which seem to me the most closely connected with the subject in question.

§ 2. The mathematical work by which these conclusions were deduced refers to material which is homogeneous, isotropic, and elastic, while the body in whose phenomena the seismologist is interested is the earth.

Now it must not be supposed that I fail to appreciate the differences between the material of theory and that of nature.

* Read December 11, 1896.

The certainty of the departure of many of the surface-strata from the attributes ascribed to isotropic elastic solids, and the uncertainty as to the density, solidity, and other physical properties of $\frac{220}{1000}$ of the earth's mass I perfectly realize. The conditions under which the deep-seated materials of the earth exist are fundamentally different from those we are familiar with at the surface. The enormous pressure, and the presumably high temperature, very likely combine to produce a state to which the terms solid, viscous, liquid, as we understand them, are alike inapplicable. But be the state what it may, the material must respond to the action of periodic forces; such forces must produce varying strains and stresses; and these strains and stresses can hardly fail to produce effects at the surface. No numerical estimate of these effects can claim to be in any sense final, as the mathematical work by which it is evolved must depend on physical data which are at best unproved. It appears desirable, however, that such numerical estimates should be made, on the least objectionable physical basis available, if only for the reason that their existence supplies a guide and incentive to direct observation.

In my opinion, for reasons previously discussed*, the treatment of the earth as an incompressible elastic solid is exposed to perhaps a minimum of objections. Most probably the material increases in density and temperature as we approach the centre, and a treatment which assumes the material to vary with the radial distance would possess higher *à priori* claims to regard than one which treats the earth as homogeneous throughout. The problem of a gravitating mass of varying elastic properties has, however, still to be published, and as the assumed law of variation would probably be mainly guess-work, the advantages for practical ends might be less than would appear at first sight.

Variation of the material with the radial distance, I may add, could hardly affect the general character of the phenomena. Surface heterogeneity, in which the material varies rapidly with latitude or longitude, is not unlikely to modify largely the magnitude of some of the results at individual stations,

* Phil. Mag. Sept. 1891, p. 232.

but is most unlikely to produce a large effect on the order of magnitude of the mean lunar or solar tidal effects at a moderate number of stations scattered over the earth's surface.

The treatment of local surface pressures in the first part of the paper is in some respects on a less uncertain basis. We can assure ourselves, if need be, of the solidity of the ground surrounding a station; and though the mathematical work treats the solid as going down to infinity, this only means in practice that the depth must be large compared to the shortest distance of the loaded area. There are reasons, however, even here for regarding the numerical results as in general but rough approximations.

They might possess high accuracy if the surface material were bare rock in horizontal strata, and the recording apparatus were supported directly on the rock; but uncertainties are introduced when the load is applied at the surface of ordinary soil, and the support of the apparatus is stonework inside a building whose foundations go to an appreciable depth.

§ 3. The observed facts on which our investigations are most likely to bear are certain slow changes in the indications of spirit-levels or delicately suspended pendulums. Some of these Prof. Milne is disposed to attribute to meteorological agencies such as rainfall or evaporation.

A relative excess of moisture to the west, say, of an observatory is, he considers, equivalent to a surface load on that side, tending to make the ground on which the building stands slope downwards from east to west. Such want of symmetry may arise from the peculiarities of the soil, or through the ground being sheltered by trees or modified by cultivation. In sunshine the shadow of the building itself, by retarding evaporation, may set up such a difference as Prof. Milne has in view.

I am not sure that an excess of evaporation from the east, say, of a building is necessarily equivalent to the withdrawal of a surface load from that side, at least to the exact extent of the surplus evaporation. The withdrawal of moisture from the soil has a decided influence on its conductivity for heat—not to speak of electricity—and so may exert a very appre-

ciable influence on the temperature near the surface, the consequences of which it would be difficult to follow. There is also, presumably, underground circulation both of air and moisture, which may not unlikely counteract to some extent differences of surface evaporation.

Though these and other uncertainties exist, it is certainly worth while considering the numerical magnitude of the results to be expected from the agencies postulated by Prof. Milne. The theoretical results will also, I hope, suggest the way in which the best use may be made of experimental determinations of the effects of surface loading over limited areas of convenient shape.

Influence of Surface Load on the observed Level.

§ 4. This influence is not so simple as might appear at first sight. The weight of the loading material is equivalent to a pressure normal to the surface, which we suppose horizontal. But, in addition, we must allow for the fact that the gravitational attraction of the loading material slightly alters the direction of "gravity" at the surface. Consider, for example, the influence of the ordinary ocean tide at a point inland near the shore. At high tide there is on the seabottom a pressure exceeding the mean by an amount corresponding to the height of the water above its mean level; this will tend to make a naturally horizontal plane dip towards the sea. At the same time the surplus volume of water will give a horizontal component to what we may regard as normal "gravity" in the neighbourhood. This second effect has been called attention to, in this very case, in Thomson and Tait's 'Natural Philosophy,' art. 818, where will be found a numerical estimate for a specified set of conditions. What a spirit-level shows is the plane perpendicular to gravity—including "centrifugal force" and all disturbing forces. We are thus obliged to consider both effects before attempting numerical estimates.

Direct Pressure Effect, Fundamental Formulæ.

§ 5. In the following calculations the earth is treated as an isotropic elastic solid, principal weight being attached to the results obtained by supposing the material incompressible.

Also, as we are primarily interested in the consequences of pressure applied over limited areas, the loaded surface is treated as a horizontal plane, on the lower side of which the material extends to infinity. On these hypotheses we are enabled to make use of the very interesting and important results established by Professors Cerruti and Boussinesq.

A convenient English abstract* of Boussinesq's work is contained in Todhunter and Pearson's 'History of Elasticity,' vol. ii. part 2, arts. 1492 *et seq.*, from which the following formulæ are quoted, the only variation being the use of Thomson and Tait's notation for the elastic constants.

The origin of coordinates lies in the undisturbed surface, taken as the plane of xy , the positive direction of the z axis being downwards into the earth. The normal pressure applied to the element $d\omega$ of surface is $p d\omega$, where p is supposed of course a known function of x, y .

u, v, w denote the components of elastic displacement, n the rigidity, and η Poisson's ratio for the material.

The displacements at any point x, y, z in the material are as follows:

$$u = -\frac{1}{4\pi n} \left\{ \frac{d^2}{dx dz} \iint r p d\omega + (1-2\eta) \frac{d}{dx} \iint \log(z+r) p d\omega \right\}, \quad (1)$$

$$v = -\frac{1}{4\pi n} \left\{ \frac{d^2}{dy dz} \iint r p d\omega + (1-2\eta) \frac{d}{dy} \iint \log(z+r) p d\omega \right\}, \quad (2)$$

$$w = \frac{1}{4\pi n} \left\{ 2(1-\eta) \iint \frac{p}{r} d\omega + z^2 \iint \frac{p}{r^3} d\omega \right\}. \quad \dots \dots \dots (3)$$

Here r is the distance between the element $d\omega$ where p is applied, and the point x, y, z where u, v, w are measured. The integration extends to all parts of the surface where p differs from zero.

The simplification in the formulæ when $\eta = \frac{1}{2}$, or the material is incompressible, should be noticed.

The *slope*—i. e., inclination to the plane of xy —introduced into any horizontal plane depends only on the vertical displacement w . In particular, the slope of the surface depends

* Chapter ix. vol. i. of Love's 'Treatise on... Elasticity' may also be usefully consulted.

only on w_0 , the value of the vertical displacement when $z=0$; and by (3) we obviously have

$$w_0 = \{(1-\eta)/(2\pi n)\} \iint (p/r) d\omega. \quad (4)$$

In (4) r is the distance between the element $d\omega$ and the point x_0, y_0 on the surface to which w_0 refers.

Relation between Pressure and Gravitational Effects.

§ 6. If we suppose t the thickness, ρ the density of the material loading the surface, its gravitational forces are derived from the potential

$$V = \gamma \iint (t\rho/r) d\omega, \quad (5)$$

where γ is the attraction between two unit masses at unit distance.

The pressure exerted by the load is

$$p = g\rho t,$$

where g is gravity at the surface. Here we may regard g as the undisturbed value prior to the application of the load, as the alteration in the vertical component is negligible for our present object. Thus

$$V = (\gamma/g) \iint (p/r) d\omega.$$

Comparing this with (4), we find for the surface value V_0 of V the simple relation

$$V_0 = 2\pi n \gamma w_0 \{g(1-\eta)\}. \quad (6)$$

This holds true of V_0 and w_0 all over the surface, and so applies likewise to their differential coefficients with respect to x_0 and y_0 —so far at least as concerns points outside the loaded area.

The direction-cosines of the normal to the surface after the application of the load are, to a first approximation,

$$\frac{dw_0}{dx_0}, \quad \frac{dw_0}{dy_0}, \quad 1;$$

so that the slope at the point x_0, y_0 is given by

$$\psi_1 = \left\{ \left(\frac{dw_0}{dx_0} \right)^2 + \left(\frac{dw_0}{dy_0} \right)^2 \right\}^{\frac{1}{2}} \quad (7)$$

Again, to a first approximation the presence of the loading-

material has altered the direction-cosines of the line of action of gravity from

$$0, 0, 1 \text{ to } \frac{1}{g} \frac{dV_0}{dx_0}, \frac{1}{g} \frac{dV_0}{dy_0}, 1.$$

Thus gravity has become inclined to the vertical at the angle

$$\psi_2 = \frac{1}{g} \left\{ \left(\frac{dV_0}{dx_0} \right)^2 + \left(\frac{dV_0}{dy_0} \right)^2 \right\}^{\frac{1}{2}} \dots (8)$$

Employing (6) in (7) and (8), we obtain the elegant relation

$$\psi_1/\psi_2 = (1-\eta)g^2/(2\pi n\gamma) \dots (9)$$

The spirit-level measures $\psi_1 + \psi_2$, which always exceeds the true change of level ψ_1 .

Since

$$\frac{dw_0}{dx_0} / \frac{dw_0}{dy_0} = \frac{dV_0}{dx_0} / \frac{dV_0}{dy_0}, \dots (10)$$

the final directions of gravity and of the normal to the surface lie in the same vertical plane (*i.e.* plane through z). This result may facilitate experimental investigations, as a rough idea of the direction of the resultant attraction of the loading-material will generally be obtainable by eye. The possible influence of want of symmetry in the contour of the ground, or variability of the surface-strata, must of course be borne in mind.

The relation (9), so far as I know, is new. Its discovery was due to a faint impression that a formula I had obtained for the effect of a loaded rectangle resembled something I had seen before, the something proving on investigation to be result (7) in Thomson & Tait's *Nat. Phil.*, art. 818.

§ 7. To form an idea of the relative importance of ψ_1 and ψ_2 in the case of the earth, I have made the following selection of hypothetical values for η and n , the latter quantity being measured in grammes weight per square centimetre:—

(i.)	$\eta = .25,$	$n = 80 \times 10^7;$
(ii.)	$.25,$	$35 \times 10^7;$
(iii.)	$.5,$	$11 \times 10^7.$

According to the table in Lord Kelvin's *Encyclopædia Article on Elasticity*, (i.) may be regarded as representing

iron or steel, (ii.) as representing brass or slate of somewhat low modulus, while (iii.) represents an incompressible material* such as seems most compatible with the hypothesis of a homogeneous earth, naturally spherical but for rotation. According to Lord Kelvin's table, the value of n in (iii.) is similar to what one should expect in rock of somewhat low elasticity.

If a be the earth's radius, $\bar{\rho}$ its mean density,

$$g/\gamma = 4\pi\bar{\rho}a/3.$$

Supposing $\bar{\rho} = 5.5$, and $a = 64 \times 10^7$ cms., we have approximately $g\bar{\rho}a = 35 \times 10^8$ grammes weight per square centimetre.

With the above figures, I find

$$\begin{array}{ll} \text{case (i.)} & \psi_1/\psi_2 = 35/16 = 2 \text{ roughly,} \\ \text{(ii.)} & 10/2 = 5, \\ \text{(iii.)} & 350/33 = 11 \text{ roughly.} \end{array}$$

The last result is likely, I think, to prove the nearest to what ordinarily occurs in practice, so that the gravitational effect may be expected to prove as a rule relatively small; still it ought not to be disregarded without due consideration of the special circumstances.

Pure Pressure Effects.

§ 8. In all cases when the largest dimension of the loaded area is small compared to its shortest distance from the point of the surface where the slope is required, a good first approximation † to the surface vertical displacement—obvious on inspection of (4)—is

$$w_0 = (1 - \eta)P/(2\pi nR), \quad \dots \dots (11)$$

where R denotes the distance from the centre of mass of the total load P .

The corresponding approximation to the slope, viz.,

$$-\frac{dw_0}{dR} = (1 - \eta)P/(2\pi nR^2), \quad \dots \dots (12)$$

shows that at considerable distances from a small loaded area the slope varies approximately as the inverse square of the

* See Phil. Mag. Sept. 1891, p. 250, remembering $E/n = 2(1 + \eta)$.

† See Todhunter & Pearson's 'History,' vol. ii. pt. 2, art. 1498.

distance. In (11) and (12) the distribution of load is not assumed uniform.

The fact that (11) holds only when the distance of the loaded area is so large that its effect is relatively small diminishes its value in practice.

§ 9. The determination of w from (3) entails the evaluation of two integrals, neither very manageable. For points on the surface there is, however, only the single integral (4). This has been converted by Boussinesq into two alternative forms—one for points outside, the other for points inside the loaded area—which are convenient when the load, though not necessarily uniform, is distributed symmetrically round a point (see Todhunter & Pearson's 'History,' arts. 1501 and 1502). In this way the depression can be easily determined at the centre and edge of a circular depressed area for a variety of laws of loading, and the depression at other points can be expressed in terms of infinite series or elliptic integrals (see Todhunter & Pearson, *l. c.*, especially art. 1504).

The slope in these cases, at any distance from the centre of the loaded area, can be obtained in the form of infinite series or elliptic functions; but results of this kind are more apt to repel than to enlighten the unmathematical reader.

Fortunately, when the load is uniform, and the loaded area rectangular, it proves possible to express the components of slope dw/dx , dw/dy at any point of the surface in terms of ordinary Napierian logarithms. I shall accordingly devote attention almost exclusively to this case.

§ 10. Returning to (3), let x' , y' be coordinates of the element $d\omega$, so that

$$d\omega = dx' dy',$$

$$r^2 = (x - x')^2 + (y - y')^2 + z^2.$$

The loading being supposed uniform, we have

$$\frac{4\pi n}{p} \frac{dw}{dx} = 2(1 - \eta) \left(\iint \frac{d}{dx} \left(\frac{1}{r} \right) dx' dy' + \iint z^2 \frac{d}{dx} \left(\frac{1}{r^3} \right) dx' dy' \right).$$

But

$$\frac{d}{dx} \left(\frac{1}{r} \right) = - \frac{d}{dx'} \left(\frac{1}{r} \right), \text{ and } \frac{d}{dx} \left(\frac{1}{r^3} \right) = - \frac{d}{dx'} \left(\frac{1}{r^3} \right);$$

hence

$$\frac{4\pi n}{p} \frac{dw}{dx} = 2(1-\eta) \int \left(\frac{1}{r_1} - \frac{1}{r_2} \right) dy' + z^2 \int \left(\frac{1}{r_1^3} - \frac{1}{r_2^3} \right) dy', \quad (13)$$

where r_1 and r_2 are the inferior and superior limits of r in the integration with respect to x' .

Suppose the origin vertically over the point where the slope is to be found, or $x=y=0$, and draw the axes of x and y parallel to the sides of the loaded rectangle. Take for the coordinates of the corners of the rectangle—

$$x_1, y_1; \quad x_2, y_1; \quad x_2, y_2; \quad x_1, y_2;$$

and suppose

$$x_2 > x_1, \quad y_2 > y_1.$$

The following result is then easily obtained from (13):

$$\begin{aligned} \frac{4\pi n}{p} \left(\frac{dw}{dx} \right)_{x=y=0}^{x=z} &= 2(1-\eta) \log \frac{(y_2 + \sqrt{y_2^2 + x_1^2 + z^2})(y_1 + \sqrt{y_1^2 + x_2^2 + z^2})}{(y_1 + \sqrt{y_1^2 + x_1^2 + z^2})(y_2 + \sqrt{y_2^2 + x_2^2 + z^2})} \\ &+ z^2 \left\{ \frac{1}{x_1^2 + z^2} \left(\frac{y_2}{\sqrt{y_2^2 + x_1^2 + z^2}} - \frac{y_1}{\sqrt{y_1^2 + x_1^2 + z^2}} \right) \right. \\ &\left. + \frac{1}{x_2^2 + z^2} \left(\frac{y_1}{\sqrt{y_1^2 + x_2^2 + z^2}} - \frac{y_2}{\sqrt{y_2^2 + x_2^2 + z^2}} \right) \right\}. \quad (14) \end{aligned}$$

This combined with the corresponding expression for dw/dy , which can be written down by symmetry, supplies complete information as to the slope at all depths. By putting $z=0$ in (14), or by direct calculation, we get

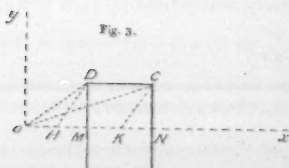
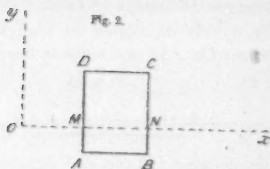
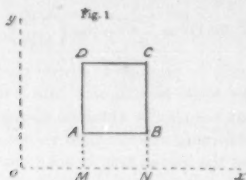
$$\frac{dw}{dx} \text{ (at origin)} = \frac{(1-\eta)p}{2\pi n} \log \frac{(y_2 + \sqrt{y_2^2 + x_1^2})(y_1 + \sqrt{y_1^2 + x_2^2})}{(y_1 + \sqrt{y_1^2 + x_1^2})(y_2 + \sqrt{y_2^2 + x_2^2})}, \quad (15)$$

where y_1, y_2 must be treated algebraically. Thus ABCD representing the loaded rectangle, DM, CN perpendiculars on O*x*, we have, in the case shown in fig. 1:—

$$\frac{dw}{dx} \text{ (at O)} = \frac{(1-\eta)p}{2\pi n} \log \frac{(OD+DM)(OB+BN)}{(OA+AM)(OC+CN)}; \quad (16a)$$

in the case shown in fig. 2 :—

$$\frac{dw}{dx}(\text{at } O) = \frac{(1-\eta)p}{2\pi n} \log \frac{(OD+DM)(OB-BN)}{(OA-AM)(OC+CN)}. \quad (16b)^*$$



In the case of symmetry shown in fig. 3, where the centre of the rectangle lies on Ox ,

$$(OD+DM)/(OA-AM) = (OD+DM)^2/OM^2, \text{ \&c.,}$$

and (16 *b*) reduces to

$$\frac{dw}{dx}(\text{at } O) = \frac{(1-\eta)p}{\pi n} \log \left(\frac{OD+DM}{OC+CN} \cdot \frac{ON}{OM} \right). \quad (17)$$

* An equivalent but somewhat longer form for the logarithm (leading directly, however, to [17]) is given in eqn. (7) art. 818 of Thomson and Tait's 'Natural Philosophy.'

In this last case of course $dw/dy=0$, and Ox is the line of greatest slope at O .

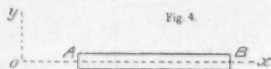
If in fig. 3 we draw DH and CK , bisecting the angles ODM , OCN , we easily throw (17) into the elegant form

$$\frac{dw}{dx} \text{ (at } O) = \frac{1-\eta}{\pi n} P \log \left(\frac{NK}{MH} \right). \quad (18)$$

It will frequently be possible to divide nearly the whole of a loaded area, not itself rectangular, into a small number of rectangles, so that the results obtained above could doubtless be utilized for obtaining approximate values of the slope in many cases where the loaded area is not rectangular.

Subcases when one Dimension of Loaded Rectangle small.

§ 11. In fig. 4, AB represents an elongated loaded area symmetrical about Ox . If we suppose the breadth $2b$ small



compared to the distance $OA=c$, and denote the length AB by $2a$, we easily deduce from (17) as a first approximation

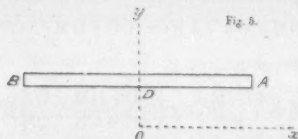
$$\frac{dw}{dx} \text{ (at } O) = (1-\eta)P \div 2\pi n OA \cdot OB, \quad (19)$$

where $P \equiv p \cdot 2a \times 2b$ is the total load over the area.

If, further, c be small compared to $2a$, we have

$$\frac{dw}{dx} \text{ (at } O) = (1-\eta)pb/\pi nc = (1-\eta)P \div 4\pi nac. \quad (20)$$

When (20) applies, the slope along the axis of symmetry varies inversely as the shortest distance from the loaded area.



§ 12. In fig. 5, AB represents an elongated area perpendicular to the axis of symmetry Oy .

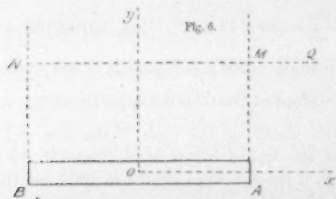
Supposing first that $OD(=c)$ and the breadth $2b$ are comparable, but both small compared to the length $2a$, we easily find from the formula corresponding to (17)

$$\frac{dw}{dy} \text{ (at } O) = \frac{(1-\eta)p}{\pi n} \log \frac{c+2b}{c}. \quad (21)$$

If, further, the breadth be small compared to the shortest distance from O , we reduce (21) to

$$\frac{dw}{dy} \text{ (at } O) = 2(1-\eta)pb/\pi nc = (1-\eta)P/2\pi nac, \quad (22)$$

where P denotes as before the total load. Comparing (20) and (22) we observe that for equal distances c , the position of the loaded area in fig. 5 is twice as effective as the position in fig. 4.



§ 13. In fig. 6 the elongated loaded area has its centre at the origin of coordinates, and the axes of Ox and Oy are along its length $2a$ and breadth $2b$ respectively. The slope is required at a point $Q(x, y)$ whose shortest distance from the area is considerable compared to b .

Draw through Q a parallel to AB cutting in M and N the lines AM and BN drawn perpendicular to AB .

From (15) and the corresponding equation we have as first approximations to the components of slope

$$-\frac{dw}{dx} = \frac{(1-\eta)pb}{\pi n} \left(\frac{1}{QA} - \frac{1}{QB} \right), \quad (23)$$

$$-\frac{dw}{dy} = \frac{(1-\eta)pbAM}{\pi n} \left\{ \frac{1}{QA(QA+QM)} - \frac{1}{QB(QB+QN)} \right\} \quad (24)$$

In (24), QM and QN are to be treated algebraically, and the formula must not be applied to cases in which either QA + QM or QB + QN tends to become very small (*cf.* § 11).

Numerical Illustrations.

§ 14. Suppose in the case of symmetry, illustrated by fig. 3, that the loaded area is a square 100 metres in the side, and that OM, its shortest distance from O, is 1 metre.

Suppose the load to arise from a sheet of water 1 cm. deep, or that

$$p = 1 \text{ gramme wt.}$$

As in case (iii.) of § 7, let us put

$$\eta = .5, \quad n = 11 \times 10^7 \text{ grammes wt. per sq. cm.}$$

Then we have approximately, in absolute measure,

$$\frac{dw}{dx} = (2\pi \times 11 \times 10^7)^{-1} \log_e(10100/163);$$

or, as unit angle = 206×10^3 seconds of arc approximately,

$$\text{slope at O} = 0''\cdot0012 \text{ approximately.} \quad \dots (25)$$

The result would be the same if the side and the least distance of the loaded square were altered in the same proportion, *e. g.* if the side were altered to 1000 and the shortest distance to 10 metres.

The slope increases directly as the load. It would, however, require an abnormally large differential rainfall or evaporation to appreciably influence by direct pressure a level inside a building situated on strata similar to the material of our calculation.

§ 15. The differential effect of barometric pressure during the approach or retirement of a deep cyclonic depression would appear a more probable disturbing cause. We might very easily have a mean differential excess or diminution of pressure of 1 or 2 cm. of *mercury* over an area whose greatest dimension was very large compared to the shortest distance from the observing station, and consequently effects 10 or 20 times that appearing in (25) might not unreasonably be expected in disturbed weather.

In the case of a large cyclonic area it would be desirable

to apply a formula applicable to a loaded *spherical* surface, but (17) would probably give a very fair idea of the order of magnitude of the result.

§ 16. As an illustration of a different kind, suppose in fig. 5 that O is a station near a long straight portion AB of a tidal river, and that we desire the difference of slope at O at high and low tide. It will suffice to take the difference of level at high and low water as the same all along AB. Suppose this difference to be 5 metres, and assume η and n to be the same as in the last example.

Taking first $c = 4 \times 2b$, we get approximately from (21)

$$\frac{dw}{dy}(\text{at } O) = \frac{5 \times 10^2 \times 7}{2 \times 22 \times 11 \times 10^7} \log_e 10 \times \log_{10}(1.25),$$

or in seconds of arc

$$\text{slope at } O = 0''.033. \quad (26)$$

Taking next $c = 2b$, we replace (26) by

$$\text{slope at } O = 0''.10. \quad (27)$$

If, for instance, the river be 100 yards broad, the first station is 400, the second 100 yards from the bank.

§ 17. As rivers are seldom straight, I have supplemented the above by calculating the slope at the centre of a semi-circular channel of width $2b$, supposed small compared to the radius R . For a difference h in the level of high and low water, I find as a first approximation

$$\text{difference of slope at centre of semicircle} = 2(1-\eta)gbh/\pi nR. \quad (28)$$

To aid the imagination, the river may be supposed to enter and leave the semicircle by straight channels forming continuations of the limiting diameter, so that the semicircular portion alone need be considered.

It will be noticed that (22) and (28) are identical if

$$c = R, \text{ and } p = gh.$$

In other words, the semicircular tidal river has exactly the same influence on the slope of the station at its centre as it would have if the channel were straight throughout and came within the same distance of the station.

§ 18. The results of the last two paragraphs point to changes

of water-level in tidal rivers and estuaries as more likely to appreciably affect the level of neighbouring observatories than any probable differential peculiarities of rainfall or evaporation. In making this observation I exclude of course the direct action of water on the foundations of the building.

In all cases similar to those treated in §§ 14-17 the direct gravitational action of the load must be taken into account to obtain the full result. For instance, in the case of an anti-cyclone, the horizontal attraction of the surplus air must be considered as well as the excess of pressure over the area covered by the anti-cyclone.

Pressure Effects below the Surface.

§ 19. As the foundations of most buildings are below the ground-level, the slope at some little depth possesses considerable interest. The general formula (14) for the depression at any depth due to a loaded rectangle, though easily evaluated for specified numerical values of z , x , &c., is somewhat complicated. Its general character will be sufficiently comprehended from the results in the case of symmetry, when

$$y_2 = -y_1 = b.$$

Putting

$$x_1 = c, \quad x_2 = c + 2a,$$

we then convert (14) into

$$\begin{aligned} \frac{4\pi\mu}{p} \left(\frac{dw}{dx} \right)_{z=z}^{z=y=0} &= 2(1-\eta) \log \frac{(b + \sqrt{b^2 + c^2 + z^2}) (-b + \sqrt{b^2 + (c+2a)^2 + z^2})}{(-b + \sqrt{b^2 + c^2 + z^2}) (b + \sqrt{b^2 + (c+2a)^2 + z^2})} \\ &\quad + 2bz^2 \{ (c^2 + z^2)^{-1} (b^2 + c^2 + z^2)^{-\frac{1}{2}} \\ &\quad - ((c+2a)^2 + z^2)^{-1} (b^2 + (c+2a)^2 + z^2)^{-\frac{1}{2}} \}. \quad (29) \end{aligned}$$

So long as z/c is small the right-hand side of (29) can be expanded in a rapidly converging series of the form

$$A + B(z/c)^2 + \dots,$$

where A and B are independent of z .

There is no tendency in B/A to become very large for finite values of a/c and b/c .

When we neglect $B(z/c)^2$ &c., we simply get the slope at

the surface. We thus see that at depths small compared to the shortest distance of the loaded area the slope is nearly the same as at the surface itself.

The value of B is easily obtained in special cases. As an example, take the sub-case of fig. 4, in which c/a and b/c are both supposed small. We then get for the slope

$$\left(\frac{dw}{dx}\right)_{z=y=0}^{z=z} = \frac{pb}{\pi nc} \left(1 - \eta + \frac{1}{2}\eta \frac{z^2}{c^2}\right). \quad (30)$$

In (30) constant terms of the order $(b/c)^2$ are omitted, though possibly more important than the variable term.

Again, in the sub-case of fig. 5, when b/c and c/a are supposed both small, we find

$$\left(\frac{dw}{dy}\right)_{z=z}^{x=y=0} = \frac{2pb}{\pi nc} \left(1 - \eta + \eta \frac{z^2}{c^2}\right). \quad (31)$$

In both these instances the slope *increases* with the depth. The formulæ hold only so long as z/c is small, so that the phenomenon is rather of theoretical than practical importance. Though somewhat opposed to *a priori* conceptions, this result would not appear exceptional. Thus, take the case of an isolated load $pd\omega$ at a point P on the surface, and consider the vertical displacement at a point Q at depth z . Join $QP=r$, and draw $QM=x'$ perpendicular on the vertical PM . Then denoting the angle QPM by α , we have at Q

$$w = (pd\omega/4\pi nr) \{2(1-\eta) + \cos^2 \alpha\}, \quad (32)^*$$

$$= (pd\omega/4\pi nx') \{2(1-\eta)(1+z^2/x'^2)^{-\frac{1}{2}} + (z/x')^2(1+z^2/x'^2)^{-\frac{3}{2}}\}.$$

Thus when z/x' is small, we find, neglecting powers of z/x' above the second,

$$w = (pd\omega/2\pi nx') \{1 - \eta + \frac{1}{2}\eta(z/x')^2\}; \quad (33)$$

whence

$$-\frac{dw}{dx'} = (pd\omega/2\pi nx'^2) \{1 - \eta + \frac{3}{2}\eta(z/x')^2\}. \quad (34)$$

Here the slope $-dw/dx'$ increases at first with the depth as in the case of (30) and (31).

§ 20. When the depth is of the same order of magnitude as the horizontal distance of the nearest point of the loaded area, individual cases of (14) or (29) require separate consideration.

* Tothunter & Pearson's 'History,' vol. ii. eqn. (xxiv.) of art. 1497.

When the depth becomes large compared to the horizontal distance of the remotest point of the loaded area, we easily find from (14) as a first approximation

$$\left(\frac{dw}{dx}\right)_{z=z}^{x=y=0} = \frac{(5-2\eta)p(y_2-y_1)(x_2^2-x_1^2)}{8\pi n z^3}, \quad (35)$$

showing that the slope now diminishes as the inverse cube of the depth.

If P denote the total load, $\bar{x}, \bar{y}$ the coordinates of the C.G. of the loaded area, we have at once from (35)

$$\left(\frac{dw}{dx}\right)_{z=z}^{x=y=0} = (5-2\eta)P\bar{x}/(4\pi n z^3), \quad (36)$$

and by symmetry

$$\left(\frac{dw}{dy}\right)_{z=z}^{x=y=0} = (5-2\eta)P\bar{y}/(4\pi n z^3). \quad (37)$$

The line of greatest slope is thus in the vertical plane which contains the C.G. of the loaded area, and if R be the horizontal distance of the C.G., the slope is given by

$$(dw/dR)_{z=z} = (5-2\eta)PR/(4\pi n z^3). \quad (38)$$

The conditions assumed in (35) are practically tantamount to those of the elementary loaded area, and (38) can in fact be deduced from (32) by supposing α small.

*Luni-Solar Effects.**

§ 21. Another possible cause affecting the indications of pendulums and spirit-levels is the gravitational action of the heavenly bodies, especially the sun and moon. If we regard the earth as a sphere of mass E and radius a , and suppose the moon's mass to be M and its distance from the earth R , there exists in the earth a system of bodily forces of which the principal come from a potential

$$g(M/E)(a/R)^2(r^2/a)(3\cos^2\theta-1)/2, \quad (39)$$

where g is "gravity" at the earth's surface, neglecting "centrifugal force." The moon is supposed to lie in the line

* Strictly the problem is a dynamical one; as yet only an "equilibrium" solution is available.

$\theta=0$, the earth's centre being origin, and r, θ ordinary polar coordinates. As explained in art. 812 of Thomson and Tait's 'Natural Philosophy,' there results at the earth's surface a component force radially outwards

$$g(M/E)(a/R)^3(3 \cos^2 \theta - 1),$$

and a component along the tangent

$$T = 3g(M/E)(a/R)^3 \sin \theta \cos \theta, \quad \dots \quad (40)$$

directed towards the point under the moon ($\theta=0$).

Both components being small compared to g , the direction of gravity is, owing to the direct attraction alone, deflected through the angle

$$\delta\psi' = \tan^{-1} \{ 3(M/E)(a/R)^3 \sin \theta \cos \theta \} \quad \dots \quad (41)$$

from the vertical. The angle being very small may be replaced by its tangent.

Thomson and Tait suppose

$$(M/E)(a/R)^3 = 10^{-5}/182, \quad \dots \quad (42)$$

and thence draw the following conclusion:—"the plummet is deflected towards the point of the horizon under either moon ($\theta=0$) or antimoon ($\theta=\pi$), by an amount which reaches its maximum value ... $0''.017$ when the altitude is 45° ." They add—"The corresponding effects of solar influence are of nearly half these amounts." According to this conclusion direct luni-solar influence should make itself felt in any system of pendulum or spirit-level observations in which the accuracy is of the order $0''.02$.

§ 22. The data on which the above calculation is based are pretty accurately known, which constitutes a reason for treating the direct effect by itself. It must, however, be borne in mind that the luni-solar influence is not confined to the pendulum bob, but extends to the material of the earth itself. Consequently the result calculated by Lord Kelvin and Prof. Tait is part only of a composite effect, which there is no very obvious way of analysing in practice into its components.

In the actual earth the most obvious consequence of luni-solar action is the ocean tides, and, as we saw in § 4, any station near the sea-shore has its apparent level affected by

these in two distinct ways. Even at an inland station ocean tides must exert some influence, though presumably it is very small.

In addition, however, to ocean tides there must be tides in the earth's mass, whether solid throughout or not, and it is to these I shall now call attention.

The potential term (39) is only one of a series. The numerical values of the coefficients diminish rapidly as the order of the harmonic increases; still it is desirable not wholly to neglect the higher harmonics, if only to make sure that the comparative smallness of the disturbing forces answering to them is not compensated in any instance by great effectiveness. I shall thus consider in the first place the results of the general problem when the degree of the harmonic appearing in the disturbing forces is unrestricted, making use of the results contained in a paper communicated to the Cambridge Philosophical Society* in 1887.

§ 23. Before entering, however, on this investigation, it is desirable to consider briefly the relation between the results of theory and the phenomena we may expect to encounter in direct observation.

Surface-points on the undisturbed surface, regarded as spherical, transform into surface-points on the strained surface; thus a very small surface-area, *e. g.* a square decimetre, may be regarded as a tangent plane in both conditions.

Suppose, now, this area to have rigidly attached to it a spirit-level, consisting of part of a circular arc filled with liquid and with a minute air-bubble. In the undisturbed condition suppose the bubble exactly at the central division O of the arc, while in the disturbed condition it is at an angular interval $\delta\theta$ from O. This an observer would naturally attribute to a change $\delta\theta$ of level. The true interpretation is that in the disturbed condition the resultant of the forces at the surface makes with the normal the angle $\delta\theta$. In a rigid earth $\delta\theta$ would be the angle of Thomson and Tait's calculation; but in an elastic earth allowance must be made for the fact that the attraction of the disturbed earth is not along the normal.

* See the Society's Transactions, vol. xiv. p. 278.

The effect on astronomical observations is still more complicated. Thus let an observer take the altitude of a star in the same vertical plane as the moon, using a mercury surface for his horizontal plane. The observed altitude will differ from the theoretical—*i. e.* the true altitude if the disturbing influence were absent—by an amount equal to the angle between the disturbed and undisturbed mercury surfaces. This is the algebraical sum of the inclination of the resultant gravitational force to the radius-vector in the disturbed condition and of the inclination of this radius-vector to its undisturbed position.

This explanation will show what the quantities are of which we require to know the theoretical values.

§ 24. To return to the problem. The earth is treated as truly spherical when undisturbed, "centrifugal force" being neglected, and as possessed when disturbed of uniform density ρ , and of uniform isotropic elastic qualities throughout, determined by the elastic constants m, n .

The assumption of natural sphericity and the neglect of the centrifugal force answer merely to the neglect of small quantities of the second order of magnitude relative to those of the first; the other assumptions have been discussed in § 2. In our ultimate applications the material will be supposed incompressible, *i. e.* $n/m=0$, but it is undesirable to introduce unnecessary limitations in the mathematical results themselves. Further, absolutely incompressible material is merely a mathematical fiction, so it is desirable to have the means ready to hand to apply a correction to mathematical results based on such an hypothesis.

Supposing the typical term in the potential of the disturbing forces to be

$$rV_i/\sigma_i, \quad \dots \dots \dots (43)$$

where σ_i is a known surface-harmonic of degree i , and V_i a given numerical magnitude, we easily see that the equation to the strained surface will take the form

$$r=a+\sum a_i\sigma_i. \quad \dots \dots \dots (44)$$

At the present stage all we know is that a_i is small compared to a , the mean radius of the strained surface.

The bodily forces consist in part of the disturbing forces, but mainly of the self-gravitational action of the "earth." The complete value of the potential V is given by

$$V = -\frac{1}{2}ga^{-1}r^2 + \sum 3g(r/a)^i a_i \sigma_i / (2i+1) + \sum r^i V_i / \sigma_i. \quad (45)$$

In ordinary circumstances we are supposed to be given the unstrained surface, with full information as to the force system, and it is customary to regard the surface equations as applying to the unstrained surface. In the present instance—and I daresay as a rule in practice—the forces depend to some extent on the disturbed form of the body. It is thus convenient, to say the least of it, to suppose that the surface equations apply in the present instance to the disturbed surface. This implies nothing more serious than the replacing the ordinary definition of strain, viz.

$$(\text{final length} - \text{initial length}) / (\text{initial length}),$$

by

$$(\text{final length} - \text{initial length}) / (\text{final length}).$$

The two definitions are equivalent so long as it is justifiable to apply the mathematical theory, which assumes the square of a strain negligible†.

§ 25. The problem whose results I am about to use was more general than the one at present before us, inasmuch as the surface was not assumed to be naturally spherical. The notation employed in its solution was also somewhat different, the potential being given in the form

$$V = -\frac{1}{2}ga^{-1}r^2 + \sum a_i V_i \sigma_i^i. \quad (46)$$

Thus in utilizing the results we must put

$$V_i = 3ga^{-i} / (2i+1) + V_i' / a_i. \quad (47)$$

In the general problem V_i was unrestricted, but I contented myself with giving the two arbitrary constants $a_i V_i$, $a_i Z_i$ explicitly in terms of $a_i V_i$ and $ga^{-i} a_i$. The expressions for the displacements freed from arbitrary constants were given (*l. c.* equations (13) to (15), pp. 280, 281) only for the case when

$$V_i = 3ga^{-i} / (2i+1),$$

or when V_i' in (47) is zero.

* See Prof. G. H. Darwin, *Phil. Trans.* 1879, Part I.

† See *Phil. Mag.* Sept. 1891, pp. 246-7.

It is easy, however, to add the terms containing V_i' . For if in the equations (11) and (12) of p. 280, *l. c.*, we substitute for V_i the right-hand side of (47) and multiply up by $a_i\sigma_i$, we notice that $V_i'\sigma_i$ appears with the same coefficients as V_i possessed in the earlier equations (32) and (33) (*l. c.* p. 264), which determined the unknowns Y_i and Z_i —treated in that instance as each the combined product of surface harmonic and arbitrary coefficient—for a perfect sphere acted on by bodily forces. Thus for terms in V_i' in the displacements, we have only to take the results (36), (37), and (38), *l. c.* pp. 264–265, and in them replace V_i by $V_i'\sigma_i$ and S_i by zero.

Doing so, we find for the components of displacement, measured respectively in the directions of the fundamental polar elements dr , $r d\theta$, $r \sin \theta d\phi$, the following results:—

$$u = \frac{g\rho r}{10a(m+n)} \left\{ r^2 - \frac{a^2(5m+n)}{3m-n} \right\} \\ + g\rho \sum_{2n(2i+1)} \frac{a_i\sigma_i}{D_i} \left[r^{i+1}a^{-i} \{ 10m^2i(i^2-1) \right. \\ \left. - mn(4i^4+4i^3+34i^2+29i+10) + n^2(8i^3+8i^2+13i-2) \} \right. \\ \left. - r^{i-1}a^{-i+2}i \{ 10m^2i(i+2) - mn(4i^3+4i^2-2i+1) - n^2(4i-3) \} \right] \\ + \frac{5(m+n)}{2n} \sum_{2n} \frac{\rho i V_i' \sigma_i}{D_i} \left[a^2 r^{i-1} i \frac{m(i+2)-n}{i-1} - r^{i+1} \{ m(i+1)-n \} \right], \quad (48)$$

$$v = \frac{d}{d\theta} \sum \Psi_i, \quad w = \frac{1}{\sin \theta} \frac{d}{d\phi} \sum \Psi_i; \quad \dots \quad (49)$$

where

$$D_i = 5(m+n) \{ m(2i^2+4i+3) - n(2i+1) \}, \quad \dots \quad (50)$$

$$\Psi_i = \frac{g\rho a_i\sigma_i}{2n(2i+1)D_i} \left[r^{i+1}a^{-i} \{ 10m^2(i-1)(i+3) \right. \\ \left. - mn(4i^3+12i^2-6i+17) - n^2(8i^2-17) \} \right. \\ \left. - r^{i-1}a^{-i+2}i \{ 10m^2i(i+2) - mn(4i^3+4i^2-2i+1) - n^2(4i-3) \} \right] \\ + \frac{5(m+n)\rho V_i' \sigma_i}{2nD_i} \left[a^2 r^{i-1} i \frac{m(i+2)-n}{i-1} - r^{i+1} \{ m(i+3)-n \} \right]. \quad (51)$$

§ 26. Before utilizing these results we must determine a_i in terms of V_i' , which is easily done as follows:—The surface being supposed originally spherical, the terms $\sum a_i\sigma_i$ in (44)

arose solely from the action of the disturbing forces, and so must be identical with the variable terms in the surface-value of u .

Thus writing $a + \Sigma a_i \sigma_i$ for r in the principal terms in (48), and a for r in the subsidiary, then equating the separate harmonic terms to the corresponding ones in $\Sigma a_i \sigma_i$ and reducing, we find

$$a_i = \frac{\rho a^{i+1} V_i / i \{ (2i+1)m - n \} \div \{ 2(i-1)n(2i^2 + 4i + 3)m - (2i+1)n \}}{1 + \frac{gpa}{n} \frac{15i(2i+1)m^2 - (8i^3 + 6i^2 - 2i - 9)mn + (4i^3 - 2i^2 - 3i - 3)n^2}{5(2i+1)(3m-n)\{ (2i^2 + 4i + 3)m - (2i+1)n \}}}. \quad (52)$$

If the self-gravitation were negligible, the denominator in (52) would become unity, the numerator remaining unchanged. Thus self-gravitation reduces the change of form produced by the disturbing forces depending on the harmonics of degree i in the ratio

$$1 : 1 + \frac{gpa}{n} \frac{15i(2i+1)m^2 - (8i^3 + 6i^2 - 2i - 9)mn + (4i^3 - 2i^2 - 3i - 3)n^2}{5(2i+1)(3m-n)\{ (2i^2 + 4i + 3)m - (2i+1)n \}} \quad (53)$$

If $n/m = 0$,

or the material be incompressible, (52) reduces to

$$a_i = \frac{\rho a^{i+1} V_i / i \{ (2i+1) \} / \{ 2(i-1)(2i^2 + 4i + 3)n \}}{1 + (gpa/n)i / (2i^2 + 4i + 3)}, \quad (54)$$

and the ratio (53) becomes

$$1 : 1 + (gpa/n)i / (2i^2 + 4i + 3). \quad (55)$$

When $i=2$, (52) becomes

$$a_2 = \frac{\rho a^3 V_2' (5m-n) / \{ n(19m-5n) \}}{1 + 3(gpa/n)(10m^2 - 5mn + n^2) / \{ 5(3m-n)(19m-5n) \}}. \quad (56)$$

A result equivalent to (56), with the notation

$$\lambda = m - n, \quad \mu = n,$$

was given by Prof. Karl Pearson, in Todhunter and Pearson's 'History,' vol. ii. part 2, p. 425. An obvious misprint of 4μ for 14μ occurs, however, in the denominator of his formula.

§ 27. We may safely assume $m-n$ positive, so the numerator in (52) has clearly the same sign as V_i' ; also for a given value of V_i' it diminishes as i increases. Thus so long as the denominator in (52) exceeds unity there is no risk lest the relative smallness of the forces proceeding from any higher harmonic may be compensated in any way. It is obvious, however, that the coefficient of gpa/n in the denominator can be made negative by taking i large enough, for ordinary values of n/m . For instance, if $n/m=1/2$,—the hypothesis of uniconstant isotropy,—the coefficient of gpa/n is negative when i exceeds 9, and with i infinite the denominator as a whole would vanish and change sign as n passed through the value $6gpa/100$.

If we take as before

$$gpa = 35 \times 10^8 \text{ grammes wt. per sq. cm.},$$

this critical value of n has the very ordinary value 21×10^7 grammes wt. per sq. cm.

Thus if such a value as $m/n=2$ were admissible the contingency of a_i/V_i' becoming enormously large for a high value of i would be quite a possible one. Unless, however, as I have previously pointed out, n/m be very small, the term in u , independent of the angular coordinates, would in a body of the earth's mass be enormously greater than is consistent with the mathematical theory of elasticity. Therefore, so long as the present calculation is justifiable, the denominator in the value of a_i/V_i' can differ but little from that occurring in (54), and we are thus thoroughly justified in neglecting all the higher harmonic terms in the potential relative to the term containing the second harmonic.

§ 28. As a small departure of n/m from 0 would exercise but little influence on numerical values, it will be best, as we are dealing with data so uncertain, to neglect n/m altogether.

Thus, putting

$$i=2, \quad \sigma_2 = P_2, \quad n/m=0, \\ V_2' = g(M/E)(a^2/R^3), \dots \dots \dots (57)$$

we have for the displaced surface

$$r = a + \sigma_2 P_2, \dots \dots \dots (58)$$

where

$$a_2/a = \frac{5}{19} (gpa/n) (M/E) (a/R)^3 \div \left\{ 1 + \frac{2}{19} (gpa/n) \right\}. \quad (59)$$

An equivalent result is given in Thomson and Tait's 'Natural Philosophy,' art. 840. The result will also be found, along with that answering to $i=2$, $m=2n$, in Mr. Love's 'Treatise on Elasticity,' vol. i. pp. 302, 303.

The corresponding surface displacements are

$$u_a = \frac{5}{19} a P_2 (gpa/n) (M/E) (a/R)^3 \{1 + 2gpa/19n\}, \quad (60)*$$

$$v_a = -\frac{9}{38} a \sin \theta \cos \theta (gpan) (M/E) (a/R)^3 \{1 + 2gpa/19n\}. \quad (61)*$$

The term in u independent of the angular coordinates absolutely vanishes for $n/m=0$, and both components of the surface displacement, and so the resultant displacement itself, are reduced owing to the self-gravitation in the common ratio

$$1 : 1 + 2gpa/(19n). \quad \dots \dots (62)*$$

The angle through which the radius-vector is rotated from its undisturbed position, in the direction away from $\theta=0$, is equal to v_a/a , and so is known from (61). As v_a/a is negative for all values of θ between 0 and $\pi/2$, this rotation is really *towards* the moon at every point of the illuminated hemisphere.

§ 29. We next require to find the inclination of the resultant force to the radius-vector over the surface.

Employing (57) and (59) in (45) we find for the complete value of the potential

$$V = -\frac{1}{2} g (r^2/a) [1 - 2P_2 (M/E) (a/R)^3 (1 + 5gpa/19n) \div (1 + 2gpa/19n)]. \quad (63)$$

The component forces along and perpendicular to the radius-vector are

$$\bar{R} = \frac{dV}{dr}, \quad \bar{\Theta} = \frac{1}{r} \frac{dV}{d\theta}.$$

* The material being as here incompressible, it may be proved that for any value of i in (45) the *displacements* are everywhere the same as in a *sphere* of radius a , over whose surface act purely normal tractions equal to $\rho a^i \sigma_i V^i \div \{1 + (gpa/n)i/(2i^2 + 4i + 3)\}$.

PROCEEDINGS
AT THE
MEETINGS OF THE PHYSICAL SOCIETY
OF LONDON.
SESSION 1896-97.

February 14th, 1896.

Captain ABNEY, President, in the Chair.

The following was elected a Member of the Society :—

Mr. G. HALLIDAY.

Prof. RAMSAY and Mr. EUMORFOPOULOS read a paper "On the Determination of High Temperatures by the Meldometer."

Prof. RAMSAY showed a Direct-vision Spectroscope of special form.

February 28th, 1896.

Prof. PERRY, Vice-President, in the Chair.

The following were elected Members of the Society :—

MESSRS. F. BEDELL, A. GRIFFITHS, D. MORGAN LEWIS,
C. RODGERS, and S. WOOD.

Sir D. SALOMONS showed some experiments with Incandescent Lamps.

Prof. FLBING and Mr. PETAVEL communicated a paper "On an Analytical Study of the Alternating-Current Arc."

March 13th, 1896.

Prof. CAREY FOSTER, Vice-President, in the Chair.

The following were elected Members of the Society:—

MESSRS. F. W. HENKEL and D. KORDA.

Mr. J. H. REEVES read a paper on "An addition to the Wheatstone Bridge for the Determination of Low Resistances."

Mr. J. H. REEVES read a note on "The Exact Value of Matthiessen's Standard."

Herr PULJY communicated a paper on "Kathode Rays," with illustrations.

Mr. W. ACKROYD and Mr. H. B. KNOWLES also exhibited some Röntgen-ray photographs.

March 27th, 1896.

Prof. CAREY FOSTER, Vice-President, in the Chair.

Prof. FLEMING read a paper on "The Edison Effect."

Mr. W. B. MORTON communicated a paper "On the Electromagnetic Effect of Moving Charges."

April 24th, 1896.

Captain ABNEY, President, in the Chair.

Mr. R. A. LEHFELDT communicated a paper on "Symbolism in Thermodynamics."

Mr. ROLLO APFLEYARD read a paper "On the Adjustment of the Kelvin Bridge."

Mr. J. FRITH read a paper "On the Effect of Wave-form on the Alternate-Current Arc."

May 8th, 1896.

Captain ABNEY, President, in the Chair.

The following were elected Members of the Society:—

MESSRS. A. THORNTON and E. TREMLETT CARTER.

MESSRS. FRITH and RODGERS gave a paper "On the True Resistance of the Electric Arc."

May 22nd, 1896.

Prof. AYRTON, Vice-President, in the Chair.

Mr. ROLLO APPLEBY read a paper on "Dielectrics."

Prof. WILLIAM JONES made a communication "On the Magnetic Field due to an Elliptical Current at a point in the Plane of the Ellipse and within it."

Mr. CAMPBELL read a paper on "New Instruments for the Direct Measurement of the Frequency of Alternating or Pulsating Electric Currents."

June 12th, 1896.

Captain ABNEY, President, in the Chair.

Mr. CAMPBELL read a paper "On the Measurement of very large and very small Alternating Currents."

Mr. GRIFFITHS exhibited some Improvements in "Apparatus for the Measurement of Electrical Resistance."

Prof. S. P. THOMPSON made a communication on "Röntgen Rays."

June 26th, 1896.

Captain ABNEY, President, in the Chair.

The following was elected a Member of the Society:—

Mr. R. H. SOBEY.

Mr. F. BEDELL read a paper on "Admittance and Impedance Loci."

Prof. S. P. THOMPSON read a paper "On the Properties of a Body having Negative Resistance."

Mr. F. W. BURSTALL gave a paper "On the Use of Bare Wire for Resistance Coils."

Mr. J. LARMOR gave a paper "On the Theory of Moving Electrons and Electric Charges."

October 30th, 1896.

Captain ABNEY, President, in the Chair.

This was a Special Meeting. The following resolutions were passed :—

- I. That the Subscription to the Society be increased to £2 2s. per annum.
- II. That all Members who have already compounded for their subscriptions be invited to contribute to the Funds of the Society voluntarily, by either subscribing One Guinea annually or compounding for such Subscription.
- III. That a Guarantee Fund be instituted.
- IV. That in future Members of the Society be styled "Fellows of the Physical Society of London."
- V. That a Foreign Secretary be appointed who shall be *ex officio* a Member of the Council.
- VI. That a Librarian be appointed who shall be *ex officio* a Member of the Council.
- VII. That an Editor of the Abstracts be appointed who shall be *ex officio* a Member of the Council.
- VIII. That the Articles of Association of the Society be and hereby are altered in the manner to give effect to the previous resolutions.

At the Ordinary Meeting of the same date a letter was read from Lord Kelvin thanking the Society for the Address which the President, on behalf of the Society, had recently presented to him.

The following were elected Members of the Society :—

Messrs. D. B. MAIR, G. H. J. HURST, and F. G. DONNAN.

Prof. STROUD and Mr. J. B. HENDERSON read a paper "On a satisfactory method of Measuring Electrolytic Conductivity by means of Continuous Currents."

Mr. ROLLO APPLEYARD exhibited a number of different forms of Electrical Trevelyan Rockers.

November 13th, 1896.

Captain ABNEY, President, in the Chair.

The following was elected a Member of the Society :—

Mr. J. W. GIFFORD.

A communication "On some Experiments with Röntgen's Radiation" was received from Prof. THRELFALL and Mr. POLLOCK.

Mr. BRYAN and Dr. BARTON gave a paper "On the Absorption of Electric Waves along Wires by a Terminal Bridge."

November 27th, 1896.

Captain ABNEY, President, in the Chair.

This was a Special Meeting at which the resolutions passed on October 30th were confirmed.

At the Ordinary Meeting of the same date the following were elected Fellows of the Society:—

MISS FANNY LOWATER, MESSRS. W. E. HARRISON and

V. A. MUNDELLA.

Professor RUCKER, Vice-President, took the Chair while the President made a communication upon "Apparatus for giving Diagrams of the Efficiency of a Photographic Shutter."

December 11th, 1896.

Prof. AYRTON, Vice-President, in the Chair.

The following were elected Fellows of the Society:—

MESSRS. C. CHREE and J. E. MYERS.

Dr. CHREE gave a paper on "Applications of Physics and Mathematics to Seismology."

Mr. R. T. RUDD communicated a paper on "Musical Tubes."

January 22nd, 1897.

Prof. AYRTON, Vice-President, in the Chair.

The following was elected a Fellow of the Society:—

Mr. J. R. ASHWORTH.

Mr. W. B. CROFT gave an exhibition of some simple Apparatus.

Mr. E. C. BALY read a paper "On the Passage of Electricity through Gases."

Annual General Meeting.

February 12th, 1897.

Captain ARNEY, President, in the Chair.

The following Report of the Council was read by the Chairman : —

In the Annual Report of the Council for the past year there has to be chronicled the greatest change which has taken place in the Society since its first establishment in 1875. With the expansion given to the scope of the work of the Society in the publication every month of the Abstracts, in the printing in advance of proofs of papers to be read before the Society, and in securing a home at Burlington House, it was found impossible to carry on the operations of the Society on a sound and substantial basis under the old regulations. The difficulties were greatly accentuated by the notable decrease in the interest for money invested, compared with the condition in this respect of the money market at the date of the Society's foundation. The Society owes much to the foresight in these financial matters of past Councils, so that on the one hand the necessary changes which the out-going Council have proposed and which the Society has loyally supported them in carrying out, have neither been hurriedly proposed nor postponed till too late, and on the other hand are believed to be sufficient for the purposes of the present day.

Though the annual subscription of Members has been raised, the Council is fully convinced that it is as low as is consistent with maintaining the efficiency of the Society, and is not higher than that of other scientific associations. The formal title of Membership is now conveyed in the word "Fellow."

There have been held fourteen ordinary business meetings since the last report, as well as the two special meetings necessary for the passing and confirming the changes in the constitution of the Society, all at Burlington House.

From a scientific point of view the past year has been marked by the development of Prof. Röntgen's great discovery of the peculiar radiations which by general consent bear his name, and Fellows of the Physical Society have, as might be expected, been forward in these investigations.

A satisfactory number of new members has joined the Society,

and the Council feels great satisfaction that only a few resignations have been sent in owing to the changes in the regulations that have been made.

Among the deaths which have taken place are those of Mr. W. H. Walenn, September 20, 1896, and Captain Marshall Hall, April 14, 1896.

WILLIAM HENRY WALENN was born in London, January 7, 1828, and died at his residence, 9 Carleton Road, N., on September 20, 1896, after a long and painful illness. He was trained as an engineer at the works of Messrs. Cottam, and received part of his education at University College, London, where he studied mathematics under Professor De Morgan. He was one of the earliest abridgers of specifications to the Patent Office, beginning under the then Comptroller, Mr. Woodcroft, for whom he compiled the series of abridgments relating to Electricity and Magnetism, Photography, and other subjects. In 1866 his book, 'Little Experiments for Little Chemists,' was published, and in it was given a new process for depositing brass upon zinc. In 1871 he contributed a paper to the 'Philosophical Magazine,' "On the Solutions for Depositing Copper and Brass by means of Electric Force." About this time he conducted some experiments for the Government of the day in electro-depositing copper upon the bottom of an iron ship. Between 1868 and 1880 he communicated through the 'Philosophical Magazine' several mathematical papers upon "Unitates," and methods of checking calculations by means of these. Mr. Walenn was known among his friends as a man always ready and willing to aid any inquirer after scientific truth, whether in his favourite study of mathematics, or in the many branches of physics and chemistry with which he was familiar. He became a Fellow of the Chemical Society in 1866, and of the Institute of Chemistry soon after its establishment.

The adoption of the Report was proposed and passed unanimously.

The election of Officers and other Members of Council then took place, the new Council being constituted as follows :—

President.—SHELFORD BIDWELL, M.A., LL.B., F.R.S.

Vice-Presidents who have filled the Office of President.—Dr. J. H. GLADSTONE, F.R.S.; Prof. G. C. FOSTER, F.R.S.; Prof. W. G. ADAMS, M.A., F.R.S.; Lord KELVIN, D.C.L., LL.D., F.R.S.; Prof. R. B. CLIFTON, M.A., F.R.S.; Prof. A. W. REINOLD, M.A., F.R.S.; Prof. W. E. AYETON, F.R.S.; Prof. G. F. FITZGERALD, M.A., F.R.S.; Prof. A. W. RUCKER, M.A., F.R.S.; Capt. W. DE W. ABNEY, R.E., C.B., D.C.L., F.R.S.

Vice-Presidents.—Major-General E. R. FESTING, R.E., F.R.S.; L. FLETCHER, M.A., F.R.S.; Prof. J. PERRY, D.Sc., F.R.S.; G. JOHNSTONE STONEY, M.A., F.R.S.

Secretaries.—T. H. BLAKESLEY, M.A., M.Inst.C.E.; H. M. ELDER, M.A.

Foreign Secretary.—Prof. S. P. THOMPSON, D.Sc., F.R.S.

Treasurer.—Dr. E. ATKINSON, J.P.

Librarian.—C. VERNON BOYS, F.R.S.

Other Members of Council.—WALTER BAILY, M.A.; L. CLARK, C.E., F.R.S.; A. H. FISON, D.Sc.; Prof. J. A. FLEMING, M.A., F.R.S.; R. T. GLAZEBROOK, M.A., F.R.S.; Prof. A. GRAY, M.A.; G. GRIFFITH, M.A.; Prof. G. M. MINCHIN, M.A., F.R.S.; Prof. W. RAMSAY, Ph.D., F.R.S.; J. WALKER, M.A.

Votes of thanks were passed to the Chemical Society; to the OFFICERS; and to the AUDITORS.

PROPERTY ACCOUNT OF THE PHYSICAL SOCIETY, DECEMBER 31, 1896.

ASSETS.		LIABILITIES.	
	£ s. d.		£ s. d.
Subscriptions due, estimated	10 0 0	Subscriptions for 1897 in advance	4 4 0
£400 Furness 4 per cent. Debenture Stock	456 0 0	Balance due to the Treasurer	3 4 10
£1000 Midland 4 per cent. Preference Stock	1194 0 0	Abstracts, Oct.-Dec.	29 5 10
£200 Metropolitan Board of Works 3½ per cent. Stock	217 0 0	Printers	127 3 9
£270 Lancaster Corporation 3 per cent. Stock	227 6 8	Balance	2448 11 11
£254 2s. 9d. New South Wales 3½ per cent. Inscribed Stock	249 7 5		
Balance in the Bank	154 6 3		
Dividend unpaid	4 6 0		
Guarantee Fund, on Deposit	100 4 0		
	<u>£2512 10 4</u>		<u>£2512 10 4</u>

EDMUND ATKINSON, *Treasurer.*

Securities examined and found correct.

FREDERICK HOVENDEN } *Auditors.*
ALFRED W. PORTER. }

February 1, 1897.

Thus at a point on the surface the principal terms, which alone we require, are

$$\bar{R} = -g, \quad \bar{\Theta} = -3g \sin \theta \cos \theta (M/E) (a/R)^3 (1 + 5gpa/19n) \\ \div (1 + 2gpa/19n), \quad . \quad (64)$$

and the inclination of the resultant force to the radius-vector is to a first approximation

$$\delta\chi = \bar{\Theta}/\bar{R} = 3 \sin \theta \cos \theta (M/E) (a/R)^3 (1 + 5gpa/19n) \\ \div (1 + 2gpa/19n). \quad . \quad (65)$$

For the apparent change of altitude, $\delta\alpha$, in a star we have, as already explained in § 23,

$$\delta\alpha = \delta\chi + v_a/a = 3 \sin \theta \cos \theta (M/E) (a/R)^3 \left\{ 1 + \frac{7}{38} (gpa/n) \right\} \\ \div (1 + 2gpa/19n). \quad . \quad (66)$$

§ 30. For the apparent change of level $\delta\psi$, we require the inclination of the resultant force to the normal. To obtain this we may employ the result (65) in conjunction with the inclination $\delta\psi_1$ of the normal to the radius-vector, the latter being given to a first approximation by

$$\delta\psi_1 = \left(-\frac{1}{r} \frac{dr}{d\theta} \right)_{r=a} = \frac{15}{19} \sin \theta \cos \theta (gpa/n) (M/E) (a/R)^3 \\ \div (1 + 2gpa/19n). \quad (67)$$

Thus we have finally

$$\delta\psi = \delta\chi - \delta\psi_1 \\ = 3 \sin \theta \cos \theta (M/E) (a/R)^3 \div (1 + 2gpa/19n). \quad (68)$$

This result can also be got by noticing that

$$\delta\psi = \tan^{-1} (T/g), \\ = T/g, \text{ to a first approximation,}$$

where

$$T \equiv \bar{R} \sin \delta\psi_1 - \bar{\Theta} \cos \delta\psi_1 \\ = 3g \sin \theta \cos \theta (M/E) (a/R)^3 \div (1 + 2gpa/19n). \quad . \quad (69)$$

is the tangential component of the surface-force.

Comparing (68) with (41) we have

$$\delta\psi : \delta\psi' : : 1 : 1 + 2gpa/19n, \quad . \quad . \quad (70)$$

or the self-gravitational forces reduce the apparent change of level, as calculated in Thomson and Tait's 'Natural Philosophy,' in precisely the same ratio as they reduce the ellipticity of the surface.

This last result might probably be deduced at once from the fact that u_a and v_a are reduced in the same proportion, but I have preferred an explicit mathematical proof.

Comparing (66) and (68), we have

$$\delta a / \delta \psi = 1 + \frac{7}{4}(2gpa/19n), \dots (71)$$

showing that the apparent change in star's altitude—the star being, it will be remembered, in the same vertical plane with the moon—is always in excess of the apparent change of level.

Numerical Estimates.

§ 31. As before, we shall take

$$a = 64 \times 10^7,$$

$$gpa = 35 \times 10^8 \text{ grammes wt. per sq. cm.,}$$

$$(M/E)(a/R)^3 = 1/(182 \times 10^3).$$

We shall consider only the greatest and least values of n specified in § 7, exhibiting the results side by side; θ , it will be remembered, is measured from the line joining the centres of the earth and moon.

Numerical Results: Lunar Influence.

$$n = \begin{array}{ll} 80 \times 10^7 \text{ grammes wt.} & 11 \times 10^7 \text{ grammes wt.} \\ \text{per sq. cm.} & \text{per sq. cm.} \end{array}$$

$2gpa/19n =$	35/76	700/209
$1 : 1 + 2gpa/19n \text{ (approx.)} =$	11 : 16	3 : 13
$u_a =$	$28 \frac{3 \cos^2 \theta - 1}{2} \text{ cms.}$	$68 \frac{3 \cos^2 \theta - 1}{2} \text{ cms.}$
$v_a =$	$-25 \sin \theta \cos \theta \text{ cms.}$	$-61 \sin \theta \cos \theta \text{ cms.}$
Polar less equatorial radius =	42 cms.	102 cms.
Apparent change level, $\delta \psi =$	$0''\cdot 012 \sin 2\theta$	$0''\cdot 004 \sin 2\theta$
Apparent change of star's altitude, $\delta a \left\{ \begin{array}{l} = \end{array} \right.$	$0''\cdot 021 \sin 2\theta$	$0''\cdot 027 \sin 2\theta$

The reduction effected by the self-gravitational forces in Thomson and Tait's estimate, $0''\cdot017 \sin 2\theta$, for the apparent change of level increases conspicuously as the rigidity diminishes. In fact, for the lower value of n , $\delta\psi$ would be insensible unless with an instrument recording to $\frac{1}{2}''$ of a second of arc.

On the other hand, the changes in the shape of the earth and in the star's apparent altitude are very decidedly larger for the lower value of n .

Corresponding results of about half the numerical size of the above would be obtained in the case of solar influence.

Final Conclusions.

§ 32. The results obtained indicate at least the directions in which luni-solar effects may be profitably looked for. If the earth's elasticity for luni-solar influence be perfect, apparent changes of level or star's altitude will be *nil* when the moon or sun, as the case may be, is either in the zenith or on the horizon, while they will be a maximum when the altitude is 45° . If the elasticity be not perfect, a lag in the tides may be expected. As regards star's altitude, a hopeful feature is that the influence, being the same for all stars in the same vertical, should be easily separable from terrestrial refraction. The further fact that the apparent change is proportional to the cosine of the star's azimuth measured from the vertical plane containing the moon, or sun, may prove of assistance.

It would appear that luni-solar effects are not unlikely to prove of as much consequence as the direct pressure or gravitational effects of any ordinary differential meteorological action in the neighbourhood of an observatory, though not nearly so important as ocean or estuary tides for observatories situated within a few hundred yards of high-water mark.

The considerable fluctuation of the calculated luni-solar effects with the value ascribed to the earth's rigidity may lead eventually to interesting speculations as to the state of the earth's interior.

Subsidiary Remarks.

§ 33. Whilst attention has been confined to surface-pressure

and luni-solar action, it is not intended to imply the non-existence of other agents capable of producing similar phenomena. The sun's direct heating effect is doubtless in some cases a most effective agent in altering the level. *A priori* one would expect a diurnal variation from this cause, most sensible at stations on rocky ground exposed to the south.

§ 34. Before quitting the subject, it is desirable to consider what light existing seismological data throw on the credibility of the hypothetical theory adopted.

It appears pretty generally believed that wave-velocities calculated from observations near and distant from the epicentre of an earthquake are usually different, and the existence of at least two widely different wave-velocities seems on some occasions well established at the distant stations. One of the two wave-velocities has been regarded (on, I think, mistaken grounds) as postulating an elasticity incredibly high for an elastic solid medium.

These phenomena are easily reconciled with the elastic solid hypothesis. When waves travel between two distant points through the interior of a sphere of large radius they may be expected to behave much as if the medium were infinite. Now in an infinite isotropic medium*, as is well known, there are two wave-velocities, v_1 and v_2 , given in our previous notation by

$$v_1 = \sqrt{(m+n)/\rho} \quad v_2 = \sqrt{n/\rho}.$$

Thus, under the conditions supposed, we should expect two earthquake-waves with velocities similar to v_1 and v_2 . For definiteness, suppose that the velocities are actually v_1 and v_2 , and suppose them to be respectively 12.5 and 2.5 kilometres per second, this appearing a fair estimate.

Then, in absolute c.g.s. measure,

$$\sqrt{(m+n)/\rho} = 125 \times 10^4, \quad \sqrt{n/\rho} = 25 \times 10^4.$$

Taking $\rho = 5.5$ for the earth, we have the approximate results,

$$n = m/24 = 35 \times 10^7 \text{ grammes wt. per sq. cm.}$$

* See, for instance, Love's 'Treatise on Elasticity', vol. i. pp. 133, 134.

For E , Young's modulus, and k , the bulk modulus (resistance to compression), we have similarly,

$$E \equiv n(3 - n/m) = 10 \times 10^8 \text{ grammes wt. per sq. cm.,}$$

$$k \equiv m - n/3 = 83 \times 10^8 \quad \text{''} \quad \text{''} \quad \text{''}$$

The rigidity and Young's modulus—the quantities from whose magnitudes our conception of a material's elasticity is usually derived—are in no ways remarkable, being much below the average magnitude observed in iron. The only abnormal feature is the enormous resistance to compression. Any one, however, who considers the enormous pressures presumably in continuous operation on the earth's deep-seated material, will appreciate the probability that it responds uncommonly little to any slight increase in pressure.

A difference between the velocities calculated at stations near and distant from the epicentre is only what we should expect. Lord Rayleigh* has shown that waves with a velocity somewhat less than $\sqrt{n/\rho}$ may be propagated through the material close to the surface of a medium bounded by an infinite plane; and a similar phenomenon may be expected in a sphere, so long at least as the distance from the epicentre is small compared to the radius. In such waves the velocity must depend mainly on the density and elastic properties of the surface material, which in general must differ largely from the corresponding quantities in the deep-seated material. Thus the velocities calculated from the observed effects must depend largely on whether the waves propagated along the surface or those propagated through the interior are the dominant ones; in other words, on whether the distance of the station from the epicentre is or is not small compared to the earth's radius.

DISCUSSION.

Prof. PERRY said he had thought of taking up the subject from an experimental point of view, and trying the effect of loading a large block of indiarubber. He had not had time to refer to the author's paper, in which the reasons were

* Proc. London Math. Soc. vol. xvii. (1886). See also Love's 'Treatise,' vol. i. pp. 328-330.

given for taking the earth as incompressible. He (Prof. Perry), however, thought that this assumption led to results in contradiction to actually observed facts. Prof. Milne had obtained results which, for want of any other explanation, he had been compelled to attribute to meteorological causes. The reason Dr. Chree had obtained so small a value for the effect of loading by surface-water might be because he had assumed erroneous values for the elastic constants. If he took a value for Poisson's ratio such as we meet with in practice, the effects would be much larger. Prof. Darwin had also investigated the folding of the surface of the earth due to loading. The results obtained by the author with reference to the velocity of waves did not seem quite satisfactory. The small waves which were found, both at Berlin and the Isle of Wight, to precede the main waves coming from an earthquake in Japan, were not accounted for. The wave-velocity in an infinite mass of steel (a very elastic material) was about 6 kilometres per second, which was very different from 12.5 kilometres per second. The author had assumed such values for the elasticity as would give the correct velocity.

The AUTHOR, in reply, said that in applying the equations of elasticity to the earth's interior, unless the material were supposed nearly incompressible, one obtained values of the strains too large to be consistent with the fundamental mathematical hypothesis that the squares of strains are negligible. In the case of surface-loading no such restriction was necessary, so far as the surface-layers at least are concerned. The differences between the several numerical estimates for the ratio of gravitational and pressure effects of a surface-load were principally due to the differences in the hypothetical values ascribed to the rigidity. It was his wish to make it clear that the pressure and gravitational agencies treated in detail in the paper were not the only ones likely to affect the level; he had specially called attention to solar heating and possible direct influence of moisture on the foundations of buildings, &c. The reason why for the one wave-velocity so much higher a value was obtained than that Prof. Perry calculated for steel was solely the high value, 24 : 1, found for the ratio of Thomson and Tait's elastic constants m and n .

He knew Prof. Darwin had treated of the phenomena met with in loose earth in some cases, but could not say whether this was what Prof. Perry referred to. He had himself once thought of attempting an application of what Prof. Karl Pearson termed the "equations of pulverulence," as treated in detail by Prof. Boussinesq, but had not done so, partly from a feeling of uncertainty as to their physical value. Supposing these equations satisfactory, they ought to give better results than the equations of elasticity when surface-load was applied to a deep alluvial soil.

V. *On the Effect of Capacity on Stationary Electrical Waves in Wires.* By W. B. MORTON, M.A.*

WHILE working recently at stationary electrical waves in wires produced in Blondlot's manner, I was led to make some measurements on the effect produced when a capacity is inserted at a point of the secondary circuit. The positions of the successive nodes were explored in the usual way by a bridge, the indicator being a vacuum tube which was placed across the wires and which showed a maximum of brightness when the bridge was at a node. When two opposite points of the parallel secondary wires were joined to the plates of a small air condenser, the effect was to bring closer together the nodes on the two sides of the condenser, the amount of this shortening of the apparent half wave-length depending on the position of the inserted capacity. The effect was nil when the condenser was at a node, and maximum when it was midway between two nodes. This influence of the increased capacity of the wires is of course of the same nature as the shortening of the wave-length when the wires pass from air into a dielectric liquid. Drude and others have made use of this way of measuring directly the index of refraction of different liquids for the electric waves; but the influence of an isolated capacity does not seem to have been much studied. Salvioni has published † some measurements on the

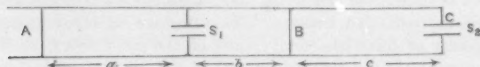
* Read April 9, 1897.

† *Rend. Acc. Linc.* 1892, pp. 250-253; *Wied. Beibl.* xvii. p. 485.

effect of capacity inserted at a point between the end condenser and a bridge. When the second condenser was put in it was necessary, in order to restore resonance, to alter the distance of the plates of the end condenser. Von Geitler* got rid of the reflected waves by using a terminal resistance so that no standing oscillations were formed in the wires. He found that when a condenser was inserted it caused reflexion of the waves, and a series of nodes and loops could be found in front of the condenser. Mazzotto† in a recent research has availed himself of the effect of an isolated capacity to produce a gradual change in the wave-length of the oscillations. He uses pieces of wire hung on to the parallel wires.

The theory of electric waves in wires has recently been treated in an exhaustive manner by Drude‡. His method consists in following out in detail the various reflexions undergone at the bridges by a wave-train which starts from the end of the wires. The state of affairs at a point of the circuit is obtained by summation of a series of separate disturbances due to the different direct and reflected trains. The calculations are rather complicated. In obtaining a formula with which to compare my observations I have used a method adapted from some work of Mr. Heaviside's§. Apart from the actual results obtained, the investigation is perhaps of some interest as showing how easily some problems connected with oscillations in wires can be attacked by this method.

The experiments were made, for the most part, at the end of the parallel secondary wires remote from the oscillator, the arrangement being as shown in the diagram.



C is the end condenser of capacity S_2 . S_1 an interposed condenser. The vacuum tube was placed across the plates of

* Wied. Ann. xlix. pp. 184-195 (1893); cf. Barton and Bryan, Proc. Phys. Soc. xv. p. 23.

† Nuov. Cim. [4] ii. pp. 296-311 (1895); Wied. Beibl. xx. p. 392.

‡ Abhandl. der Sächs. Ges. der Wiss. xxiii. pp. 64-168 (1896); Wied. Ann. lx. pp. 1-46 (1897).

§ 'Electrical Papers,' ii. p. 194 et seq.

S_2 . The plates of S_1 and S_2 were kept at a constant distance apart while the position of S_1 was varied. A nodal position B of the bridge having been found so that the tube shone, B was left on the wires, and A laid on and adjusted so that the tube remained bright. Then A, B are nodes of the same system. Owing to the finite length of the bridge the potential-difference at its ends is not quite zero, the true node falling at the centre of the bridge. Accordingly, if the bridge B is taken off the wires the nodes will fall a short distance to the left of B—roughly half the length of the bridge. This distance Drude has called the “bridge-shortening.” In order to restore maximum brightness in the tube, the bridge A must now be displaced in the same direction through *twice* the “bridge-shortening.” The difference in the positions of A, according as B is on or off the wires, gives us therefore a means of finding the correction to be applied to the observed bridge-positions to get the true nodes in the wires.

We want to express that the circuits AB and BC are in resonance. In order to find approximately the period of oscillation of such circuits we can proceed as follows:—Suppose a simple harmonic potential-difference $V_0 \sin nt$ to be kept up between the wires at one end of the circuit, and find an expression for the oscillations produced in the circuit. The amplitude of these oscillations will become infinite when n corresponds to the natural period of the system. A formula has been obtained by Cohn and Heerwagen for a circuit like BC. I have not found any discussion of the circuit AB.

We neglect the resistance of the wires and put S for their capacity and L for their induction per unit-length, both supposed constant. This will be only approximately true as we approach the ends of the circuits. The equations connecting the current C and potential-difference V are

$$-\frac{dV}{dx} = L \frac{dC}{dt}$$

$$-\frac{dC}{dx} = S \frac{dV}{dt},$$

dx being the element of length,

$$\therefore \frac{d^2 V}{dx^2} = LS \frac{d^2 V}{dt^2};$$

or if V varies as $\sin nt$,

$$\frac{d^2 V}{dx^2} = -LSn^2 V = -q^2 V,$$

$$\text{where } q = n \sqrt{LS} = \frac{n}{v} = \frac{2\pi}{\lambda};$$

v is the velocity of radiation, and λ the wave-length along free wires;

$$\therefore V = (A \cos qx + B \sin qx) \sin nt.$$

$$\text{This gives } -C = \frac{q}{Ln} (A \sin qx - B \cos qx) \cos nt.$$

Take first the circuit BC. Here we have

$$V = V_0 \sin nt \text{ when } x=0,$$

$$\text{and } C = S_2 \frac{dV}{dt} \text{ when } x=c,$$

$$\therefore A = V_0,$$

$$\text{and } -\frac{q}{Ln} (A \sin qc - B \cos qc) = S_2 n (A \cos qc + B \sin qc).$$

Putting

$$\tan \alpha = \frac{LS_2 n^2}{q} = LS_2 v^2 q = \frac{S_2}{S} q$$

we get

$$B = A \tan (qc + \alpha),$$

and

$$V = V_0 \frac{\cos \{q(c-x) + \alpha\}}{\cos (qc + \alpha)} \sin nt.$$

The amplitude becomes infinite when $qc + \alpha = \frac{\pi}{2}$,

$$\cot qc = \tan \alpha;$$

$$i. e. \cot \frac{2\pi c}{\lambda} = \frac{2\pi}{\lambda} \cdot \frac{S_2}{S}. \quad \dots \quad (1)$$

This is Cohn and Heerwagen's formula.

Taking now circuit AB, suppose the impressed potential-difference to act at A.

Let $V_1 = (A_1 \cos qx + B_1 \sin qx) \sin nt$ between $x=0$ and $x=a$,

$V_2 = (A_2 \cos qx + B_2 \sin qx) \sin nt$ between $x=a$ and $x=b$.

The conditions to be satisfied are :—

$$\text{at } x=0, \quad V_1 = V_0 \sin nt,$$

$$\text{at } x=a, \quad V_1 = V_2,$$

$$\text{and} \quad C_1 - C_2 = S_1 \frac{dV}{dt}$$

$$\text{at } x=a+b, \quad V=0.$$

Putting in the values we get

$$A_1 = V_0,$$

$$A_1 \cos qa + B_1 \sin qa = A_2 \cos qa + B_2 \sin qa$$

$$= \frac{1}{t} \{ (-A_1 + A_2) \sin qa + (B_1 - B_2) \cos qa \},$$

$$A_2 \cos q(a+b) + B_2 \sin q(a+b) = 0,$$

$$t \text{ has been put for } \frac{qS_1}{S}.$$

On solving these equations for the constants we find

$$V_1 = \frac{\sin q(a+b-x) - t \sin q(a-x) \sin qb}{\sin q(a+b) - t \sin qa \sin qb} V_0 \sin nt,$$

$$V_2 = \frac{\sin q(a+b-x)}{\sin q(a+b) - t \sin qa \sin qb} V_0 \sin nt.$$

Equating the denominator to zero we find

$$\cot qa + \cot qb = t,$$

or

$$\cot \frac{2\pi a}{\lambda} + \cot \frac{2\pi b}{\lambda} = \frac{2\pi}{\lambda} \cdot \frac{S_1}{S}. \quad (2)$$

This formula, which connects the frequency of the oscillation with the position and capacity of the condenser, might have been deduced from formula (1). For we can imagine the capacity S_1 divided into two parts, σ and σ' , in such a way that when these parts are attached to the parts a and b of the circuit respectively, the two partial systems oscillate independently with the same frequency. We have thus from formula (1),

$$\cot \frac{2\pi a}{\lambda} = \frac{2\pi}{\lambda} \frac{\sigma}{S},$$

$$\cot \frac{2\pi b}{\lambda} = \frac{2\pi}{\lambda} \frac{\sigma'}{S}.$$

On adding we get equation (2).

In seeking to test the agreement between the theory and the observations, formula (2) was written

$$\frac{\lambda}{2} \left\{ \cot \frac{2\pi a}{\lambda} + \cot \frac{2\pi b}{\lambda} \right\} = \frac{\pi S_1}{S} = \text{constant}.$$

A small error in observation of the node position causes an error in the left-hand side very large in proportion, and of amount varying with the position of the condenser in the circuit. In view of this the method adopted was to find the mean value of $\frac{S_1}{S}$ from the observations and, using it, to calculate the values of b corresponding to each a . A comparison of the observed and calculated values of b shows a sufficiently good agreement, the discrepancy being greatest when the condenser is too near a bridge.

One set of observations involving only the circuit AB and formula (2) were taken on a small apparatus at the end near the oscillator. One bridge was kept fixed and the wave-length was the same throughout. Thus only the quantity b was liable to the error in determining the node. The uncertainty in this determination amounted to about 5 millim. The wires were about 15 millim. apart, and the condenser consisted of two small copper strips, 1 centim. by 5 centim., hung on the wires. The half wave-length was 37.5 centim. The following table gives the results measured in centimetres:—

TABLE I.

a .	b . Calc.	b . Obs.
2.0	34.7	33.1
6.0	19.8	17.5
8.0	13.4	12.9
10.0	10.3	10.5
12.0	8.5	8.7
14.0	7.7	7.9
20.0	5.9	5.9
30.0	3.7	3.9
33.0	2.7	3.1

The other observations were made at the end of the wires of a large oscillator in the manner already described. In the calculation both formulæ had to be used. First, a measurement without the interposed condenser gave directly a value

of λ for a given value of c . Then from formula (1) by use of these values the constant $\frac{S_2}{S}$ was found. In working up the subsequent observations with condenser S_1 in position, knowing c gave λ from formula (1), then λ , a , b in (2) gave a value of $\frac{\pi S_1}{S}$. Using the mean of such values the calculated b 's were got as already indicated.

The parallel wires were about 20 m. long, 2 millim. diameter, and 10 centim. apart. The plates of condensers S_1 and S_2 were of $8\frac{1}{4}$ and 20 centim. diameter respectively. Uncertainty of node position from one to two centimetres.

TABLE II.

Distance of plates of S_1 , 0.5 centim.; of S_2 , 3.7 centim.

Without S_1 , $c=44.5$; $a+b=\frac{\lambda}{2}=302.5$. Hence $\frac{S_2}{S}=13.2$.

c . Obs.	$\frac{\lambda}{2}$. Calc.	a . Obs.	b . Calc.	b . Obs.
45.1	304.6	4.4	299.5	296.0
51.3	326.6	54.9	82.5	77.8
50.7	324.5	79.7	55.3	54.7
50.1	322.4	113.5	43.3	42.9
48.5	316.8	161.5	34.4	35.7
46.3	308.9	224.3	28.1	27.9
44.7	303.2	290.3	9.3	10.1

TABLE III.

The same capacities, a different node-system.

Without S_1 , $c=54.5$; $\frac{\lambda}{2}=337.5$ giving the same value, $\frac{S_2}{S}=193.2$.

c . Obs.	$\frac{\lambda}{2}$. Calc.	a . Obs.	b . Calc.	b . Obs.
61.5	360.6	54.9	179.5	169.9
62.9	365.0	79.7	55.3	54.7
62.7	364.4	100.9	65.7	67.3
60.9	358.7	149.1	49.0	50.1
56.3	343.3	278.7	26.4	27.2
55.3	340.0	305.7	19.0	19.9

TABLE IV.

Distance of plates of $S_1 = 1$ cm., of S_2 unchanged = 3.7 cm.
Same node-system as in Table III.

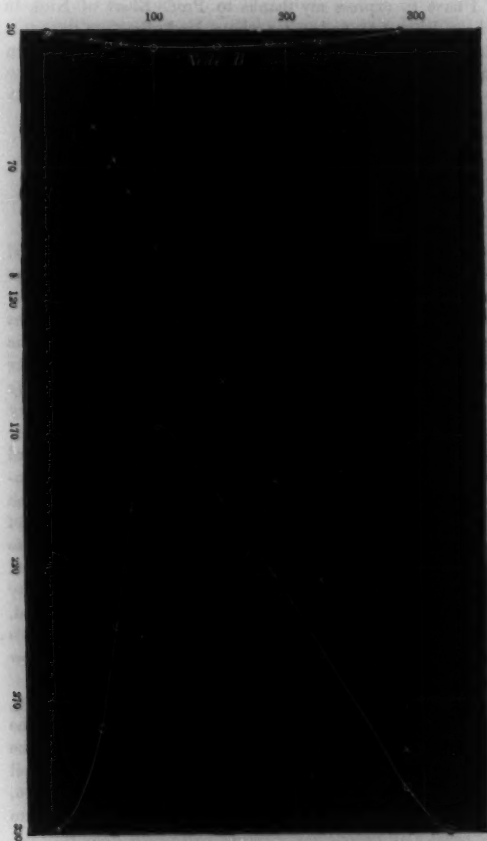
c. Obs.	$\frac{\lambda}{2}$ Calc.	a. Obs.	b. Calc.	b. Obs.
58.1	349.4	83.9	192.8	187.6
58.5	350.6	148.1	110.7	111.5
57.5	347.1	194.5	81.3	81.0
56.5	344.0	240.5	60.0	61.0
55.3	340.0	288.2	36.4	36.8
55.0	339.0	321.3	15.4	15.2

TABLE V.

Distance of plates of $S_1 = .5$ cm., of $S_2 = 6.2$ cm.
Without S_1 , $c = 60.0$; $\frac{\lambda}{2} = 300.0$; $\frac{S_2}{S} = 131.4$.

c. Obs.	$\frac{\lambda}{2}$ Calc.	a. Obs.	b. Calc.	b. Obs.
62.8	307.8	27.6	233.1	231.7
65.9	316.4	36.6	186.5	180.0
65.5	315.3	49.1	113.2	122.9
67.0	319.4	68.0	77.2	73.5
66.0	316.7	118.8	48.3	49.7
65.5	315.3	157.1	40.4	40.5
63.9	310.9	190.6	33.7	34.6
60.5	301.4	262.9	19.6	20.8

The general course of the changes referred to is best seen from the annexed figure, which corresponds to the last series of observations. The ordinates are distances of the inserted condenser from an arbitrary origin near the end of the wires. the abscissas of the points on the two curves are the distances of nodes B and A from the same origin on twice the scale, the crosses between the curves showing the position of the condenser. It will be seen that the apparent half wave-length, or distance A B, is least when the condenser bisects the distance. When the condenser coincides with either node, A B is the full half-wave. When the condenser passes outside A B, the curve of A turns in again.



I have to express my thanks to Prof. Ebert of Kiel, in whose laboratory and under whose kind direction the experiments were carried out of which these observations form part.

Queen's College, Belfast,
27th February, 1897.

VI. *Liquid Coherers and Mobile Conductors.*

By ROLLO APPLEYARD*.

IN a communication† made three years ago to the Physical Society, I described some experiments illustrating the change in electrical resistance of certain complex bodies under the influence of oscillatory discharges. All the substances dealt with were solids; and the coherence was invisible. The change of condition had therefore to be demonstrated either by measuring the resistance before and after discharge, or by connecting the coherers permanently in series with a battery and galvanometer.

The three experiments now brought before you have regard to "coherers" formed of liquid dielectrics, and mobile conductors. By choosing a transparent dielectric and an opaque conducting substance, it is possible to examine the process of coherence by direct observation. But it may be well to premise that the similarity of *results* obtained with solid and liquid "coherers," respectively, in no way proves a similarity of *process*. The two sets of phenomena are probably related, but are not necessarily identical. The term "dielectric" is here to be understood as signifying merely a substance of low conductivity.

Experiment 1.—A glass tube about eighteen inches long, and half-an-inch wide, is sealed at one end and corked at the other. Platinum electrodes are inserted at each end. The tube is nearly filled with about equal volumes of paraffin-oil and mercury. If it is laid upon a flat table and shaken, horizontally, for a few minutes, the mercury breaks up into

* Read March 26, 1897.

† "Dielectrics," *Phil. Mag.* xxxviii. p. 396 (1894); *Proc. Physical Society*, xiii. p. 155 (1895).

small spheroids; and, by a little manipulation, these can be disposed as a chain of particles lying evenly between the platinum electrodes. The resistance of the chain of mercury spheroids, measured under these conditions, is several megohms.

If we now connect the electrodes to a battery of about two hundred volts, the whole regime is suddenly altered. At the moment of applying the current, the spheroids of mercury, within the tube of oil, are visibly impelled, as though a mechanical tap had been administered to the glass; and, almost simultaneously, they coalesce into large globules. The resistance is now represented by a few ohms.

Exactly the same result can be brought about by supporting the tube near a Hertz oscillator; or, still more simply, by passing a spark into one or other, or both, of the electrodes. In order to retard the spontaneous coherence of the mercury, resulting from mutual pressure of the spheroids, it is well to keep the tube horizontal. If, however, it is desired quickly to convert a body of mercury from the subdivided to the ordinary state, sparks may be passed into the tube while it is more or less vertical. The running-together of subdivided mercury is more leisurely to be observed with large globules. These form separate, elongated, conductors. The way in which they unite will be referred to in describing Experiment 3.

Experiment 2.—A glass tube, similar to the first, but somewhat wider, is nearly filled with a mixture of paraffin-oil and water, and vigorously shaken. I propose to call this a "rain" tube. If it is kept at rest, the oil, in the common course of events, floats to the top in a few minutes. The "rain" tube, however, shows that the separation, especially towards the final stage, is accelerated by the passage of a spark, or by a direct current from a battery of about a thousand volts. If the conditions are right, the water particles suspended in the oil cohere, at the moment of electrification, to form larger drops. The frictional resistance to falling is thereby diminished, and the water is consequently precipitated in and through the oil. It may sometimes be seen descending in a rapid succession of globules, precisely as large rain-drops are precipitated after thunder. About equal parts of oil and water is a good

proportion. The containing vessel may be either a tube or a flask; it should not be more than three-quarters full of liquid. This free space facilitates the mixing when the tube or flask is shaken. The phenomenon is rendered much more striking by colouring the oil with alkanet-root. I have to thank Prof. McLeod for suggesting this pigment.

Experiment 3.—The behaviour of a mobile conductor, when electrified in a partially conducting liquid, is readily examined by pouring a little mercury into a flat photographic dish containing a stratum of paraffin-oil and water. The presence of the oil is necessary to prevent the mercury from running together too freely of its own accord. A battery of from, say, one volt to two hundred is required, and a pair of wires to dip into the dish. A reversing-key, such as is used for cable transmission, may be included in the circuit.

Suppose we begin with a large globule of mercury in each of any pair of corners of the photographic dish, several inches apart; and let the globules be connected one to each pole of the battery, by means of the dipping wires. A momentary tap of the key causes instantaneous deformation of the mercury in each corner, especially of that connected to the negative pole; and there is evident attraction between the globules. Sometimes the mercury gets into a lethargic condition; but it can always be roused by mechanical agitation of the surface. Now let the current be kept on for a few seconds; the negative globule sends forth a tentacle towards the positive globule, the length of the tentacle depending upon the current and the distance between the globules. Under favourable circumstances it may extend from corner to corner, and thus establish contact; or *fissure* may occur, the tentacle breaking into spheroids; and these spheroids may cross over between the globules. This is the order of things usually to be observed, but the action is sometimes erratic.

Let us now bring back the scattered globules of mercury to their respective corners, and distribute a few isolated spheroids in the interspace. In addition to the effects previously noticed at the terminal globules, we now see that, when the current is applied, each intermediate spheroid extends a "finger" towards the positive globule. This is the process of mobile coherence; the short "fingers," or long

"tentacles," form links between consecutive spheroids, and finally a complete conducting circuit is established.

By successive applications of the current, any elongated bodies of mercury between the terminal globules can be made to creep along like caterpillars; the successive forward motions of the tentacle, or tail, cause a corresponding retrogression of the globule as a whole.

Any small spheroids scattered about the dish may be urged in a direction depending upon the direction of the successive current impulses; and a "finger" will always appear on the side towards the positive electrode. So that by choosing a convenient stray spheroid, and operating a battery-reverser as a transmitting-key, a telegraphic receiver is improvised from no other apparatus than a drop of mercury and a little oil. By some such means the awakening genius of primitive man may have contrived all the subtle machinery of a telegraph-instrument upon the smooth surface of an oyster-shell.

DISCUSSION.

Prof. RAMSAY said he had once attempted to facilitate churning by the application of 8 or 9 volts to some milk. He thought the cream came a little faster, but it turned sour very quickly.

Prof. FITZGERALD thought that the effects observed in experiment 3 were the result of current, and not of electrostatic changes; and he would like to know the value of the actual current used. There was no doubt that the motions were due to variations in capillarity.

Mr. SHELFORD BIDWELL asked how the mercury was formed into spheroids in the tube in experiment 1.

Mr. APPLEYARD, in replying to Prof. Fitzgerald, said it was not easy to define the circuit, as the terminal-globules were rather capricious, but he would try and measure the current in some particular case. The mercury-tube in experiment 1 was shaken in a horizontal plane; the operation took about ten minutes. Equal volumes of mercury and oil was a good proportion. One quarter of the length of the tube should be left as an air-space.

VII. *The Thermo-electric Properties of some Liquid Metals.*

By WILLIAM BECKIT BURNIE, lately 1851 Exhibition
Science Research Scholar, Nottingham*.

THE object of the experiments here to be described was to compare the thermo-electric properties of solid metals with those of the same metals when melted. The four metals employed, tin, lead, bismuth, and mercury, were each thermo-electrically compared with copper, the tested metal being contained in a hard glass tube, so that the observations could be pushed to temperatures considerably above those of the melting metals, and the changes in the thermo-electric properties during the process of melting observed. Two sets of experiments were made, the first set with the greater part of the metal under test at ordinary temperatures, and the second set with all the metal under test at high temperatures.

In the first set of experiments with tin, lead, and bismuth, the glass tube containing the metal was W-shaped, the metal filling the central part, but only rising about one-third of the height in the outside limbs. To fill this tube one end was dipped into a crucible full of the melted metal, which was then allowed to cool. The crucible and tube were immersed in a bath of linseed oil, which was raised to a temperature above that required to melt the metal. When the metal was quite liquid, air was withdrawn from the upper end of the tube till the metal had risen to the right height. The tube was next slowly withdrawn from the still hot oil, to prevent cracking due to the freezing of the metal, and allowed to cool. The superfluous metal was then melted out of the open limbs. The two copper-metal junctions were to be in the two open limbs of the tube, and a number of thick copper wires were so arranged round the outside of the tube as to conduct away the heat from all parts not in the immediate neighbourhood of the hot junction. By this means all the metal except a small portion was kept solid, and the level of the melted metal was always higher in the open than in the

* Read February 26, 1897.

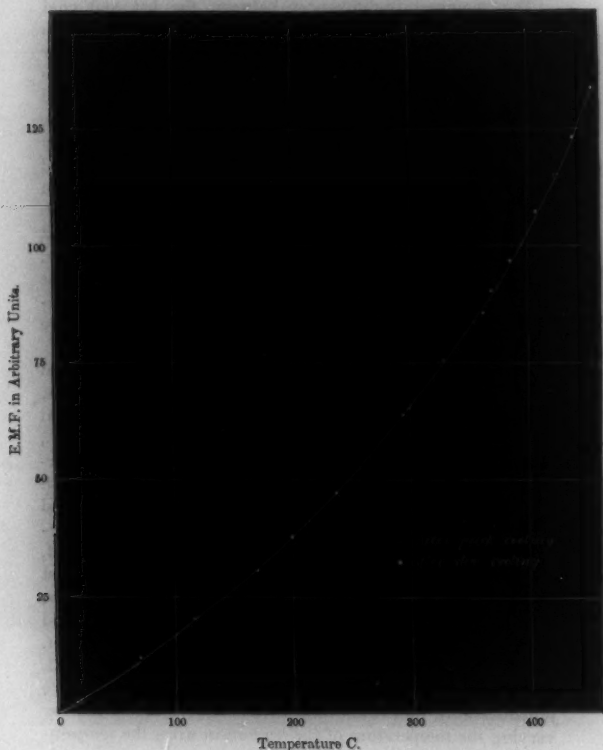
closed limb. This was necessary, as the solid metal was not a satisfactory cork for the tube, and a slight liquid pressure was needed to keep good contact between the liquid and solid metal.

The temperature-differences between the hot and cold junctions were measured with a thermo-element of platinum and a platinum-rhodium alloy (Pt 90 per cent., Rd 10 per cent.). This thermo-element was calibrated between the temperatures of 0°C. and $263^{\circ}\cdot 5\text{C.}$ by comparison, in linseed oil, with a Reichsanstalt standardized thermometer. The calibration curve was then extrapolated as far as 430°C. by means of an equation. For reasons which will be explained later, it was useless to adopt any more accurate method than this for the first set of experiments. The thermo-element was insulated within and from without with asbestos paper, and each junction was bound to one of the standard copper wires. These copper wires were also insulated with asbestos, so that only their ends, which were in close proximity to the junction of the temperature-measuring thermo-element, were uncovered.

Some experiments were made to determine the electrical insulating properties of asbestos paper at high temperatures, and it was found that the errors introduced by the electrical conductivity of the supposed insulator were, up to 600°C. , negligibly small. Other experiments were made to determine whether or not the temperature of the junction in asbestos differed greatly from the temperature of the metal surrounding the asbestos. One junction of the temperature-determining thermo-element was immersed in melted lead, and the other in melting ice. The parts of the wires just outside the lead were alternately heated with a burner, whereby the temperature-difference between the junction and the metal would be decreased, and cooled with an air blast, whereby the temperature-difference would be increased. Readings of the temperature-difference between the two junctions were meanwhile taken; and it was found that by varying the temperature of the leading-in wires the temperature of the hot junction was not altered by more than $\frac{1}{10}$ of a degree. As the leading-in wires were hotter, when warmed by the flame, than the melted lead, $\frac{1}{10}$ degree was

greater than the greatest error introduced during the experiments by the heat-insulating capability of the asbestos paper.

Fig. 1.—Copper-Lead Thermo-element.

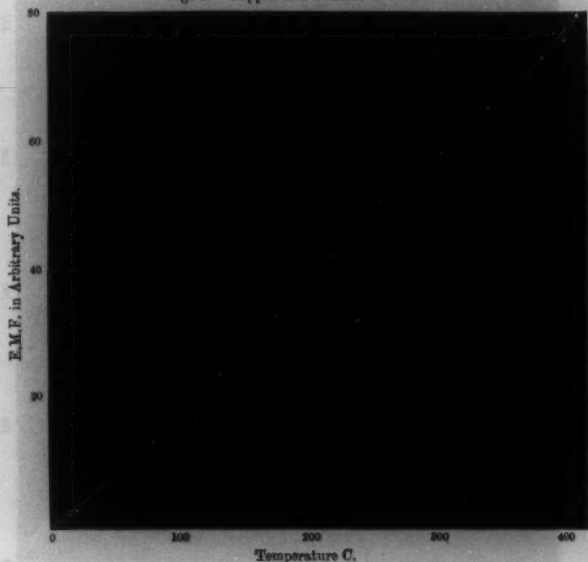


Each of the copper wires with its thermo-element junction was melted into the metal in its respective limb, and held in

the centre of the tube by asbestos paper. One limb of the W was heated in a sand bath, and the other was kept cold in melting ice.

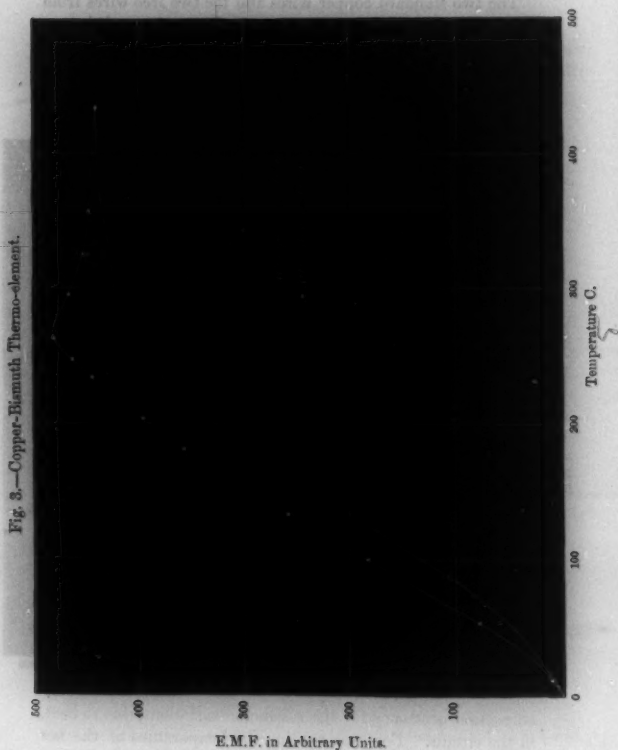
The two standard copper wires and the two free wires from the temperature-determining thermo-element were led to a paraffin switch-block, so that either pair could be connected, through a reversing key and a resistance, to a reflecting galvanometer. This resistance was so great as to render no

Fig. 2.—Copper-Tin Thermo-element.



correction necessary for the resistance variations, due to heat, in the circuit. For each reading the temperature of the hot junction was made nearly constant; then, using each thermo-electric combination alternately, galvanometer deflexions were noted with the current direct, and the current reversed. After the necessary correction, the mean of the readings of

the platinum-platinum-rhodium thermo-element gave the temperature, and the mean of the readings of the thermo-element under test gave the E.M.F. in arbitrary units.

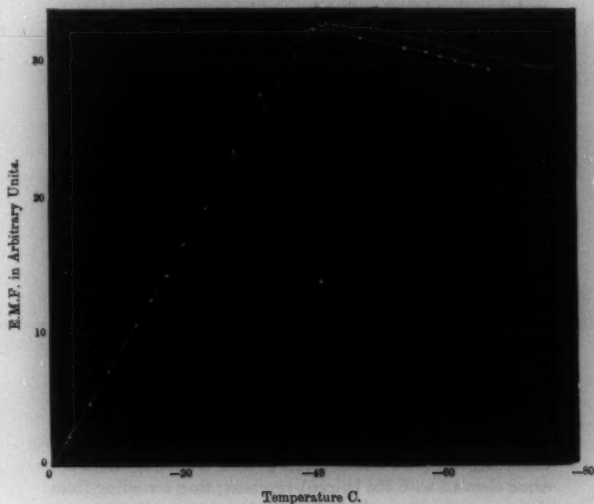


After each complete experiment, the constant of the temperature-measuring thermo-element was determined by removing

the crucible containing sand, putting in its place one containing melted lead, and allowing the lead to cool, meanwhile taking readings.

The curves for copper-lead, copper-tin, and copper-bismuth, which were obtained by this method, are given in figs. 1, 2, and 3 respectively. The abscissæ are temperatures and the ordinates E.M.F.'s in arbitrary units. These curves will be discussed when the mercury experiments have been described.

Fig. 4.—Copper-Mercury Thermo-element.



The W-shaped tube was not suitable for the application of a freezing-mixture, and so in the mercury experiments the junctions were each in a small test-tube about half filled with the metal. An inverted U-tube also full of mercury, with one end dipping into the metal in each of the test-tubes, connected the two. One of the test-tubes was surrounded by

melting ice, and the other was cooled with a mixture of solid carbon dioxide and ether. A mass of copper wire was wrapped round the tube to be cooled, in order that it might warm up slowly after the evaporation of the carbon dioxide.

As the temperatures to be measured approached the neutral point of the thermo-element used in the last experiments, an iron-constantan thermo-element was employed to measure the temperature-differences between the two junctions. This thermo-element was calibrated by the observation of the galvanometer deflexions when one junction was in melting ice and the other junction first in melting mercury and then in boiling carbon dioxide and ether. As these two deflexions were nearly proportional to their respective temperature-differences, the calibration curve equation was assumed to be of the form, deflexion $= at + bt^2$, where t is the temperature, and a and b constants. These two constants were determined from the two observations, and the calibration curve for the thermo-element was plotted from the equation. The element and the standard copper wires were insulated as in the last experiment, and bound to the glass tubes dipping into the mercury. The freezing-mixture was contained in a double-walled test-tube, the inner and outer walls being separated by cork distance-pieces, and the whole being wrapped in flannel. The electrical connexions were the same as in the last experiment.

In the experiments, the tube containing the cold junction was cooled as far as possible with the freezing-mixture, and allowed to warm slowly up. Readings of the galvanometer were meanwhile taken every thirty seconds, as follows:—First, a reading from the temperature-determining thermo-element with the current direct; then a reading from the same element with the current reversed; then a reading from the copper-mercury element with the current direct; and lastly, a reading from this element with the current reversed. The cycle then began again. From the means of each pair of readings, reduced, two curves were plotted, one giving the relation between temperature and time, and the other giving the relation between E.M.F. in arbitrary units and time.

The curves (fig. 4) give the relation between E.M.F. and

temperature deduced from these two curves, for two experiments.

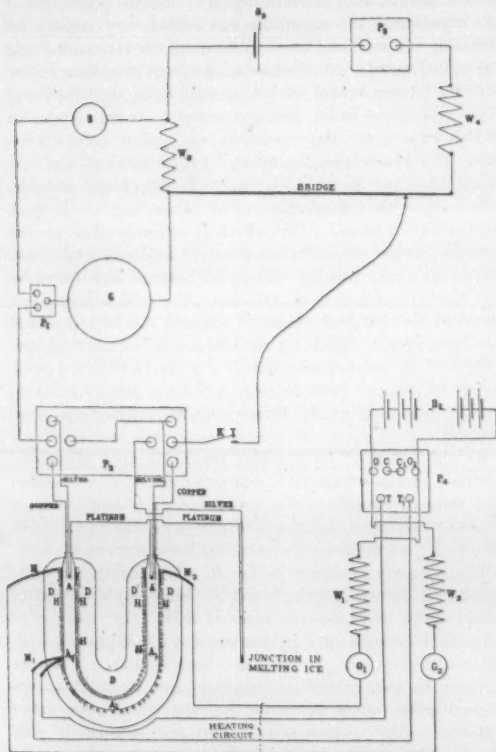
In figs. 2, 3, 4 two curves are in each case given. These two curves were obtained from the same metal. The effect is most marked with bismuth (fig. 3). At the conclusion of one experiment the apparatus was cooled very rapidly by removing the sand and blowing air on to the tube containing the melted metal. Another set of readings was then taken, with the temperature of the hot junction rising, and the lower of the two curves in fig. 3 was obtained. At the conclusion of this experiment the apparatus was cooled very slowly, about two hours being occupied, and the result of the next experiment was the upper curve. It was found possible, moreover, with intermediate rates of cooling to produce intermediate curves. This effect is probably due to the variations in the crystalline structure of the metal under test, dependent on the rapidity of its solidification; and it may be that the various thermo-electric properties which have been observed for different pieces of bismuth are largely due to this same cause. With tin the effect was less marked, and with lead it was unnoticeable. In the mercury experiments it was not possible to regulate the rate of cooling; but the two most widely differing curves obtained are given in fig. 4.

As this effect of its previous treatment upon the metal rendered accuracy impossible, the experiments were repeated with three of the metals, viz. tin, lead, and bismuth, with a method so arranged that the whole mass of the metal to be tested was in a molten state, and thus homogeneous.

The apparatus is shown in fig. 5. The metal was contained in a U-tube A A₁ A₁ A, and was heated by a current of electricity in german-silver wires H H H. The metal was introduced into the tube by means of a glass pipette, both tube and pipette being warmed in a bunsen flame. The part of the glass tube where the junctions were to be, A A, was covered with copper to render the heat distribution more uniform. The german-silver wire was insulated with asbestos paper, and wound spirally round the tube in two sections, M M₁, M₁ M₂. Over this were wrapped several layers of asbestos paper and asbestos string, DD. The

connexions of the heating circuit are shown in fig. 5. In the paraffin switch-block, P_4 , if $T C$ and $T_1 C_1$ were connected, the same current from the heating battery passed through

Fig. 5.



both ammeters, $G_1 G_2$, both resistances, $W_1 W_2$, and both sections of the heating coil. This caused a saving in current

when it was merely required to melt the necessary connexions into the metal, or to burn out the asbestos insulation. If, however, T O was connected the current passed through the ammeter G_1 , the resistance W_1 , and the long section of the heating coil; and if $T_1 O_1$ was connected the current went through the other ammeter, resistance, and heating-coil section. By adjusting the resistances W_1 and W_2 the temperatures at the junctions could be varied as required. The diameter of the german-silver wire was 0.5 mm., and the largest current required was 4 amperes. The U-tube is shown about one half actual size.

For measuring the temperatures a platinum-silver thermo-element was employed, as this gave E.M.F.'s of the same order of magnitude as those to be measured. This element had three junctions, as shown in fig. 5, so that either the temperature of one of the junctions or the temperature-difference between the two junctions of the thermo-couple under test could be measured. The thermo-element was calibrated with linseed oil up to 288°C. ; and one point on the calibration curve, at 441.4°C. , was determined with sulphur vapour. Each limb was calibrated separately, but they were found to be similar. The part of the calibration curve from 288°C. to 430°C. was plotted from the equation:—

$$\text{E.M.F. (in volts)} \times 10^3 = 0.1407 t + 0.001227 t^2.$$

This equation was obtained from the point on the curve determined with sulphur, and the point where $t = 220.7^\circ \text{C.}$ on the curve as drawn in between 0°C. and 288°C. By the application of the method of least squares to all the calibration observations a somewhat different equation was obtained, viz. :—

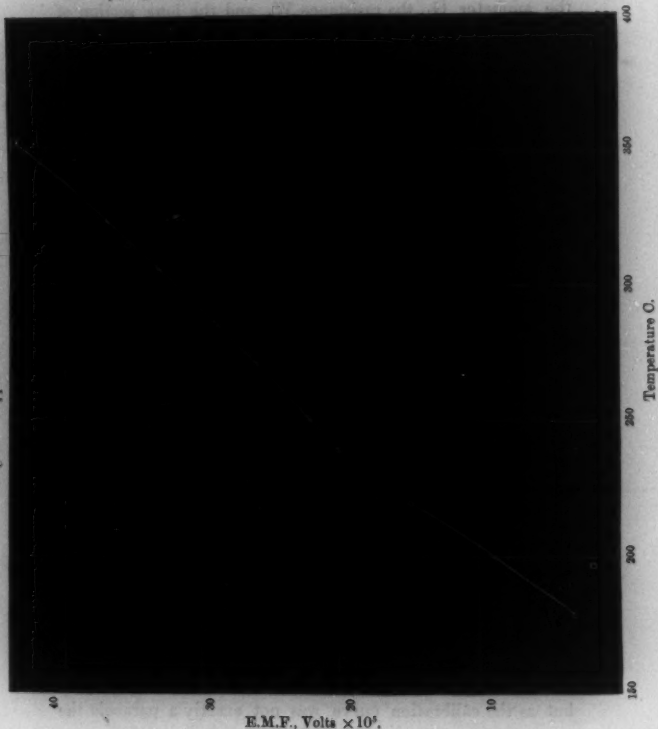
$$\text{E.M.F. (in volts)} \times 10^3 = 0.1463 t + 0.001238 t^2;$$

but as the calibration curve was not exactly a parabola the upper equation was judged the better.

The junctions and standard copper wires were insulated as in previous experiments and melted into the metals. The two standard wires and the three free wires from the temperature-determining thermo-element were led to a paraffin

switch-block, P_3 , which was connected to an arrangement of a galvanometer, G , a standard cell, S , a key, K , a bridge battery, B_2 , resistances, W_4 and W_3 , switch-blocks, P_1 and P_3 ,

Fig. 6.—Copper-Tin Thermo-element.

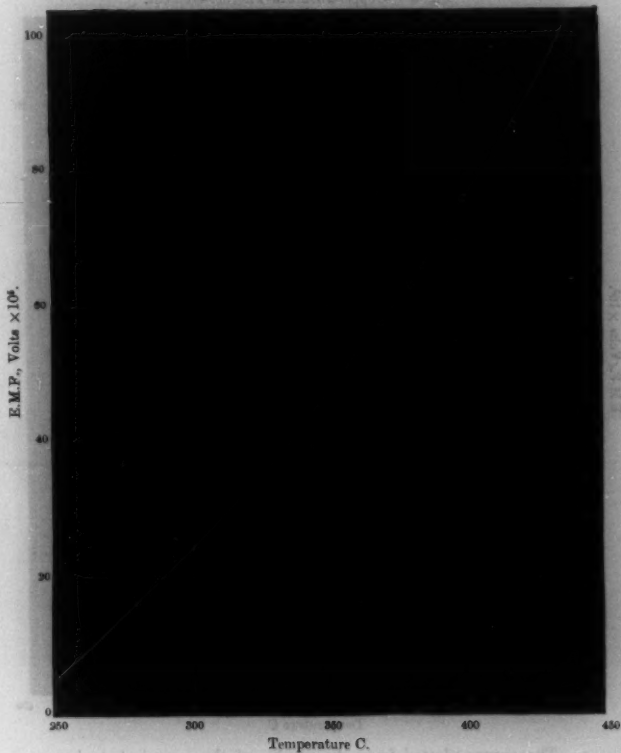


and a metre bridge, as shown in fig. 5, for measuring the E.M.F.'s.

The current in the long heating section was adjusted before each reading till the temperature of the fixed temperature

junction was at a certain point, slightly above the melting-point of the tested metal. Readings were taken with the temperature of the variable temperature junction both above

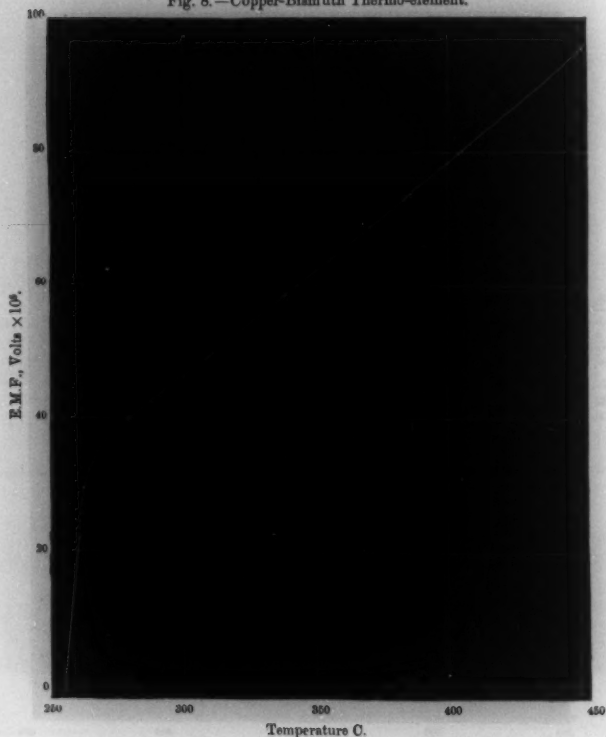
Fig. 7.—Copper-Lead Thermo-element.



and below the melting temperature of the metal. With tin and lead this latter was easy, but with bismuth, owing to

the crystallization effect before mentioned, the readings were rather uncertain. Above the melting-point, however, they were perfectly constant.

Fig. 8. — Copper-Bismuth Thermo-element.



The curves plotted from the results of the experiments by this method are given in figs. 6, 7, and 8 respectively.

In each of these curves we see that in a small variation of temperature, about the melting-point, there is a considerable change in the direction of the thermo-electric curve. The effect is smallest with lead, with tin it is larger, and with bismuth it is very remarkable; that metal changing, during melting, from an exceedingly active thermo-electric metal to one very similar to lead in its thermo-electric properties. With mercury also (fig. 4) we see that a great change takes place at the melting-point. This indicates that there is a difference between the Peltier effect for the solid and for the melted metal. But the change in curvature is not sufficiently marked for us to be able to say whether the specific heat of electricity remains proportional to the absolute temperature. To decide this point a direct determination of the Thomson effect is needed.

The worked-out observations from which the curves in figs. 6, 7, and 8 were plotted are given below.

Copper-Tin Element. (Fig. 6.)

Temperature of constant temperature junction 239°C .

Temperature of variable temperature junction.	E.M.F. in volts $\times 10^6$.
255.8	2.86
276	7.01
286.1	9.0
306.1	13.2
323.2	16.92
353.7	23.63
251	2.0
272.2	6.35
315.1	15.45
219.7	-5.1
200.1	-11.15
179.5	-16.86

2 *Copper-Lead Element.* (Fig. 7.)

Temperature of constant temperature junction $336^{\circ}\pm 1$ C.

Temperature of variable temperature junction.	E.M.F. in volts $\times 10^3$.
346.1	5.215
357	11.13
370.6	17.92
385.2	26.5
400.5	35.86
418.3	47.4
440.3	63.2
359.1	12.25
352.8	8.5
380.2	23.75
310.3	-11.9
307.3	-13.4
280.5	-23.6
251	-35

Copper-Bismuth Element. (Fig. 8.)

Temperature of constant temperature junction $278^{\circ}\pm 5$ C.

Temperature of variable temperature junction.	E.M.F. in volts $\times 10^3$.
287	2.6
291.4	3.97
308	9.01
315.7	12.05
356.7	24.5
385.1	33.68
405.9	40.15
453.6	56.25
388.3	34.18
339.2	18.94
309.2	9.73
259	-18.01
254.6	-41.2

In each case a constant has been added to the E.M.F.'s before plotting.

These experiments were conducted in the Physical Laboratory of the Eidgenössisches Polytechnikum, Zürich, under the supervision of the director, Prof. Dr. H. F. Weber, to whom my best thanks are due.

VIII. *On the Photography of Ripples.* By J. H. VINCENT, B.Sc., A.R.C.Sc., Assistant Demonstrator in Physics at the Royal College of Science, London, S.W.*

[Plates I.-III.]

MANY of the phenomena described in this paper have been exhibited at public lectures by Mr. C. V. Boys, using the stroboscopic method, which was first applied to the study of ripples by Lord Rayleigh. I am indebted to Mr. Boys for having recommended to me the work of photographing these effects, and also for many valuable suggestions.

Lord Kelvin defines a ripple as a wave whose length is less than that of the wave which is propagated with the minimum velocity. For ordinary mercury, waves less than 1.3 centim. long are ripples. Capillary ripples are those whose length is so small as to render negligible, in the value of the velocity squared, the term due to gravity. These definitions are rendered clearer by reference to Mr. Boys's Logarithmic Wave Chart. The portion of the curve, representing the relation of the velocity and wave-length, to the left of the point of minimum velocity, refers to ripples. The straight-line portion to the left of the chart represents capillary ripples.

In order to obtain ripples it is necessary to use vibration-frequencies above a certain value. Thus in the case of ordinary clean-looking mercury, with its damp and probably greasy surface, the surface-tension of which may be between 300 and 400 C.G.S. units, a frequency of about 15 per second

* Read February 26, 1897.

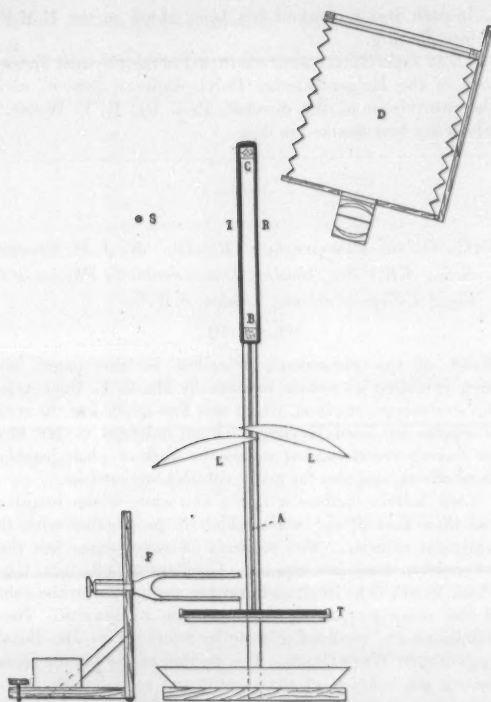


Diagram of Apparatus. $\frac{1}{16}$ nat. size.

- T. Mercury trough.
- A. Standard of retort-standard.
- B. Wooden block through which passes the bent down portion of A.
- C. Upper bar of wooden stand.
- IR. India-rubber loop.
- S. First spark-gap.
- L.L. Lenses.
- D. Camera.
- F. Tuning-fork and stand.

causes the biggest waves which the above definitions include as ripples; while frequencies of about 200 and upwards give rise to waves whose propagation is practically controlled by surface-tension, and these waves are capillary ripples.

Now the duration of the sensation produced by a luminous impression on the retina lasts for about one eighth of a second; thus we are unable to see ripples on the surface of mercury. The frequencies employed are generally many times the maximum visible frequency. It is not the high velocity of propagation which renders ripples invisible; ripples produced by a disturbance of a frequency of about 200 do not travel very quickly, a foot a second being about the order of magnitude of the velocity on the surface of ordinary mercury.

Description of the Apparatus.

A rectangular wooden trough, about 1 centim. deep and 12×15 centim. area, contains the mercury upon the surface of which the ripples are produced. This trough rests upon the rectangular base of a retort-stand, the upper portion of whose standard is bent at right angles over the base so as to be parallel with the latter. This horizontal portion is passed through a rectangular block of wood, and the whole is then slung from a gallows-like wooden structure one metre high.

The spark-gap from which the light proceeds to illuminate the mercury surface is placed near the top and to one side of the wooden stand. Light from this gap falls upon a lens to the left of the stand, so that the emergent light is parallel; after reflexion the light is collected by a second lens similar in all respects to the first, so that an image of the spark would be produced at the primary focus of the second lens; the focal length of these lenses was 44 centim. The camera is placed in such a position as to enable the first achromatic lens of the combination to collect the rays and converge them so as to come to a focus at the aperture in the lens stop. The stop used was the smallest of the set belonging to the camera (F 64). The camera is then focussed upon a fine thread laid on the surface of the mercury.

In addition to the first spark-gap, which was about .5 centim. across, a second gap was used in order to increase the brightness of the spark in the first gap. The second gap was

varied from time to time, but was generally 1.5 centim. across. The first gap was shunted by a piece of stout thread soaked in calcium-chloride solution. This prevented small sparks due to induction. The knobs of a Wimshurst machine were connected, one with a terminal of the second spark-gap and the inside coats of a battery of four half-gallon leyden-jars; the other Wimshurst terminal was connected to the outer coats, one side of the first spark-gap, and to earth. A wire joining the other terminals of the two gaps completes the spark arrangements.

Method of Causing the Ripples.

The ripples are due to the agitation of the surface of the mercury by a style of glass attached to one prong of a vibrating tuning-fork. The fork was in most cases struck with a rubber-shod hammer; but in the last two experiments, in which a strip of cover-glass attached to the fork acts as a line-source, it was found that the irregular large waves caused by the concussion entirely masked the phenomena which it was sought to photograph. In these cases the fork was maintained in synchronous vibration with another similar fork which was electrically excited. The two forks, placed approximately parallel, are tied together by a piece of thread about two feet long, so that the thread is at right angles to both forks. This thread is then adjusted to an appropriate tension by trial, when the maintained fork causes the other to vibrate for any length of time. This simple device, which has been used in the Royal College of Science laboratory for some time, is due to Mr. W. Watson.

Description of Photographs.

The plates used were of various kinds, and any of the well-known plates gave good results. Some of the negatives were intensified previous to printing. The figures are about $\frac{1}{2}$ natural size. My best thanks are due to Mr. R. Chapman, who has assisted me throughout with great zeal.

Photographs showing a series of circular waves set up by a single style attached to a fork of known frequency were taken with a view to quantitative measurements of surface-tension, wave-length, and velocity. A bar of wood was so

placed that two needle-points which it carried nearly touched the surface of the mercury. These points are a known distance apart, and by measuring the negatives we may find the scale of reduction of lengths along this line. The particulars of the motion are obtained from the equations

$$v^2 = n^2 \lambda^2 = \frac{g\lambda}{2\pi} + \frac{2\pi T}{\lambda\rho};$$

from which we find, in a particular experiment,

$$\begin{aligned} n &= 180 \text{ per second,} \\ \lambda &= .165 \text{ centim.,} \\ v &= 29.7 \text{ centim. a second,} \\ T &= 306 \text{ dynes per linear centim.} \end{aligned}$$

This low value of the surface-tension was obtained from mercury which had stood in the apparatus for some days. The values obtained in a similar way previously were 420, 421, 365. The tension falls as the mercury gets more contaminated. The value of the surface-tension of pure dry mercury is usually quoted at 540.

Fig. 1. Two styles are attached to the same prong of a fork, the frequency of which is 120. One centre is unfortunately hidden by the fork. The approximately straight dark lines which are seen to radiate from the region between the centres of oscillation are lines of minimum disturbance; they are hyperbolas of which the centres of disturbance are the foci. This photograph illustrates the interference phenomena in Optics produced by Young's or Fresnel's methods.

Fig. 2. The frequency of the fork is 256. Both styles are attached to the same prong. The photograph shows two series of interference-curves, one a family of hyperbolas analogous to those shown in fig. 1, and the other a family of ellipses.

The hyperbolas are the radiating light lines seen on the side of the photograph remote from the fork. They are fixed in position, the little dark facets moving along between pairs of hyperbolas.

The light oval curves in the region between the centres of disturbance are ellipses, since they are the loci of points of intersection of the two series of circles whose radii grow uniformly, and at the same rate with time. Their method

of production here is similar to a well-known geometrical construction for ellipses. Unlike the system of hyperbolas, these ellipses are not at rest. They travel outwards in such a way that any ellipse occupies a position which was filled previously by its predecessor a whole period before. That semiaxis of any ellipse which passes through a centre of disturbance grows with the same velocity as that with which the ripples are propagated. The other semiaxis grows with a velocity which is infinite at the commencement, but which gradually decreases to the same uniform velocity of growth as that of the first. The law of decrease of velocity is the same as the law of decrease of the lengths of whole-period elements of a linear wave with respect to a point.

In order to render these ellipses stationary it would be necessary to change one of the sources into a sink to which the circular waves converge. This could be experimentally realized with ripples by causing a circular arc and a style to be agitated by the same prong of a fork, when the effects would be analogous to M. Meslin's experiment in Optics.

Fig. 3. Frequency 256.

This photograph is very similar to fig. 1; but in addition to showing interference phenomena like those of Fresnel and Young, it also illustrates interference effects in which the direction of propagation of light is parallel to the line joining the point-sources. Thus in the photograph, if we consider the disturbance anywhere on a right line drawn perpendicular to a line joining the two point-sources produced, we see that the places of no disturbance are symmetrical about the line joining the sources. They are points on the system of hyperbolas already mentioned.

In M. Meslin's method of producing interference-fringes the screen is placed between the two point-centres, one a source and the other a sink. The bands are circular, and are sections of ellipsoids of revolution, and not of hyperboloids, such as the fringes in the photograph would become if the whole picture were rotated about the line joining the point-sources. These fringes are not seen on the screen in M. Meslin's experiment with the split lens, even when the screen is placed beyond the second focus, because the pencils do not there overlap. It seems that modifications of



Fig. 1.



Fig. 2.

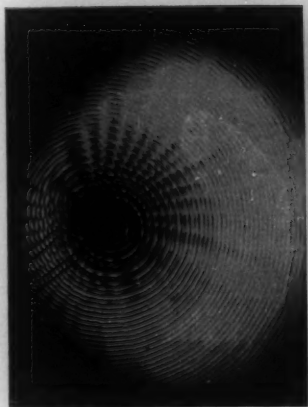


Fig. 3.



Fig. 4.



Fig. 5.



Fig. 6.

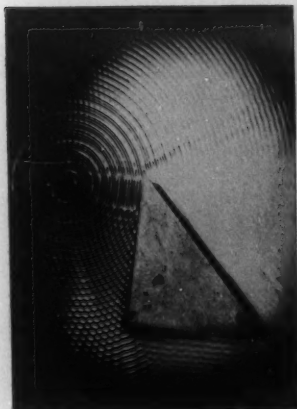


Fig. 7.

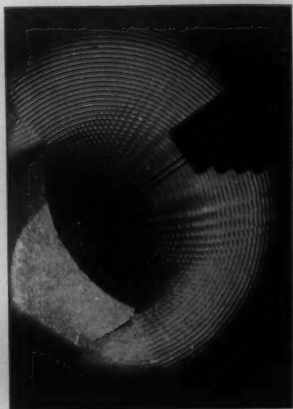


Fig. 8.

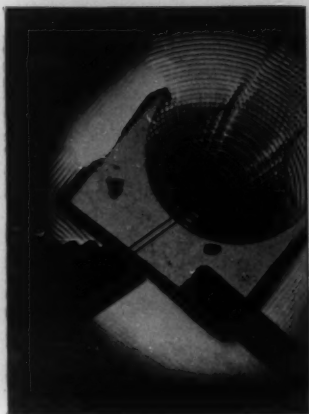


Fig. 9.



Fig. 10



Fig. 11.



Fig. 12



M. Meslin's experiment could be devised so as to enable complete circular fringes to be seen, and also to render the sections of hyperboloids visible. For example, it appears probable that if a circular portion of a convex lens were cut out and the central portion moved towards the original point-source, the sections of hyperboloids of revolution would be visible on a screen placed beyond the second focus.

Fig. 4. The two sets of ripples are produced by a fork of frequency 128 and another of frequency about 112. These two forks then produce 16 beats a second. The curved light lines represent places of minimum disturbance at the instant when the spark occurred. These lines are not stationary as in No. 3, but rotate towards their convexities. The centre of disturbance from which they move is the one of higher frequency. If we consider a point anywhere on the surface of the mercury, beats occur at that point with the same frequency as the passage of these lines of minimum disturbance takes place over the point. Thus, 5 of these lines cross any point per second.

Fig. 5. This shows ripples produced by two forks, the higher of which has a frequency four times as great as the lower, the frequency of which is 128. If we neglect the effect of gravity,

$$v^2 = n^2 \lambda^2 = \frac{2\pi T}{\lambda \rho},$$

from which it follows that the wave-length of the ripples from the higher fork should be half that due to the lower. This relation is approximately true for these ripples.

Fig. 6. Frequency 180.

This photograph shows a point-source and a reflecting-line, the latter is a side of a triangular piece of microscope cover-glass, which is kept in position by a small splinter of wood. The interference-lines which are shown are due to the mutual action of the primary and the reflected waves. The phenomena exhibited are analogous to Lloyd's single-mirror fringes in Optics.

Faint signs of diffraction invading the geometrical shadow of the obstacle can be seen. The region of shadow is covered by faint lines parallel to the nearest side of the triangle acting

as a line-source. The wave-length is the same as that of the primary waves, and the effect is due to forced vibrations.

Fig. 7. This photograph illustrates reflexion and forced vibrations. The light curved lines in the region between the source and the nearest side of the triangle are similar to those between the two sources in fig. 2. They are due to the interference between the source and its virtual image.

Fig. 8. Frequency 256.

Here we have a shallow circular reflector with the source placed approximately at the principal focus. The reflected waves are circles of large radii; the very slight outward curvature at the ends shows that even when the reflector has an arc of about 60° the effect of spherical aberration is small. Since the reflected waves come from a virtual point-source, we have, as interference-lines, a series of confocal ellipses and confocal hyperbolas; the latter are fixed, but the former travel away from the line joining the source and its image. If the reflected waves had been rectilinear, both these sets of curves would have become parabolas.

Fig. 9. Frequency 256.

The centre of disturbance here coincides very closely with the principal focus of the central portion of the semicircular reflector. The reflected ripples are straight lines in the middle, but are bent outwards from the reflector towards the ends. This illustrates spherical aberration.

Fig. 10. Frequency 256.

The obstacle is a small round cover-slip floating on the mercury. The ripple-shadow is slightly encroached upon by the waves bending round the edge of the obstacle. One side of the disk acts as a convex circular mirror, and the interference-fringes are due to the mutual action of the source and its virtual image situated within the circumference of the disk.

Fig. 11. The frequency in this and fig. 12 is 120.

Here straight-line waves are originated by the agitation of a slip of cover-glass, one side of which dips into the mercury. The waves are reflected from the shallow circular mirror, and converge to the principal focus. Two series of parabolic interference-fringes are shown. They are confocal, and have

their concavities directed towards the source and reflector respectively.

Fig. 12. Similarly excited waves are reflected at an angle of about 45° from a straight edge. The long black mark running from one end of the dipping edge to the corner of the print is due to a depression in the surface caused by a floating needle, put there to screen off the circular waves coming from the end of the strip of glass.

Diffraction is well shown in this photograph.

DISCUSSION.

Mr. Boys congratulated the author upon the way in which the experimental difficulties had been overcome. The results would bear a good deal of close examination, and they would be found to present analogues of the greatest service in demonstrating the phenomena of acoustics and optics. Such photographs were far better than geometrical pictures drawn by instruments. For example, in the photograph illustrating the regions of minimum disturbance by lines radiating from a two-point source, it was easy to make out the positions where the two series of waves were half a period behind one another. The crests and troughs appeared as a set of dark and light concentric alternating circles, broken up into short arcs by radiating lines—the loci of minimum disturbance; all the crests on one side of any particular radiating line were seen to correspond to troughs on the other side, so that the field of disturbance was mapped out as in acoustics. One set of phenomena yet awaited illustration by this photographic method, and that was “diffraction” from a grating. It might be possible to use as an exciter a comb with chisel-shaped points. He did not think it would be possible to go quite so far as to reproduce analogues of spectral analysis. Since wave-length varies with surface tension, it was possible to vary the wave-length by dropping a little ox-gall or soap solution upon the mercury surface.

Mr. BLAKESLEY asked why no reflections occurred from the sides of the mercury retainer.

Mr. BOYS said the waves were lost at the edges of the meniscus. The mercury was kept in position by an annular ring of thin glass.

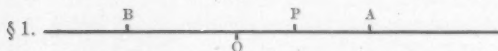
Mr. APPELYARD suggested that the analogue of refraction might be obtained by an alteration of the surface-tension over a small area by amalgamation or other means.

Mr. VINCENT thought this could be done, but that it would be very difficult.

IX. *Alternating Currents in Concentric Cables.*

*By W. A. PRICE, M.A.**

Fig. 1.



AOB is part of an infinite electrical conductor possessing resistance and capacity, the inductance being negligible. The characteristic differential equation is

$$\frac{d^2u}{dx^2} = \rho\gamma \frac{du}{dt},$$

ρ being the resistance, and γ the capacity per unit of length. At A, B are inserted equal alternating electromotive forces in the same phase, each represented by $2E \sin \omega t$, so that there is no current at O the middle point of AB. The distance AB is $2L$. At any point P distant x from O the potential due to the electromotive force acting at A is

$$E e^{-\alpha(L-x)} \sin \{\omega t - \alpha(L-x)\}$$

where

$$\alpha = \sqrt{\frac{\omega\rho\gamma}{2}},$$

and the current at P due to this force acting at A is

$$\frac{E}{\sqrt{2}} \frac{\alpha}{\rho} e^{-\alpha(L-x)} [\sin \{\omega t - \alpha(L-x)\} + \cos \{\omega t - \alpha(L-x)\}].$$

* Read April 9, 1897.

The current at P due to the electromotive force acting at B is

$$\frac{E}{\sqrt{2}} \cdot \frac{\alpha}{\rho} e^{-\alpha(L+x)} [\sin \{\omega t - \alpha(L+x)\} + \cos \{\omega t - \alpha(L+x)\}]$$

acting in an opposite direction to the first.

The potential at P due to the electromotive force acting at B is

$$E e^{-\alpha(L+x)} \sin \{\omega t - \alpha(L+x)\}.$$

Then the resultant potential at A is

$$E \{\sin \omega t + e^{-2\alpha L} \sin (\omega t - 2\alpha L)\};$$

the potential at O is

$$2E e^{-\alpha L} \sin (\omega t - \alpha L);$$

the current at A is

$$\frac{E}{\sqrt{2}} \cdot \frac{\alpha}{\rho} \{1 - 2e^{-2\alpha L} \cos 2\alpha L + e^{-4\alpha L}\}^{\frac{1}{2}} \cos \omega t.$$

Changing the time from which t is reckoned, so that ωt becomes $\omega t + \phi$, where

$$\tan \phi = \frac{1 - e^{-2\alpha L} (\cos 2\alpha L + \sin 2\alpha L)}{1 - e^{-2\alpha L} (\cos 2\alpha L - \sin 2\alpha L)},$$

and writing

$$\frac{E}{\sqrt{2}} \cdot \frac{\alpha}{\rho} \{1 - 2e^{-2\alpha L} \cos 2\alpha L + e^{-4\alpha L}\}^{\frac{1}{2}} \text{ as } A, \text{ and } L\gamma \text{ as } \Gamma,$$

the current at A becomes $A \cos \omega t$;

the potential at A becomes

$$\frac{A}{\omega \Gamma} \alpha L \frac{(\sinh 2\alpha L + \sin 2\alpha L) \sin \omega t + (\sinh 2\alpha L - \sin 2\alpha L) \cos \omega t}{\cosh 2\alpha L - \cos 2\alpha L};$$

and the potential at O becomes

$$\frac{2A}{\omega \Gamma} \alpha L \frac{(\cos \alpha L \sinh \alpha L + \sin \alpha L \cosh \alpha L) \sin \omega t + (\cos \alpha L \sinh \alpha L - \sin \alpha L \cosh \alpha L) \cos \omega t}{\cosh 2\alpha L - \cos 2\alpha L}.$$

The form and notation of these expressions are those given in 'Alternating Currents of Electricity,' T. H. Blakesley, London, 1889.

Consider the circular electrical conductor AB (fig. 2), having a total resistance $2\pi r$ and a total capacity $2\pi c$: the characteristic equation being

$$\frac{d^2 u}{d\theta^2} = rc \cdot \frac{du}{dt}.$$

At a time $t=0$, when the conductor is uncharged, a charge $2\pi S$ is introduced at A. The charge at any point P at a subsequent time t is

$$s = S \sum_{-\infty}^{\infty} \cos i\theta e^{-\mu^2 t}, \text{ where } \mu = \frac{1}{rc}:$$

$s d\theta$ being the charge at P, where $AOP = \theta$, of a small arc subtending $d\theta$ at the centre.

If a series of charges, $2\pi S \cos \omega \tau \cdot d\tau$, is introduced at A at intervals of time $d\tau$ beginning when $\tau=0$, s , the charge at P at a time t ,

$$= S \sum_{-\infty}^{\infty} \cos i\theta \left\{ \frac{\mu^2 \cos \omega t + \omega \sin \omega t}{\omega^2 + \mu^2 t^4} - \frac{\mu^2}{\omega^2 + \mu^2 t^4} e^{-\mu^2 t} \right\}.$$

If this succession of charges equivalent to a permanent alternating current $2\pi S \cos \omega t$ be continued for a long time, the term involving $e^{-\mu^2 t}$ becomes very small and we have

$$s = S \sum_{-\infty}^{\infty} \cos i\theta \frac{\mu^2 \cos \omega t + \omega \sin \omega t}{\omega^2 + \mu^2 t^4}.$$

The potential at A is

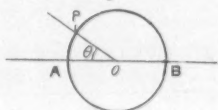
$$\frac{S}{\omega c} \sin \omega t \sum_{-\infty}^{\infty} \left(1 + \frac{\mu^2 t^4}{\omega^2}\right)^{-1} + \frac{S}{\omega c} \cos \omega t \sum_{-\infty}^{\infty} \frac{\mu^2}{\omega} \left(1 + \frac{\mu^2 t^4}{\omega^2}\right)^{-1};$$

the potential at B is

$$\frac{S}{\omega c} \sin \omega t \sum_{-\infty}^{\infty} (-1)^i \left(1 + \frac{\mu^2 t^4}{\omega^2}\right)^{-1} + \frac{S}{\omega c} \cos \omega t \sum_{-\infty}^{\infty} (-1)^i \frac{\mu^2}{\omega} \left(1 + \frac{\mu^2 t^4}{\omega^2}\right)^{-1}.$$

It may be seen that the electrical actions in the two cases discussed are identical so far as the portion AB of the conductor in the first case is concerned.

Fig. 2.



Writing $L\gamma = \pi c$, $L\rho = \pi r$, $\pi S = \Lambda$, and $\sqrt{\frac{\omega}{2\mu}}$ as β , so that $\alpha L = \beta\pi$,

$$\sum_{-\infty}^{\infty} \left(1 + \frac{i^4}{4\beta^4}\right)^{-1} = \beta\pi \frac{\sinh 2\beta\pi + \sin 2\beta\pi}{\cosh 2\beta\pi - \cos 2\beta\pi}$$

[this expression I shall refer to as $f_1(\beta)$];

$$\sum_{-\infty}^{\infty} \frac{i^2}{2\beta^2} \left(1 + \frac{i^4}{4\beta^4}\right)^{-1} = \beta\pi \frac{\sinh 2\beta\pi - \sin 2\beta\pi}{\cosh 2\beta\pi - \cos 2\beta\pi}$$

[this expression I shall refer to as $f_2(\beta)$];

$$\sum_{-\infty}^{\infty} (-1)^i \left(1 + \frac{i^4}{4\beta^4}\right)^{-1} = 2\beta\pi \frac{\cos \beta\pi \sinh \beta\pi + \sin \beta\pi \cosh \beta\pi}{\cosh 2\beta\pi - \cos 2\beta\pi}$$

[this expression I shall refer to as $f_3(\beta)$];

$$\sum_{-\infty}^{\infty} (-1)^i \frac{i^2}{2\beta^2} \left(1 + \frac{i^4}{4\beta^4}\right)^{-1} = 2\beta\pi \frac{\cos \beta\pi \sinh \beta\pi - \sin \beta\pi \cosh \beta\pi}{\cosh 2\beta\pi - \cos 2\beta\pi}$$

[this expression I shall refer to as $f_4(\beta)$].

In the particular case where β is integral

$$\sum_{-\infty}^{\infty} \left(1 + \frac{i^4}{4\beta^4}\right)^{-1} = \sum_{-\infty}^{\infty} \frac{i^2}{2\beta^2} \left(1 + \frac{i^4}{4\beta^4}\right)^{-1} = \beta\pi \coth \beta\pi,$$

$$\sum_{-\infty}^{\infty} (-1)^i \left(1 + \frac{i^4}{4\beta^4}\right)^{-1} = \sum_{-\infty}^{\infty} (-1)^i \frac{i^2}{2\beta^2} \left(1 + \frac{i^4}{4\beta^4}\right)^{-1} = \beta\pi \operatorname{cosech} \beta\pi.$$

These last expressions may also be obtained by expanding $(\cos \beta\theta \pm \sin \beta\theta)e^{\beta\theta}$ in Fourier series of cosines.

For

$$\begin{aligned} \beta\pi(\cos \beta\theta - \sin \beta\theta)e^{\beta\theta} \\ = \sum_{-\infty}^{\infty} (-1)^i \{(-1)^{\beta} e^{\beta\pi} - (-1)^i\} \left\{1 + \frac{i^4}{4\beta^4}\right\}^{-1} \cos i\theta, \end{aligned}$$

$$\begin{aligned} \beta\pi(\cos \beta\theta + \sin \beta\theta)e^{\beta\theta} \\ = \sum_{-\infty}^{\infty} (-1)^i \{(-1)^{\beta} e^{\beta\pi} + (-1)^i\} \frac{i^2}{2\beta^2} \left\{1 + \frac{i^4}{4\beta^4}\right\}^{-1} \cos i\theta. \end{aligned}$$

Putting θ equal to 0 and π successively in these series, and adding and subtracting the series thus obtained in pairs, the four last expressions are obtained. The method is applicable only when β is integral.

In a paper by Dr. Glaisher ('Proc. London Math. Soc.,' vol. vii.) the value of $\sum_{-\infty}^{\infty} \frac{1}{r^2 + x^2}$ is determined by another method which gives the same result as the above.

The physical significance of β being integral is that in that case the value of ω is such that the alternations at the sending and receiving ends of the cable are in the same phase. The charge at the sending end being

$$s = \frac{S}{\omega} \beta \pi \cosh \beta \pi (\sin \omega t + \cos \omega t)$$

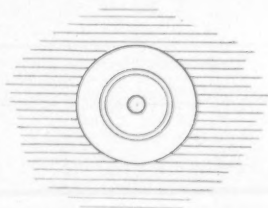
and at the receiving end

$$s = \frac{S}{\omega} \beta \pi \operatorname{cosech} \beta \pi (\sin \omega t + \cos \omega t),$$

the ratio of the amplitudes being $\cosh \beta \pi$.

§ 2. A cable, shown in section in fig. 3, contains two conductors, an inner central conductor and an outer concentric conductor, insulated from one another and the water in which the whole is immersed. The whole cable is supposed to be arranged in the circular form of fig. 2.

Fig. 3.



$2\pi r$, $2\pi r'$ are the total resistances of the inner and outer conductors;

$2\pi m$ the total capacity of the inner dielectric measured between the two conductors;

$2\pi n$ the total capacity of the outer dielectric measured between the outer conductor and the water.

Then $2\pi \frac{mn}{m+n}$ is the capacity of the central conductor, the outer one being insulated; and $2\pi n$ the capacity of the outer conductor, the inner one being insulated.

u , u' , CC' are the potentials of and the currents in the two conductors at point P and time t , and s , s' the density of the

charges on the conductors. All these quantities being referred to a unit of length subtending unit angle at the centre, then

$$rC = -\frac{du}{d\theta}, \quad r'C' = -\frac{du'}{d\theta};$$

$$-\frac{dC}{d\theta} = m\left(\frac{du}{dt} - \frac{du'}{dt}\right), \quad -\frac{dC'}{d\theta} = m\left(\frac{du'}{dt} - \frac{du}{dt}\right) + n\frac{du}{dt};$$

$$s + s' = nu', \quad \text{and} \quad s = m(u - u').$$

Also
$$\left(\frac{d^2}{d\theta^2} - mr\frac{d}{dt}\right)s + mr'\frac{d}{dt}s' = 0,$$

and
$$\left(\frac{d^2}{d\theta^2} - (m+n)r'\frac{d}{dt}\right)s' + mr\frac{d}{dt}s = 0.$$

Hence s and s' satisfy the equation

$$\left(\mu\frac{d^2}{d\theta^2} - \frac{d}{dt}\right)\left(\nu\frac{d^2}{d\theta^2} - \frac{d}{dt}\right)s = 0,$$

where μ, ν are roots of $mr r' x^2 - (mr + (m+n)r')x + 1 = 0$.

§3. A uniform conductor Q, along which the propagation of charges due to any electrical disturbance is determined by the differential equation

$$\left(\mu\frac{d^2}{d\theta^2} - \frac{d}{dt}\right)\left(\nu\frac{d^2}{d\theta^2} - \frac{d}{dt}\right)s = 0,$$

is arranged in a circle as in fig. 2. Its total resistance is $2\pi r$, and its total capacity $2\pi c$. Any disturbance will be propagated along the conductor in two distinct systems, one determined by the differential equation

$$\left(\mu\frac{d^2}{d\theta^2} - \frac{d}{dt}\right)s = 0, \quad \text{the other by} \quad \left(\nu\frac{d^2}{d\theta^2} - \frac{d}{dt}\right)s = 0;$$

and the disturbance produced at any given point will be the sum of two disturbances conveyed by the two systems.

Since each propagation is determined by a differential equation of the same form as that which applies to a single isolated conductor, such as is discussed in § 1, the action of Q is equivalent to that of a pair of single conductors, and the propagation along Q is the sum of the propagations along the equivalent pair.

Let the total resistances of the two equivalent cables be $2\pi r_1$ and $2\pi r_2$, and their capacities $2\pi c_1$ $2\pi c_2$. Then $\mu r_1 c_1 = 1$ and $\nu r_2 c_2 = 1$.

Since the pair is electrically equivalent to Q, they must convey the same steady current as Q, when subject to a steady E.M.F., and must be charged to the same potential by a given charge. So $r_1^{-1} + r_2^{-1} = r^{-1}$, and $c_1 + c_2 = c$. From these four equations r_1 r_2 c_1 c_2 can be determined.

Then if a charge $2\pi S$ be introduced into this conductor at any point, the charge at any point after a time t will be the sum of the charges at the corresponding points of the equivalent pair, if the same charge had been introduced into them in the proportions $2\pi S \frac{c_1}{c}$, $2\pi S \frac{c_2}{c}$.

If an alternating current $2\pi S \cos \omega t$ be introduced into the conductor at A where $\theta = 0$, the charge at P will be

$$s = \frac{S c_1}{\omega c} \sum_{-\infty}^{\infty} \cos i\theta \frac{\mu^3 \cos \omega t + \omega \sin \omega t}{\omega^2 + \mu^2 i^4} \\ + \frac{S c_2}{\omega c} \sum_{-\infty}^{\infty} \cos i\theta \frac{\nu^3 \cos \omega t + \omega \sin \omega t}{\omega^2 + \nu^2 i^4} = s_1 + s_2.$$

§ 4. Applying this argument to the conductors of the cable described in § 2, let r_1 r_2 c_1 c_2 be the resistances and capacities of the pair equivalent to the central conductor, and r_1' r_2' c_1' c_2' of the pair equivalent to the concentric; so that

$$r_1^{-1} + r_2^{-1} = r^{-1}, \quad r_1'^{-1} + r_2'^{-1} = r'^{-1};$$

$$r_1 c_1 \mu = r_2 c_2 \nu = r_1' c_1' \mu = r_2' c_2' \nu = 1;$$

$$c_1 + c_2 = \frac{mn}{m+n} = c, \quad c_1' + c_2' = c' = n.$$

Then
$$\frac{c_1}{c} = \frac{c_1'}{c'} = \frac{\mu}{\mu - \nu} (1 - m\nu r),$$

and
$$\frac{c_2}{c} = \frac{c_2'}{c'} = \frac{\nu}{\nu - \mu} (1 - m\mu r).$$

Then the charge at any point P due to an alternating current $2\pi S \cos \omega t$ introduced at A, where $\theta = 0$, is $s = s_1 + s_2$,

where

$$s_1 = S \frac{\mu}{\mu - \nu} (1 - m\mu r) \sum_{-\infty}^{\infty} \cos i\theta \frac{\mu i^2 \cos \omega t + \omega \sin \omega t}{\omega^2 + \mu^2 i^4},$$

$$s_2 = S \frac{\nu}{\nu - \mu} (1 - m\nu r) \sum_{-\infty}^{\infty} \cos i\theta \frac{\nu i^2 \cos \omega t + \omega \sin \omega t}{\omega^2 + \nu^2 i^4}.$$

§ 5. If an alternating current $2\pi S \cos \omega t$ be introduced into the concentric conductor at A, the charge s' at any point P $= s_1' + s_2'$, where

$$s_1' = \frac{S \nu (1 - m\nu r)}{\omega} \sum_{-\infty}^{\infty} \cos i\theta \frac{\mu i^2 \cos \omega t + \omega \sin \omega t}{\omega^2 + \mu^2 i^4},$$

$$s_2' = \frac{S \mu (1 - m\mu r)}{\omega} \sum_{-\infty}^{\infty} \cos i\theta \frac{\nu i^2 \cos \omega t + \omega \sin \omega t}{\omega^2 + \nu^2 i^4}.$$

§ 6. Consider the charge s' induced in the concentric conductor by the charge $s_1 + s_2$ in the central conductor.

From § 2,

$$\left(\frac{d^2}{d\theta^2} - m\mu r \frac{d}{dt} \right) (s_1 + s_2) + m\nu r \frac{d}{dt} s' = 0,$$

$$\frac{d^2}{d\theta^2} s_1 = \frac{1}{\mu} \frac{d}{dt} s_1, \quad \frac{d^2}{d\theta^2} s_2 = \frac{1}{\nu} \frac{d}{dt} s_2;$$

hence
$$\frac{1 - m\mu r}{\mu} \frac{ds_1}{dt} + \frac{1 - m\nu r}{\nu} \frac{ds_2}{dt} + m\nu r \frac{ds'}{dt} = 0.$$

Integrating, $m\nu r s' = -\frac{1}{\mu} (1 - m\mu r) s_1 - \frac{1}{\nu} (1 - m\nu r) s_2 + \text{some expression } \Theta \text{ independent of } t.$

$$\Theta \text{ is periodic in } \theta, \text{ satisfies } \left(\mu \frac{d^2}{d\theta^2} - \frac{d}{dt} \right) \left(\nu \frac{d^2}{d\theta^2} - \frac{d}{dt} \right) \Theta = 0,$$

and has a mean value zero. Hence $\Theta = 0$, and

$$m\nu r s' = -\frac{1}{\mu} (1 - m\mu r) s_1 - \frac{1}{\nu} (1 - m\nu r) s_2.$$

In the same way the charge s induced in the central conductor by the distribution $s_1' + s_2'$ in the concentric con-

ductor is given by the equations

$$\left\{ \frac{d^2}{d\theta^2} - (m+n)r' \frac{d}{dt} \right\} (s_1' + s_2') + mr \frac{d}{dt} s = 0,$$

$$\frac{d^2}{d\theta^2} s_1' = \frac{1}{\mu} \frac{d}{dt} s_1', \quad \frac{d^2}{d\theta^2} s_2' = \frac{1}{v} \frac{d}{dt} s_2',$$

and
$$mrs = \frac{1}{v} (1 - mrv) s_1' + \frac{1}{\mu} (1 - mr\mu) s_2'.$$

§ 7. Suppose an alternating current $2\pi A \cos \omega t$ to be introduced into the central conductor at A where $\theta=0$, and a current $2\pi B \cos (\omega t + \phi)$ into the same conductor at B where $\theta=\pi$.

The charge at B on the central conductor due to the current introduced at A is, writing

$$\sqrt{\frac{\omega}{2\mu}} \text{ as } \beta, \text{ and } \sqrt{\frac{\omega}{2v}} \text{ as } \gamma,$$

$$\begin{aligned} \frac{A c_1}{\omega c} \{ \sin \omega t f_3(\beta) + \cos \omega t f_4(\beta) \} \\ + \frac{A c_2}{\omega c} \{ \sin \omega t f_3(\gamma) + \cos \omega t f_4(\gamma) \}. \end{aligned}$$

The charge at B on the central conductor due to the current introduced at B is

$$\begin{aligned} \frac{B c_1}{\omega c} \{ \sin (\omega t + \phi) f_1(\beta) + \cos (\omega t + \phi) f_2(\beta) \} \\ + \frac{B c_2}{\omega c} \{ \sin (\omega t + \phi) f_1(\gamma) + \cos (\omega t + \phi) f_2(\gamma) \}. \end{aligned}$$

The charge at B induced on the concentric conductor by the current introduced at A is

$$\begin{aligned} - \frac{1}{m\mu r'} (1 - m\mu r) \frac{A c_1}{\omega c} \{ \sin \omega t f_3(\beta) + \cos \omega t f_4(\beta) \} \\ - \frac{1}{mvr'} (1 - mvr) \frac{A c_2}{\omega c} \{ \sin \omega t f_3(\gamma) + \cos \omega t f_4(\gamma) \}. \end{aligned}$$

The charge at B induced on the concentric conductor by

the current introduced at B is

$$-\frac{1}{m\mu r'}(1-m\mu r)\frac{B}{\omega c}c_1\{\sin(\omega t+\phi)f_1(\beta)+\cos(\omega t+\phi)f_2(\beta)\} \\ -\frac{1}{m\mu r'}(1-m\mu r)\frac{B}{\omega c}c_2\{\sin(\omega t+\phi)f_1(\gamma)+\cos(\omega t+\phi)f_2(\gamma)\}.$$

The potential of the central conductor at the point B is $\frac{(m+n)s+ms'}{mn}$, where s s' are the total charges on the two conductors respectively, and is

$$\mu r'\frac{A}{\omega c}c_1\{\sin\omega t f_3(\beta)+\cos\omega t f_4(\beta)\} \\ +\mu r'\frac{A}{\omega c}c_2\{\sin\omega t f_3(\gamma)+\cos\omega t f_4(\gamma)\}, \\ +\mu r'\frac{B}{\omega c}c_1\{\sin(\omega t+\phi)f_1(\beta)+\cos(\omega t+\phi)f_2(\beta)\} \\ +\mu r'\frac{B}{\omega c}c_2\{\sin(\omega t+\phi)f_1(\gamma)+\cos(\omega t+\phi)f_2(\gamma)\}.$$

If a known current $2\pi A \cos \omega t$ be introduced at A, and the point B be kept at zero potential, the equation of the last expression to zero for every value of t gives two equations to determine B and ϕ , the amplitude and phase of the current at the receiving end of the cable. These are

$$A\{\mu c_1 f_3(\beta)+\nu c_2 f_3(\gamma)\}+B[\mu c_1\{\cos\phi f_1(\beta)-\sin\phi f_2(\beta)\} \\ +\nu c_2\{\cos\phi f_1(\gamma)-\sin\phi f_2(\gamma)\}]=0; \\ A\{\mu c_1 f_4(\beta)+\nu c_2 f_4(\gamma)\}+B[\mu c_1\{\sin\phi f_1(\beta)+\cos\phi f_2(\beta)\} \\ +\nu c_2\{\sin\phi f_1(\gamma)+\cos\phi f_2(\gamma)\}]=0.$$

Similarly, if the currents had been introduced into the concentric conductor, the central one being insulated throughout, we should have for the potential of the concentric conductor at B, $\frac{1}{n}(s+s')$, which is exactly the same expression as for the central conductor substituting r' for r , $\frac{c_1'}{c}$ for $\frac{c_1}{c}$, and

$\frac{e_2'}{c}$ for $\frac{e_2}{c}$. Remembering that $\frac{c_1}{c} = \frac{c_2'}{c}$ and $\frac{c_2}{c} = \frac{c_1'}{c}$, it is seen that the equations for determining the phase and amplitude of the current received at B when that end of the concentric conductor is earthed are the same as in the case of the central conductor, only interchanging c_1 and c_2 .

§ 8. So if an alternating current, $2\pi A \cos(\omega t + a)$, be introduced into the central conductor at A, and a current $2\pi B \cos(\omega t + b)$ pass out of the concentric conductor at B, the potentials of the conductors at A, B may be determined.

The potential of the central conductor at A is

$$\frac{1}{mn} \{ (m+n)s + ms' \}$$

where s, s' are the charges on the two conductors.

$$\begin{aligned} \text{This} &= \frac{A}{\omega} \cdot \frac{\mu^2 r}{\mu - \nu} (1 - m\nu r) \{ \sin(\omega t + a)f_1(\beta) + \cos(\omega t + a)f_2(\beta) \} \\ &+ \frac{A}{\omega} \cdot \frac{\nu^2 r}{\nu - \mu} (1 - m\mu r) \{ \sin(\omega t + a)f_1(\gamma) + \cos(\omega t + a)f_2(\gamma) \} \\ &- \frac{B}{\omega} \cdot \frac{\mu}{n(\mu - \nu)} \{ \sin(\omega t + b)f_3(\beta) + \cos(\omega t + b)f_4(\beta) \} \\ &- \frac{B}{\omega} \cdot \frac{\nu}{(v - \mu)n} \{ \sin(\omega t + b)f_3(\gamma) + \cos(\omega t + b)f_4(\gamma) \}. \end{aligned}$$

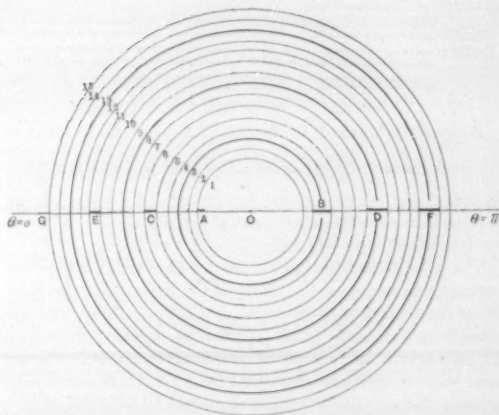
The potential of the concentric conductor at B is $\frac{1}{n}(s + s')$.

$$\begin{aligned} \text{This} &= \frac{A}{\omega} \cdot \frac{\mu}{n(\mu - \nu)} \{ \sin(\omega t + a)f_3(\beta) + \cos(\omega t + a)f_4(\beta) \} \\ &+ \frac{A}{\omega} \cdot \frac{\nu}{n(\nu - \mu)} \{ \sin(\omega t + a)f_3(\gamma) + \cos(\omega t + a)f_4(\gamma) \} \\ &+ \frac{B}{\omega} \cdot \frac{\mu\nu r'}{\mu - \nu} (1 - m\mu r) \{ \sin(\omega t + b)f_1(\beta) + \cos(\omega t + b)f_2(\beta) \} \\ &+ \frac{B}{\omega} \cdot \frac{\mu\nu r'}{\nu - \mu} (1 - m\nu r) \{ \sin(\omega t + b)f_1(\gamma) + \cos(\omega t + b)f_2(\gamma) \}. \end{aligned}$$

§ 9. Consider a system of circular conductors, of which fifteen are shown in fig. 4.

Of these Nos. 1, 5, 6, 10, 11, 15 have each a resistance $2\pi r$; they represent the central conductors of such concentric cables as we have been considering. Nos. 2, 4, 7, 9, 12, 14 have each a resistance $2\pi r'$. They represent the outer or concentric conductors. Nos. 3, 8, 13, have no resistance, and are maintained always at zero potential throughout. They represent the water or sheathing of the cables.

Fig. 4.



The capacity measured between each of the pairs 1, 2 : 4, 5 : 6, 7 : 9, 10 : 11, 12 : 14, 15 respectively is $2\pi c$, where $c=m$; and between the pairs 2, 3 : 3, 4 : 7, 8 : 8, 9 : 12, 13 : 13, 14 is $2\pi c'$, where $c'=n$. The capacity between the pairs 5, 6 : 10, 11 is zero.

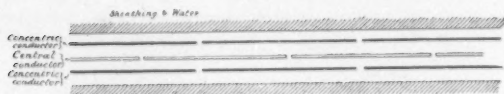
At B conductors 2, 4 are connected by a piece having no resistance or capacity.

Similarly at C, conductors 5, 6 are connected; at D, 7, 9; at E, 10, 11; at F, 12, 14, and so on. The series may be indefinitely extended.

The system is electrically equivalent to a length of concentric cable of which the two conductors are cut alternately

at equidistant points in the way shown in fig. 5, representing a longitudinal section of the cable. If an alternating current be introduced into the system of fig. 4 at any link, say E, and the point A be kept at zero potential, alternating currents of the same period, but of different phases and amplitudes, will be found at every point of both conductors.

Fig. 5.



Suppose the currents at the points A, B, C, D, . . . G to be

$$A \cos(\omega t + a), B \cos(\omega t + b), \dots G \cos(\omega t + g),$$

so that the values of Gg being known, and the point A being kept at zero potential, the values of $Aa, Bb, Cc, \dots Ff$ are to be determined, and especially the values of Aa .

The potential at the point C on the conductor 5 has been expressed in terms of Cc, Bb ; and the potential at the point C on the conductor 6 in terms of Cc, Dd . These two are equal for all values of t , and we have an equation which can be separated into two others between Bb, Cc, Dd . In the same way we obtain two equations at A, B, D, E, and F, and between these all the quantities required can be obtained.

In the particular case where ω has such a value that β and γ are integral, and the currents at all the points are in the same phase, let us write in § 8,

$$\frac{\mu^2 r}{\mu - v} (1 - mvr) f_1(\beta) + \frac{v^2 r}{v - \mu} (1 - m\mu r) f_1(\gamma) \text{ as } P,$$

$$\frac{\mu}{n(\mu - v)} f_2(\beta) + \frac{v}{n(v - \mu)} f_2(\gamma) \text{ as } Q,$$

$$\frac{\mu vr'}{v - \mu} (1 - m\mu r) f_1(\beta) + \frac{\mu vr'}{\mu - v} (1 - mvr) f_1(\gamma) \text{ as } R,$$

remembering that when β, γ are integral, $f_1(\beta) = f_2(\beta)$, and $f_3(\beta) = f_4(\beta)$, and the same with γ .

The condition at A gives $AP=BQ$,
 " " B " $(A+C)Q=2BR$,
 " " C " $(B+D)Q=2CP$,
 " " D " $(C+E)Q=2DR$, and so on.

If the known current is introduced at B, and A is kept to earth, the ratio of the amplitude of the received current to the sent current is given by

$$\frac{B}{A} = \frac{P}{Q} \dots \dots \text{Case I.}$$

If the known current is introduced at C, we have

$$\frac{C}{A} = 2 \frac{PR}{Q^2} - 1 \dots \dots \text{Case II.}$$

If the known current is introduced at E, we have

$$\frac{E}{A} = 8 \left(\frac{PR}{Q^2} \right)^2 - 8 \frac{PR}{Q^2} + 1 \dots \text{Case III.}$$

If the known current is introduced at G, we have

$$\frac{G}{A} = 32 \left(\frac{PR}{Q^2} \right)^3 - 48 \left(\frac{PR}{Q^2} \right)^2 + 18 \frac{PR}{Q^2} - 1 \dots \text{Case IV.}$$

§ 10. Applying these results to a cable having the following constants, represented by half the circle of fig. 4:

Length 4000 kilometres = $4 \cdot 10^8$ units.

Resistance of each conductor, 5 ohms per kilometre, *i.e.* $2 \cdot 10^{13}$ units for the whole cable.

Capacity measured between the conductors, $\frac{5 + \sqrt{7}}{18} 10^{-16}$

units per kilometre (about $\cdot 425$ mfd.), *i.e.* $\frac{5 + \sqrt{7}}{18} 4 \cdot 10^{-12}$ units for the whole cable.

Capacity measured between the outer conductor and the sheathing, $\frac{5 - \sqrt{7}}{9} 10^{-18}$ units per kilometre (about $\cdot 26$ mfd.),

i.e. $\frac{5 - \sqrt{7}}{9} 4 \cdot 10^{-12}$ units for the whole cable.

Then

$$m = \frac{2 \cdot 5 + \sqrt{7}}{\pi} 10^{-12}, \quad n = \frac{4 \cdot 5 - \sqrt{7}}{\pi} 10^{-12},$$

$$r = r' = \frac{2}{\pi} 10^{13}, \quad \mu = \frac{\pi^2}{80}, \quad \nu = \frac{9\pi^2}{80},$$

$$\frac{c_1}{c} = \frac{c_2'}{c'} = \frac{3 + \sqrt{7}}{16}, \quad \frac{c_2}{c} = \frac{c_1'}{c'} = \frac{13 - \sqrt{7}}{16}.$$

Consider the cable in four cases :—

Case I. when both conductors are continuous.

Case II. when the inner conductor is cut in the middle so that the cable is in two sections.

Case III. when the inner conductor is cut in two places, and the outer in one, so that the cable is in four sections.

Case IV. when the inner conductor is cut in three places, and the outer in two, so that the cable is in six sections.

Take a value of $\omega \cdot \frac{9 \cdot 144 \cdot \pi^2}{80}$ which makes $\beta \gamma$ integral in all cases. This corresponds to about 25 alternations per second.

Then if the cable be divided into n equal sections the new values of $rr'mn\beta\gamma$ are inversely proportional to n , while the values of μ, ν are proportional to the square of n . The value of $\frac{PR}{Q^2}$ is unchanged in every case.

In Case I. $\beta = 30 \gamma = 12$, and $A = 1 \cdot 26 e^{-12\pi} B$;

Case II. $\beta = 18 \gamma = 6$, and $A = 1 \cdot 07 e^{-12\pi} C$;

Case III. $\beta = 9 \gamma = 3$, and $A = \cdot 57 e^{-12\pi} E$;

Case IV. $\beta = 6 \gamma = 2$, and $A = \cdot 31 e^{-12\pi} G$.

Thus the effect of each successive subdivision is to further reduce the amplitude of the received current.

§11. Returning to the equation of § 2 for determining $\mu\nu$, viz. :—

$$murr'x^2 - x\{m + \alpha\nu' + m\nu'\} + 1 = 0,$$

write in this

$$\frac{1}{r} = \gamma, \quad \frac{1}{r'} = \gamma', \quad \frac{1}{m} = k, \quad \frac{1}{n} = k', \quad \text{and } \gamma + \gamma' = \Gamma,$$

so that γ, γ' are the conductivities of the two conductors, k, k' the rigidities to electrostatic stress of the two dielectrics, and Γ the total conductivity of the two conductors.

The equation becomes

$$x^2 - x\{\gamma k + \Gamma k'\} + \gamma\gamma'kk' = 0,$$

and the roots of this equation represent the signalling speeds of the two equivalent cables.

If k be very small, $x = \Gamma k'$ or 0. Hence if the layer of dielectric between the conductors be very thin, though at the same time perfectly insulating, the speed through the central conductor, however small its section may be, is the same as if the whole of the two conductors were solid, and the whole used for the conducting circuit.

DISCUSSION.

Mr. BLAKESLEY said he was sorry the result did not indicate a successful type of cable. He would have been inclined to predict that the amplitude would have decreased with the number of sections. If a number of condensers were joined in series, and one end was subjected to a periodic E.M.F., the amplitude would fall off inversely as the square of the number of the condensers, the total capacities remaining the same.

Mr. PRICE then exhibited a galvanometer support. The instrument is suspended from two indiarubber cords attached at the top and bottom to cross-bars of metal, thus forming a rectangle. The cross-bars are provided with knife-edges in such a way as to compensate for unequal stretching of the indiarubber. Weights can be added, if necessary, to the support, so as to increase its inertia.

*X. A Nickel Stress Telephone. By T. A. GARRETT, M.A.,
and WILLIAM LUCAS, M.A.**

It is well known that, if a magnetized nickel wire be subjected to longitudinal stress, variations of this stress produce variations in the magnetization of the wire. If the stress be a pressure, an increase in the pressure produces an increase in the magnetization, and a decrease in the pressure a decrease in the magnetization.

Suppose then that a magnetized piece of nickel wire, round which a coil of insulated wire has been wound, is fixed in a vertical position by a clamp at its lower end, and that its upper end is fastened to the centre of a light horizontal plate, which serves as a diaphragm. On speaking against the top of the diaphragm variations of longitudinal pressure and consequently of magnetization will be produced in the nickel wire, and hence a varying current will be induced in the coil. If the coil be connected with any kind of receiving telephone the speech will be reproduced.

We have made a large number of experiments with the object of determining the form of instrument which would give the best results; but owing to the difficulties of comparing the speaking powers of different instruments we found it impossible to say with certainty which form was the best. The details of one of the forms with which good results were obtained are as follows:—The nickel wire was 10 cm. long and 1 mm. in diameter. The clamp was formed of a massive piece of gunmetal (an ordinary workshop vice will do quite as well) and the diaphragm was a circular disk of pine wood 3.5 mm. thick and 12.5 cm. in diameter. A hole was made through it in the centre into which the nickel wire was stuck with sealing-wax. This diaphragm was entirely supported by the nickel wire. The coil consisted of 3.9 grms. of No. 40 B.W.G. silk-covered wire wound directly on the nickel wire. The length of the coil was about 7.5 cm. An ordinary watch telephone, the resistance of the coils of which was about 136 ohms, was used as a receiver. The

* Read April 9, 1897.

resistance of the line was quite small, as all the experiments were made from one room to another in the same building. The nickel wire was sometimes magnetized by stroking it with a magnet, sometimes by sending a current from a few Leclanché cells through the coil. Whether the wire was annealed or not appeared to be immaterial. Wires of various dimensions were tried, but the results obtained were not very different. With a diaphragm made of pine wood, the results were decidedly better than with a metallic diaphragm articulation then being very clear. It was found to be better to use a clamp of considerable mass.

When the nickel wire was replaced by an iron wire the results obtained were very feeble.

The same instrument was also used as a receiver. With an iron wire it worked very badly, with a nickel wire it worked rather better but not at all well. Since beginning our experiments we have found that M. Ader* has used as a receiver an instrument somewhat similar to the above, except that the wire was of iron instead of nickel.

In order to show that the current induced in the coil is at any rate chiefly due to variations in the magnetization of the nickel wire produced by the variations of stress, and not to the relative motion of the nickel wire and the coil, the following experiment was made:—A steel wire and a nickel wire of the same dimensions were attached to similar wooden diaphragms. These wires having been magnetized by stroking them with a permanent magnet were in turn inserted into the same solenoid and clamped as before at their lower ends. The same watch telephone was used as a receiver with each. The results obtained with the weakly magnetized nickel wire were enormously better than those obtained with the strongly magnetized steel wire. If the induced currents were chiefly due to the relative motion of the coil and magnetized wire the best results would have been obtained with the strongly magnetized steel wire, since it can hardly be supposed that the relative motions of the coil and magnetized wire differed so much in the two cases as to cause such an enormous difference in the results.

* *Comp. Rend.* 1879, t. lxxxviii. p. 575

DISCUSSION.

Dr. S. P. THOMPSON said that some years ago he had worked with a somewhat similar apparatus, using it as a "receiver," with wires of nickel, cobalt, and iron. Cobalt gave the best results; the metallic strips in his experiments dipped into the solenoids without contact with them. This arrangement did not work well as a "transmitter," even when a battery was included in the circuit. In some cases the rods were cut into short lengths separated by brass.

Mr. BOYS asked how the nickel "stress" instrument compared in clearness and loudness with an ordinary telephone.

Mr. SHELFORD BIDWELL had tried a nickel telephone with a mica diaphragm; depending not upon mechanical stress, but magnetic strain. It did not work well.

Dr. CHREE thought the "stress" telephone might possibly be improved by choosing the right strength of magnetic field.

Mr. APPLEYARD said the arrangement was interesting historically, because it was, mechanically, almost identical with the original instrument used by Philip Reis as a "receiver." The authors had succeeded in getting it to work as a "transmitter." Their success was probably due to the rapidity with which the magnetization of nickel responded to very small changes of stress or current. The Post-Office electricians had tried to introduce nickel cores into relays, on account of its magnetic sensitiveness; the results, he believed, had not been very satisfactory.

Mr. T. A. GARRETT, in replying, said the "stress" telephone gave better articulation than an ordinary "watch" telephone, but the sounds were feebler. There seemed to be a field-strength proper to the instrument; he had noticed that the articulation was clearer with three cells than with six.

XI. *The Formation of Mercury Films by an Electrical Process.* By ROLLO APLEYARD*.

If a sheet of gelatine, damp leather, or similar permeable substance is used as a separating diaphragm between two bodies of mercury, and a current is sent through it, a film of mercury is deposited upon the surface connected to the positive pole of the battery. And the film remains on the diaphragm after removal from the apparatus.

If the diaphragm is replaced in the apparatus and subjected to a current in the reverse direction, the film vanishes from that surface, and a second film appears on the other side. That is to say, the film is always on the side of the diaphragm connected to the positive pole of the battery, and there is no film on the negative surface.

In this way I have deposited mercury-films upon such different substances as filter-paper, plaster-of-paris, Woodbury-type-gelatine, porous earthenware, asbestos-paper, and sheep's skin. As a rule, the only preparation of the substances is damping with distilled water. Asbestos mill-board is one of the best materials to operate upon.

The coating of mercury formed by this electrical process is far more perfect than any I have been able to obtain by the methods of "pickling" or contact-pressure. I have no definite views as to the cause of the phenomenon, but I am inclined to regard it as a secondary action, the result of electrolysis, aided to some degree by electric osmosis. The experiments of Mr. C. K. Falkenstein† upon the electric tanning of leather are evidence in favour of the osmosis theory. I understand also that electric osmosis has lately been used for administering cocaine, replacing the ordinary hypodermic injections.

It will be remembered that in some experiments‡ I recently described to the Physical Society a globule of mercury, electrified in a partially conducting liquid, sent forth a tentacle in a direction towards the positive electrode. It is perhaps

* Read May 14, 1897.

† Journ. Inst. Elec. Engineers, vol. xxii. p. 225 (1893).

‡ "Liquid Coherers and Mobile Conductors," *ante*, p. 72.

significant that in the present experiments the film is always on the positive surface. In the case of liquid "coherers" it is possible to conceive that, under the influence of steady or transitory currents, mobile conductors attempt to creep in this fashion through the dielectric films. And, again, though the effect is probably very slight and entirely masked by electrolytic disturbances, sea-water may thus penetrate the dielectric at a "fault" in a cable. The film-forming process may be of practical utility where large surfaces are required to be coated with mercury, as in gold extraction; and possibly in the preparation of plates for primary or secondary batteries.

The following experiments are given in illustration of the action:—

(1) A sheet of filter-paper, damped with distilled water, is folded upon itself four or five times. It is then laid flat upon a conducting surface. An indiarubber ring is pressed down on the folded paper, and mercury is poured in to fill the ring. A battery of 200 volts is then connected for about three minutes between the mercury and the lower conducting plate, the mercury being positive. After stopping the current the ring is removed from the filter-paper and superfluous mercury is poured off, leaving a circular film upon the upper surface of the paper. On unfolding the sheet, a blackish substance is found distributed through it, graduated from the positive towards the negative pole.

(2) If a sheet of tinfoil is included within the folds of filter-paper, while the battery is applied, the tinfoil becomes perforated with pin-holes. This, however, must not be taken *per se* as a proof of any direct transition of metallic mercury through the diaphragm; for if, instead of the mercury contained in the ring, a sheet of metal is used as the upper electrode, pinholes still appear in a sheet of tinfoil included in the folds of damp filter-paper. Moreover, if a sheet of tinfoil is used as the upper electrode, this also becomes perforated.

(3) Again, if the sheets of tinfoil are removed, and a gold coin is used for the upper (positive) electrode, upon the top of the damp filter-paper, the current produces a gold discoloration which penetrates the folds. I have not yet had time

to examine this result, but it seems to throw some light upon the formation of metallic lodes and veins. And although the discoloration is the effect of a steady current, and not sparks it may help to explain the "inductoscripts"* of Mr. F. J. Smith.

DISCUSSION.

Dr. S. P. THOMPSON said he did not know of any other examples of an *anode* being more active—mechanically—than the *kathode*, except the electric arc. He was surprised that the film should appear on the positive surface.

Mr. SHELFORD BIDWELL thought selenium presented in some of its actions an example of the *anode* being thus active.

Prof. AYRTON said that if a vessel containing a substratum of mercury amalgam was filled up with water in which gold crushings were washed, the gold descended into the amalgam. This, however, might be partly due to gravity, and partly to simple electrolysis.

Mr. APPLEYARD said he had no definite views as to the formation of the films. He believed it to be a secondary effect of electrolysis, aided by electric osmosis. The experiments of Mr. C. K. Falkenstein upon the electric tanning of leather, and the early results of M. Perret, helped the idea of electric osmosis; they were not sufficient, however, to justify that theory without further research. A careful chemical analysis of the deposits left in the folds of filter-paper would be the best guide.

* "Inductoscripts," by F. J. Smith, Proc. Phys. Soc. vol. xi. p. 353 (1892).

XII. An Instrument for the Comparison of Thermometers.

By W. WATSON, B.Sc., A.R.C.Sc., Assistant Professor of Physics at the Royal College of Science, London.*

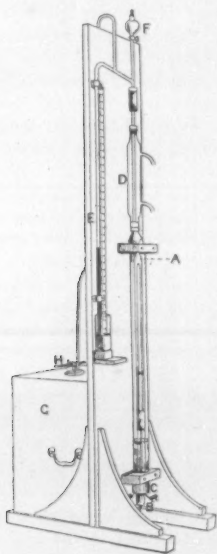
IN most investigations where it is necessary to measure temperature it is convenient to use a small thermometer, often of short range, particularly suited for the purpose. The only practicable method of determining the errors of such a thermometer is by comparing its readings with those of a standard thermometer. This comparison is in general rendered very difficult on account of the two thermometers differing both as to the length of the scale and the capacity of the bulb. Hence if, as is essential, the bulbs are placed close together in a water-bath, a considerable length of one of the mercury columns must be out of the bath, involving an uncertain correction for cool column. Again, the difference in size of the bulbs renders the lag of the thermometers different, so that unless the temperature of the bath is kept absolutely constant for some time we cannot be sure that *both* thermometers have reached a steady temperature. These objections make the use of a water-bath at temperatures over about 20°C . both troublesome and unsatisfactory, and this paper is written in order to describe an instrument which the author has designed for comparing some thermometers required for some other work, and which has worked so satisfactorily that it may be of some use to others.

The principle employed is that of using a vapour-jacket in order to obtain different constant temperatures, as recommended by Ramsay and Young (*Journ. Chem. Soc.* xlvii. p. 640, 1885). A general view of the instrument is shown in fig. 1. A glass tube AB, closed at the top and open below, about 76 cm. long and 2.5 cm. in diameter; passes up inside a second glass tube of about the same length and 4.7 cm. in diameter. The space between these two tubes is at the bottom closed by an indiarubber cork C (figs. 1 and 2), while at the top the inner tube is kept in place by three small pieces of glass rod fused on and forming a triradiate

* Read May 14, 1897.

star. The upper end of the outside tube is fused on to a Liebig condenser D, the upper end of the condenser being connected by a rubber joint fitted with a glycerine seal to a manometer E, a stoppered funnel F, and a large glass bottle which acts as a reservoir. This bottle is packed round with cotton-wool and is enclosed in a wooden box G. The air can be exhausted from or admitted to this bottle by means of a three-way tap H. Some mercury is placed on the top

Fig. 1.

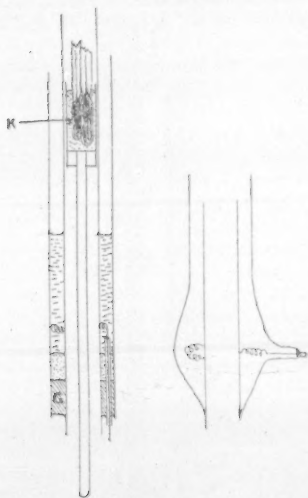


of the cork C (fig. 2) to prevent the liquid used to form the vapour touching the cork. The liquid is heated by means of a spiral of fine uncovered platinum wire, the terminals being fused into glass tubes which pass down through the mercury and the cork.

Attempts to boil the liquid in an auxiliary bulb were rendered futile on account of the excessive bumping which took place after the liquid had been boiling for a day or two. The heating by a platinum wire, however, entirely obviates all bumping, and might be of use in determining the boiling-point or in distilling liquids which are inclined to bump, so long as they are not electrolytes.

Fig. 2.

Fig. 3.



The vapour of the boiling liquid rises between the inside and outside tube, is condensed in the condenser D, and runs down to the bottom again.

The thermometers are fastened together, their bulbs resting in a small glass vessel K (fig. 2) which is filled with mercury. This vessel is supported by a glass rod which rests on the table on which the instrument stands. To remove the thermometers the instrument is drawn to the edge of the

table till this rod can be lowered and with it the thermometers. The scales are read by means of two telescopes, it being quite easy to see through the vapour if suitable liquids are used. The three liquids recommended by Ramsay and Young, viz. carbon bisulphide (20° to 46°), ethyl alcohol (46° to 79°), and chlorobenzene (79° to 120°) do very well. Water cannot be used as it forms drops on the inside of the glass so that the thermometers cannot be read. For temperatures higher than 120° the indiarubber cork closing the tube will not do, but the arrangement shown in fig. 3 may be used. The disadvantage of this form is that it can hardly be made by any but a professional glass-blower; the first form, on the other hand, can easily be put together by anyone who can do the most rudimentary glass-blowing.

In order that the indiarubber cork may form a thoroughly air-tight joint, it must be well cleaned with benzene, coated with indiarubber solution, and put in place while the solution is wet. When the solution has got thoroughly dry, which takes four or five days, the joint will be quite air-tight.

With such an arrangement the thermometers can easily be maintained at a temperature constant to within $0^{\circ}01$ for three or four hours together. The manometer is only used to adjust the temperature to the desired point, a table of vapour-pressures being employed. In about half an hour after altering the pressure, and hence the temperature, the thermometer readings become quite constant. The instrument once started can be left entirely to itself; the only thing the observer has to do is to read the thermometers, then let a little more air in to get the next higher temperature. He may then go away for half an hour, and when he returns will find the temperature constant and can take the new readings.

DISCUSSION.

Prof. AYRTON thought the apparatus would come into extensive use; it did away with errors arising from differences of length of thermometer stems, it left no question as to the equality of temperature of the two bulbs, and there was no probability of error due to a difference of thermal "lag" in any two thermometers.

Mr. WATSON, in replying to a question of Prof. Perry's, said the fact of using indiarubber joints limited the available range of temperature. Working with blown joints, Ramsay and Young had found no difficulty in their vapour-density experiments at higher temperatures.

XIII. *On the Isothermals of Isopentane.*

By J. ROSE-INNES, M.A., B.Sc.*

[Plate IV.]

THE recent publication by Prof. Sydney Young of a long series of researches on isopentane (Proc. Phys. Soc. Session 1894-95, pp. 602-657) offers a great opportunity to those interested in the theory of gases. Since isopentane is a saturated hydrocarbon, there seems to be a reasonable hope that we are here dealing with a substance which will not tend to form complex molecules at low volumes, and whose behaviour may therefore be treated as normal; and this fact, together with the wide range of volume over which the experiments have been conducted, renders Prof. Young's results well fitted to test the various formulæ that have been from time to time proposed. Among these formulæ there is none more important than that formerly suggested by Prof. Young himself, in conjunction with Prof. Ramsay, that the pressure of a gas kept at constant volume is a linear function of the temperature; and the bearing of the experimental results with isopentane upon this formula is fully considered by Prof. Young in his paper. Accepting the formula, and writing it as

$$p = bT - a,$$

where b and a are functions of the volume only, the values of b and a for a large number of volumes are given in the paper (*loc. cit.* pp. 650-655), and they are sufficiently numerous to enable us to fully test any algebraic expression that endeavours to represent them. I spent a considerable amount of time examining the values of a and b , testing the formulæ that

* Read May 28, 1897.

have been proposed by various physicists, as well as others of my own devising, without arriving at any that gave complete satisfaction; and it occurred to me afterwards that possibly more definite results could be secured by examining some physical quantity which depended upon both a and b than by examining b and a separately by themselves. Thus if we accept Ramsay and Young's linear law, there will be one and only one temperature for each volume at which the gas has its pressure equal to that given by the laws of a perfect gas. In effect, if we put

$$p = bT - a$$

as giving the actual pressure, we may also write

$$p = \frac{RT}{v} + \left(b - \frac{R}{v}\right)T - a,$$

and this shows that $p = \frac{RT}{v}$ if we take $T = \frac{a}{b - R/v}$. Call the temperature so found τ ; the values of τ have been calculated and are given in the following table (p. 128); the results above vol. 400 are not included, as they vary within such wide limits owing to experimental errors.

In calculating this table, the value of R was taken $= \frac{1}{0.001158}$.

An examination of the table shows that the temperature in question is very much the same for all large volumes down to about vol. 8. Of course the actual numbers obtained vary a good deal, but these variations are without method, sometimes in one direction and sometimes in another, and when the numbers are plotted against $v^{-\frac{1}{2}}$ it appears to me impossible to tell from an inspection of the diagram whether the value above vol. 8 is on the whole increasing or decreasing. These variations may therefore be attributed to experimental error; and they may be to a large extent got rid of by employing "smoothed" values of b and a , as was done in Ramsay and Young's paper on ether (Phil. Mag. xxiii. p. 441). Prof. Young has not given any smoothed values of b for isopentane, however, and I have thought it better not to use my own smoothed values, so as to employ numbers that are above the suspicion of bias towards any particular theory.

Volume.	τ .	Volume.	τ .	Volume.	τ .
350.....	851.9	22.....	835.8	4.3 ...	804.6
300.....	835.4	20.....	845.9	4.0 ...	799.1
260.....	820.8	19.....	854.7	3.8 ...	792.3
230.....	825.6	18.....	848.7	3.6 ...	777.3
200.....	969.6	17.....	850.9	3.4 ...	764.0
180.....	935.6	16.....	845.3	3.2 ...	737.2
160.....	846.7	15.....	854.1	3.0 ...	715.5
140.....	787.0	14.....	848.9	2.9 ...	696.5
120.....	808.9	13.....	847.8	2.8 ...	675.9
100.....	804.1	12.....	844.1	2.7 ...	656.4
90.....	793.8	11.....	840.1	2.6 ...	637.7
80.....	773.4	10.....	843.5	2.5 ...	615.2
70.....	834.7	9.....	846.7	2.4 ...	593.7
60.....	876.2	9.....	838.3	2.3 ...	565.9
50.....	871.6	8.....	834.5	2.2 ...	541.1
45.....	853.1	8.....	833.7	2.1 ...	511.2
40.....	816.2	7.....	833.2	2.0 ...	478.7
36.....	832.8	7.....	832.2	1.9 ...	446.3
35.....	820.0	6.....	828.1	1.8 ...	408.6
30.....	833.7	6.....	827.2	1.7 ...	367.2
28.....	835.1	5.....	819.8	1.6 ...	318.4
26.....	846.7	5.....	813.7		
24.....	848.0	4.....	809.5		

The mean value of τ for all volumes above 8 is 842.4; and this number could be taken as the true value for all such volumes without introducing any serious error. Below vol. 8, however, the value of τ distinctly diminishes with the volume; and by the time the critical point is reached, τ is not more than about 804. It is not easy to find a formula for τ which will give the experimental results throughout the whole range of volume; but whatever may turn out to be its proper algebraic expression, there appears to be little doubt that its initial decrease from the maximum value varies as a higher power of the density than the first.

The result appears to me to be of importance, since it is by itself sufficient to reveal the inadequacy for isopentane of many of the gas-formulæ that have been proposed. We have seen that

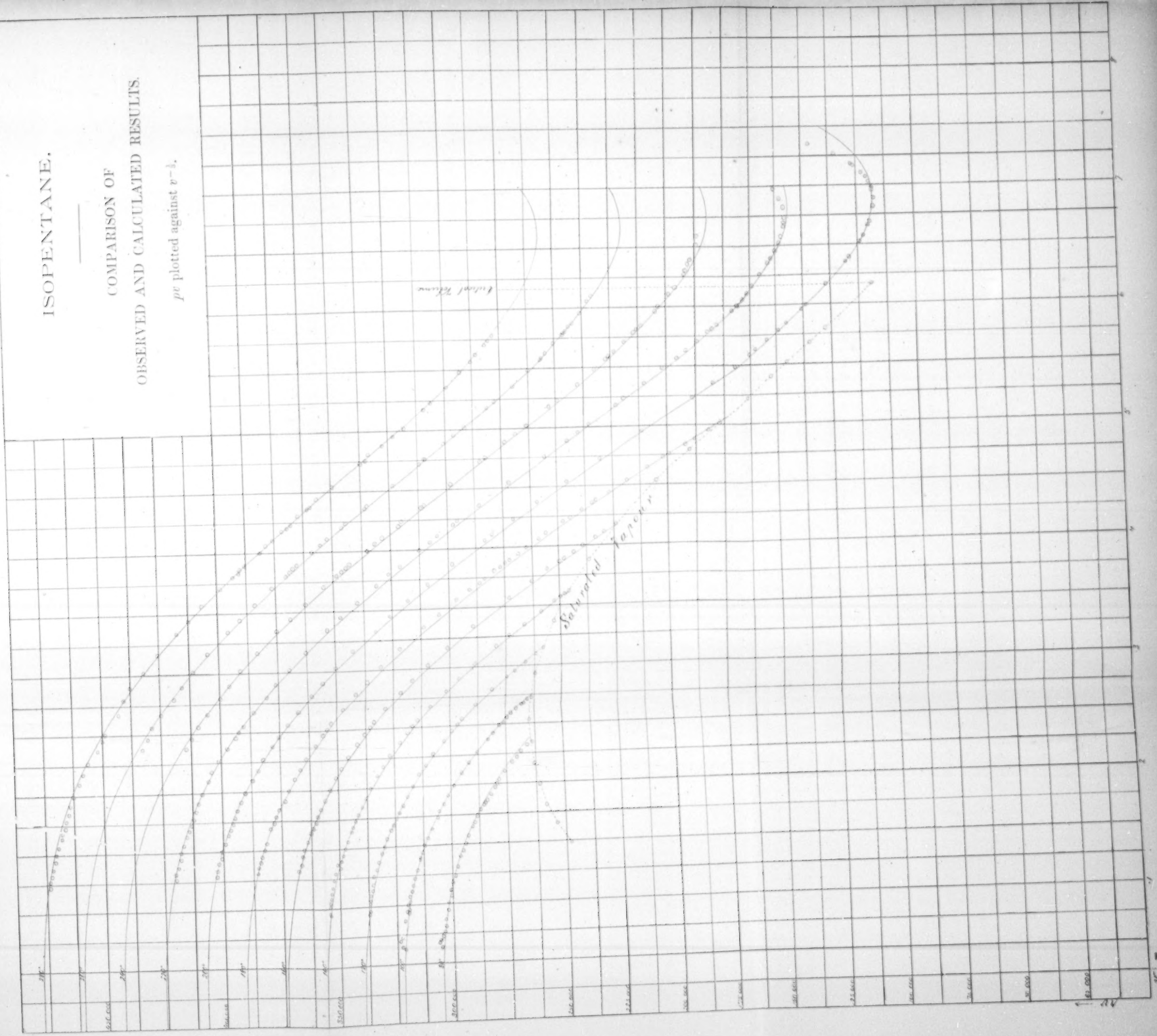
$$\tau = \frac{av}{bc - R}.$$

Suppose that we can put

ISOPENTANE.

COMPARISON OF
OBSERVED AND CALCULATED RESULTS.

pv plotted against v^{-1} .



$$b = \frac{R}{v} + \frac{B_2}{v^2} + \frac{B_3}{v^3} + \text{higher powers of } \frac{1}{v},$$

$$a = \frac{A_2}{v^2} + \frac{A_3}{v^3} + \text{higher powers of } \frac{1}{v};$$

Then

$$\tau = \frac{A_2 + A_3 v^{-1} + \dots}{B_2 + B_3 v^{-1} + \dots},$$

$$= \frac{A_2}{B_2} + \frac{A_3 B_2 - A_2 B_3}{B_2^2} v^{-1} + \text{higher powers of } \frac{1}{v}.$$

If b and a in any gas-formula can be expanded in powers of v^{-1} as assumed, and if $A_3 B_2 - A_2 B_3$ is different from zero, which will in general happen, the decrease of τ from the maximum value will be as v^{-1} ; so that the proposed formula is inadmissible.

Let us now return to the quantities a and b . The values of $\frac{1}{av^2}$ were plotted against $v^{-\frac{1}{2}}$ by Prof. Young, and the resulting diagram is given in his paper (*loc. cit.* p. 653). The curve determined by the points plotted would seem to be of a somewhat complicated character, and I do not think it possible to obtain any simple formula that will reproduce it entirely. On the other hand, there seems to be considerable evidence of discontinuity in the neighbourhood of vol. 3.4; and even if there is not discontinuity in the true mathematical sense of the word, there appears to be such a rapid alteration of behaviour as to amount in practice to the same thing. The easiest plan is to treat the curve as consisting of two parts, the formula passing abruptly from one expression to another somewhere in the neighbourhood of vol. 3.4. Whether this accurately represents what takes place in nature is uncertain, but there is no doubt that it immensely simplifies the problem. We may therefore confine our attention to volumes above 3.4; and of the formulæ already proposed I found the best to be that suggested by Mr. W. Sutherland, who has given a gas-formula equivalent to putting

$$a = \frac{l}{v(v+k)},$$

where l and k are two constants characteristic of the gas

(Phil. Mag. xxxv. p. 215). I have taken the following values of the constants

$$l = 5,420,800, \quad k = 3.636.$$

The next step was to find a formula for b . Now

$$b = \frac{R}{v} + \frac{a}{\tau},$$

and the conditions already discovered with respect to the algebraic expression for τ considerably restrict the field of research; this is an advantage, as it lessens the amount of arithmetical work to be performed. I found that fairly good results could be obtained by putting

$$b = \frac{R}{v} \left(1 + \frac{e}{v + k - \frac{g}{v^2}} \right),$$

where R and k have the values already given and

$$e = 7.473, \quad g = 6.2318.$$

By combining the formulæ given above for a and for b , we obtain as the formula for the isothermals

$$p = \frac{RT}{v} \left(1 + \frac{e}{v + k - \frac{g}{v^2}} \right) - \frac{l}{v + k},$$

where R , e , k , g , and l are constants, and have the values already given.

In order to test this formula it is desirable to draw a system of isothermals, but if this be done in the ordinary Andrews' diagram the result is not satisfactory, as the range of p is so large. It was found possible to obtain a good diagram, however, by calculating pv and plotting it against v^{-1} ; the calculated isothermals are shown as continuous lines, while the experimental values are put in as dots. It will be seen that there is a fair agreement between calculation and experiment down to about vol. 3.4. Below this volume there is no longer any agreement; we should naturally expect such a result, since the formula for a admittedly holds up to the neighbourhood of vol. 3.4 only. Therefore, even if the calculated isothermal were to agree with the found isothermal below

vol. 3.4 for some one temperature, this would only happen by a compensation of errors, and could not occur for any second temperature.

It will be noticed that in the neighbourhood of vol. 16 there is a sensible divergence between the calculated and found isothermals amounting to slightly over 1 per cent. This divergence is certainly unsatisfactory as far as it goes; but differences just as great have occurred in the past in inquiries of this kind between the results of independent observers experimenting on the same substance. For the present, then, and pending the confirmation of Prof. Young's results by some other observer, we may take the simple formula given above as representing all that we certainly know concerning the behaviour of isopentane under the conditions of volume specified.

The formula proposed may be employed to calculate the critical constants; this may be done by a method depending solely on Ramsay and Young's linear law. Let us take the equation

$$p = bT - a,$$

and differentiate it with respect to v , keeping T constant,

$$\frac{dp}{dv} = T \frac{db}{dv} - \frac{da}{dv}.$$

Differentiate again

$$\frac{d^2p}{dv^2} = T \frac{d^2b}{dv^2} - \frac{d^2a}{dv^2}.$$

At the critical point $\frac{dp}{dv}$ and $\frac{d^2p}{dv^2}$ vanish together, so we have

$$T \frac{db}{dv} - \frac{da}{dv} = 0,$$

$$T \frac{d^2b}{dv^2} - \frac{d^2a}{dv^2} = 0.$$

Eliminating T this gives

$$\frac{db}{dv} \frac{d^2a}{dv^2} - \frac{d^2b}{dv^2} \frac{da}{dv} = 0.$$

When v has been calculated from this equation we may

obtain T by putting

$$T = \frac{da}{dv} \div \frac{db}{dv},$$

and when v and T are known the original isothermal equation will give p .

In this way we may obtain the following results :—

$$\begin{cases} \text{critical volume} = 4.5, \\ \text{critical temperature} = 191^{\circ}.7 \text{ C.}, \\ \text{critical pressure} = 26250, \end{cases}$$

which agree fairly well with the numbers found experimentally by Prof. Young.

There is one of the isothermals in the set determined by our formula which is of peculiar interest. The biggest value of τ occurs when v is indefinitely large, its value being then $\frac{l}{Re}$. Let us call this quantity τ_0 for shortness, and let us see what form the isothermal for this particular temperature takes. We shall easily obtain

$$p = \frac{R\tau_0}{v} + \frac{lg}{v^3(v+k-\frac{g}{v^2})(v+k)},$$

so that the pressure for a long range of volume is practically given by Boyle's law ; and when an increase does occur, it varies as the *fifth* power of the density. In the case of isopentane this is merely a deduction from theory, since the temperature required is 567° C. ; much too high to allow of any experiments being carried out for verification. Indeed, for most other substances, the temperature of minimum deviation from Boyle's law, being much higher than the critical temperature, is much too high to allow of any experiments being performed there. An exception may be made, however, in the case of nitrogen, which has a very low critical temperature. The following are the values of pv at 16° C. for nitrogen under different pressures determined by M. Amagat ("Mémoires sur l'élasticité et la dilatabilité des Fluides jusqu'aux très-hautes Pressions," *Ann. Chim. Phys.* 6th series, vol. xxix. p. 107).

Pressure in metres of mercury.	<i>p_v</i> .	Pressure in metres of mercury.	<i>p_v</i> .
76	1.0000	45	.9895
20	.9930	50	.9897
25	.9919	55	.9902
30	.9908	60	.9908
35	.9899	65	.9913
40	.9896		

From this table it is evident that at 16° C. for nitrogen we are only a short way off the temperature of minimum deviation from Boyle's law. At temperatures higher than the temperature of minimum deviation the gas would have *p_v* increasing with pressure from the first, and this is what happens with hydrogen at ordinary temperatures.

DISCUSSION.

Prof. YOUNG said the diagrams representing the observed and calculated isothermals were probably the best ever obtained. Divergence among the values of τ was explained in part by the smallness of the angle between the theoretical isochor for a perfect gas, and the real isochor. The point of coincidence was difficult to define. Moreover, the values of τ were obtained from "unsmoothed" values of v . The evidence against the linear law consisted in a certain similarity in the shape of the different curves. It was not easy to see where experimental errors could come in. The peak was a very striking feature of the curves, and the agreement between the results with ether and those of isopentane was very remarkable. These two substances had their boiling-points close together, their critical temperatures close together, and their molecular weights nearly alike. The two substances not only agreed in each giving a peaked curve, but the peak corresponded to almost identical volumes. Prof Young hoped at some future time to examine normal pentane, and to determine whether τ was a constant for this substance also.

XIV. *On the Magnetic Properties and Electrical Resistance of Iron as dependent upon Temperature.* By DAVID K. MORRIS, Ph.D., formerly 1851 Exhibition Scholar, University College, London*.

[Plates V. & VI.]

THE changes brought about by heating in the various physical properties of iron have been, since the first investigations into the nature of the phenomenon of recalcence by Gore and by Barrett, the subject of much experimental work, which, during the last ten years, has been largely upon the relation of the magnetic properties of iron to temperature.

As long ago as 1879, however, Baur of Zürich † described experiments on an iron bar inserted while hot into a long magnetizing helix, which served to determine its permeability at any instant, the temperature being estimated from a knowledge of the law of cooling. This method, although rough, enabled him to show that in weak magnetic fields the permeability of iron rises, and that in strong ones it falls, with rise of temperature; and that as the critical point is reached the magnetic qualities of iron in fields of all strengths very rapidly disappear.

These results have been confirmed and extended by the researches of Ledebøer in 1888 ‡, and by those of Dr. Hopkinson, communicated to the Royal Society in 1889 §. In the latter, a ring-magnet was used, furnished with primary and secondary coils insulated with asbestos, and capable of withstanding a red heat, which temperature was obtained by means of a gas-furnace. Measurements could thus be made of the permeability of the iron core by the ballistic method,

* Read May 14, 1897. Inaugural Dissertation for the Degree of Ph.D. at the University of Zürich.

† O. Baur, "Neue Untersuchungen über den Magnetismus," *Wied. Ann.* 1880, p. 394.

‡ Ledebøer, *Journal de Physique* (2) vol. vii. p. 199.

§ Hopkinson, "Magnetic and other Physical Properties of Iron at a High Temperature," *Phil. Trans. Roy. Soc.* 1889, A. p. 443.

and the temperature was deduced from the resistance of the copper secondary winding.

M. Curie, in a more recent investigation *, used a method by which the permeability could be obtained in intense magnetic fields. He experimented on a sample of iron in the interior of an electrically-heated porcelain furnace, and deduced its magnetization from the forces acting on it when placed in a non-uniform magnetic field. A thermo-electric couple served to measure the temperature.

The variations of magnetic hysteresis with temperature have been studied by W. Kunz of Darmstadt †.

The relation of the electrical resistance of iron to temperature has also received the attention of several experimenters; curves expressing this relation, and extending to beyond the critical temperature, having been given by Kohlrausch in 1887 ‡, by Hopkinson in 1889 §, and by Le Chatelier in 1891 ||.

General Description of the Method of Experiments.

By the method used in the present work, measurements of the electrical resistance and of the magnetization of the iron could be made simultaneously.

It consisted in the employment, as in Dr. Hopkinson's experiments, of a ring-magnet, whose insulation was calculated to withstand a white heat. The core of this ring-magnet was formed of a length of *insulated* iron wire or strip, whose ends projected out of the ring, so that the resistance of the iron in the core could be measured; and *in this core* was imbedded an insulated platinum wire, from the resistance of which the temperature of the iron could be deduced.

The heating was carried out electrically. For this purpose the ring (which was made quite small, about an inch in diameter) was covered with a non-inductively wound layer of insulated platinum wire. By passing a suitable current through this wire the temperature could be raised to any

* P. Curie, "Propriétés magnétiques des corps à diverses températures," *Ann. de Chim. et de Phys.* July 1895, p. 289.

† W. Kunz, *Elektrotechnische Zeitschrift*, 1894, p. 194.

‡ W. Kohlrausch, *Wied. Ann.* vol. xxxiii. p. 42.

§ Hopkinson, *Proc. Roy. Soc.* vol. xlv. p. 457.

|| Le Chatelier, *Comptes Rendus*, vol. cx. p. 283.

required extent. The heat was thus generated exactly where wanted; and by wrapping the ring thickly with asbestos the loss of heat by radiation was made small, and the temperature in the interior fairly uniform.

In the experiments of M. Curie, 1500 watts were absorbed in obtaining a temperature of 1350° : by the above method, 73 watts (a current of 4.6 amperes with 16 volts across the terminals) was found sufficient to maintain the ring at 1150° ; and 45 watts sufficient at 800° , *i. e.*, above the critical temperature of the iron. Dealing thus with comparatively small currents, the accurate regulation of the temperature was rendered simple; whilst absolute constancy for long periods of the heating current, and therefore of the temperature, was, with a set of accumulators in good order, not difficult to attain.

To avoid oxidation of the iron, the ring-magnet was placed in a glass vessel, and the electrical connexions brought through a well-fitting cork rendered air-tight by a thick layer of sealing-wax poured over it while hot. The oxygen contained in this closed space could, previous to the experiments, be absorbed by an auxiliary coil of bare iron wire, heated to bright redness by an electric current. In the later experiments the jar was, as a preliminary, exhausted by a small air-pump to about a third of an atmosphere; during the heating, the pressure might rise above that of the atmosphere, in which case the excess of heated air was allowed to escape from a glass tube dipping into mercury; and this at other times served as a gauge to show whether the glass vessel was really air-tight.

Details of the various Ring-Magnets.

During the course of the experiments, four ring-magnets were made.

The first came quickly to grief through accidental overheating and partial oxidation of the iron. The oxide seems to have combined with the silicates of the insulating materials, forming a kind of slag, for the ring, on taking to pieces was simply a collection of platinum wires buried in a brown glassy substance with a little iron left in the core.

The experiments with the second ring (whose core was

formed of a length of ordinary soft iron wire) were more successful; but the results, though interesting, have not been thought worth including in this paper. The experience gained in the construction and use of the first two ring-magnets was turned to account in the two later ones.

Platinum wires were exclusively employed for the windings of the ring-magnets, as no substance either magnetic or fusible below 1200° was admissible. These wires, after use in one ring, were used again, with fresh insulation, in the next.

Iron cores.—The cores of the third and fourth ring-magnets (referred to in this paper as specimens A and B respectively), were formed from strips, one chosen from each of two groups of iron samples kindly procured for the author by Mr. R. Jenkins from Messrs. Jos. Sankey and Sons, of Bilston. The sample strips were 1 cm. in width; they were described as follows:—

Specimen A.—“Charcoal Iron, No. 4 Quality; thickness .02 in.”

„ B.—“Best Transformer Quality ~~xx~~; thickness .014 in.”*

These specimens proved, after careful annealing, to be exceptionally good both as regards high permeability and low hysteresis.

Platinum Thermometer Wire. Method of deducing Temperatures.—The specimen of pure annealed platinum wire used in the measurement of temperature was from the firm of W. C. Heräus, of Hanau, near Frankfurt-a.-M. The same piece of wire was used in both the later ring-magnets. Its diameter was .0453 cm., its specific resistance at 0° was 10,110 C.G.S. units, and its temperature-coefficient, which was remarkably high, was $\alpha_{0^{\circ}-100^{\circ}} = .00386$.

The resistance-temperature curve was taken by mounting the platinum wire (carefully re-annealed by heating to bright redness in a mass of asbestos) on a small cylindrical mica frame, and measuring its resistance at intervals of about 30°

* This iron, according to the above firm, gives on analysis 99.925 per cent. of iron by difference, the impurities (.075 per cent.) being distributed between carbon, phosphorus, and silicon, with a trace of manganese. It is as pure Swedish iron as can be produced commercially

up to 200° C. in a bath of linseed oil. The temperature was measured by a standard thermometer which was carefully checked at 0° and 100° . The oil was vigorously stirred, and observations of resistance were only made when the temperature was very nearly constant.

The experiments of Callendar and Griffiths have established the fact that, for pure annealed platinum wire, if $\frac{1}{100}$ th part of the increase of resistance which it experiences in passing from 0° to 100° be called the increment for one "platinum degree," so that the resistance of a platinum wire is a linear function of its "platinum temperature," pt° ; then, if t° is the actual temperature on the air-thermometer scale, the difference $t^{\circ} - pt^{\circ}$ can be expressed over wide ranges of temperature by the form $a + bt + ct^2$, which in this case must be reducible to the form $t^{\circ} - pt^{\circ} = \delta \{ (t/100)^2 - t/100 \}$.

It was thought that, in the present investigation, an error in the determination of the absolute values of the higher temperatures of even several degrees was of comparatively small moment where the recalescence point for different specimens of iron is known to differ by 50° or more. The resistance-temperature curve for the platinum thermometer wire was not determined directly above 200° , but was extrapolated by the help of Callendar's formula. The value of δ for this wire was $2 \cdot 10$.

The *absolute* values of the temperatures given must therefore be accepted with caution. They may be wrong by as much as 5° in the neighbourhood of 800° , the error rising with the square of the temperature. For the present purpose, however, it is the *relative* accuracy which is of importance, and this is of a much higher order.

To obtain reliable measurements of temperature, it was necessary that the thermometer-wire should satisfy the following conditions:—(1) Its temperature must not differ sensibly from that of the iron which it is intended to measure; (2) its constants must not alter in any way during the heating of the ring; (3) its resistance must be measured with sufficient precision.

(1) The first condition was the most difficult to satisfy: and it was only by actually burying the thermometric wire in the core itself, and by having the temperature very nearly

constant before taking any observations, that the temperature of the thermometer-wire and that of the iron core were brought to a satisfactory coincidence.

(2) With regard to the second condition: the resistance-temperature curve of the thermometer-wire, besides being taken previous to experiments, was also taken after those with each of the specimens A and B; in neither case could any alteration either in the resistance-coefficient or in the value of δ be detected. (Alterations of these constants which had taken place with former thermometer-wires were thought to be due to the fact that the insulation in contact with them had not been previously decarbonized (see p. 142).)

One alteration, however, did take place, particularly during the last annealing of specimen B, when a temperature of 1150° was reached and maintained for some time: the resistance of the thermometer-wire rose (on this occasion as much as 1 per cent.); though, as stated above, the form of its resistance curve remained throughout unchanged.

Messrs. Heycock & Neville* have repeatedly noticed such changes, though slight in amount, in their resistance pyrometers; and conclude that they were due to strains set up by the permanent expansion of the mica frames on which their wires were supported. It is possible that such increase of resistance may also be due, as Prof. Weber has suggested, to evaporation of the platinum at these high temperatures; none of the author's platinum wires retained, after *prolonged* heating, their bright metallic surface†.

* Heycock and Neville, "On the Determination of High Temperatures by means of Platinum Resistance Pyrometers," Trans. Chem. Soc. 1895, p. 160.

† Since the above was written, Mr. E. H. Griffiths, F.R.S., has kindly drawn my attention to the fact that his own platinum thermometer wires on mica frames, when heated for considerable periods at temperatures of 1200° – 1400° , do not thus lose their former bright surface. It is therefore rather improbable that in my case the dull surface should point to any serious evaporation of the platinum. I have in this connexion one or two observations which I had not given as I was in doubt as to their reliability:—The mean *sectional area* of the platinum thermometer wire, as measured by the specific gravity method, diminished from $\cdot 001182$ cm.² to $\cdot 001162$ cm.² during use in Rings A and B; the *length* increased from 31.33 to 31.56 cm. during the heating in Ring B, and during this same heating the *weight* of the wire did not alter.—D. K. M.

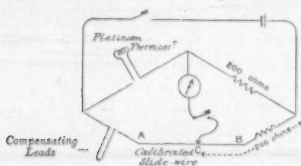
In reducing the results, it has been assumed that this alteration of resistance, which was generally very small, took place entirely during the time that the ring was at its highest temperature.

(3) In order to satisfy the third condition, it was not only necessary that precise measurements of resistance should be possible ($\frac{1}{20}$ per cent. represents more than $\frac{1}{2}^{\circ}$ at 1000°), but also that the resistance of the thermometer-wire and that of its compensating leads should be measured very nearly simultaneously.

To obtain the nett resistance from a *single observation*, the following arrangement (fig. 1) was employed, which is similar to, though not identical with, that used in Callendar's platinum thermometers.

The platinum thermometer-wire and leads formed one arm of the Wheatstone's bridge, and the compensating leads (made exactly equal to those of the thermometer), plus a

Fig. 1.—Form of Bridge for Temperature Measurements.



portion AC of the slide-wire, formed a second arm equal to the first. The other two arms contained equal resistances; the 200-ohm coil of a resistance-box in the one; and in the other the smaller coils of the box (together making 200 ohms) with just so many tenths of an ohm short-circuited as were in the remainder CB of the slide-wire. It is clear then that, if C is the position of balance, the resistance AC is equal to the nett resistance of the thermometer-wire.

The resistance of the slide-wire, whose calibration curve was very uniform, was about 1.4 ohms. This was approximately that of the thermometer-wire at 1200° , so that the balancing point for all lower temperatures was within the range of the metre-long slide-wire. The resistance per unit

length of the latter was conveniently obtained by inserting a standard ohm in the place of the thermometer-wire.

The above method was also applied to the measurement of the resistance of the iron in the core.

Insulation.—The chief difficulty of the method of these experiments lies in so insulating the wires themselves and the various layers of the ring-magnet from one another, that the insulation shall still remain good at the highest temperatures reached.

The only material which is practically applicable to the insulation of the wires is *asbestos paper*; this paper, which though fairly thin is by no means uniform, was used in the form of long narrow strips wound spirally over the wires to be insulated. After considerable practice, it was found possible to cover wires even as small as .3 mm. in diameter in this way with a reliable, and fairly thin and uniform covering; for such wires, a strip about 4 mm. wide was found to be most suitable.

Asbestos paper must of necessity contain starch or something similar to hold it together; and experiments proved that, when heated in a non-oxidizing atmosphere, the resulting carbon deposit reduced the insulating power of the asbestos to a very low value. This difficulty was overcome by carefully burning out all the hydrocarbons in one layer of insulation, leaving, so to speak, nothing but the asbestos 'ash,' a white and very brittle covering; and then cautiously over-covering this with a layer of ordinary asbestos paper.

In the first experiments, the leakage between the heating-coil (with 10 volts across its terminals) and the secondary winding was sufficient, when the ring was at 1000°, to drive the astatic needle of the sensitive galvanometer included in the secondary circuit almost round to its stops; and though the needle could be easily brought back artificially to its zero position again, the leakage was uncertain enough in amount to render ballistic observations at this temperature quite out of the question. The introduction of the above-described improvement in the asbestos insulation reduced this leakage to about $\frac{1}{100}$ th part of its former amount; but it still represented a not very constant zero error of several centimetres.

With the object of still further reducing the leakage, mica was tried as insulation between the layers. This plan ultimately succeeded very well; but the difficulty of applying a reliable layer of mica insulation to a surface of "double curvature," such as that of a small ring, without the assistance of any adherent whatever, appeared at first insurmountable. Latterly, however, it was found that if the mica insulation for the "doubly-curved" parts was applied in the form of small suitably-shaped pieces, previously heated in a Bunsen flame* and then bent or continuously broken by pressing over a rounded edge, then, by putting each piece of mica into place as the coil of insulated wire above it was wound, a reliable layer of mica insulation could be obtained without much trouble.

Mica was also used in the later ring-magnets both for the insulation of the iron strip and for that of the thermometer-wire, in a way about to be described; it was not only put between the turns of the iron strip, but was also placed next to it, beneath the asbestos covering; since any carbon deposited from the latter would, if in contact with the glowing iron, combine with it and alter its character.

It is easily shown that no appreciable error in the measured resistance of the iron or thermometer-wire would arise from a conductivity of the insulation sufficient to give rise to even a large deflexion of the ballistic galvanometer needle.

Construction of Ring-Magnets.—The rings containing specimens A and B were almost identical in construction, differing mainly in the superiority of the insulation of the later ring. They were made in the following way (see figs. 2-4):—

A sample iron strip was chosen, and a narrow band 2 mm. wide was cut by a cold chisel out of the middle, leaving a specimen consisting of two parallel strips each 4 mm. in width joined at one end. The mean section of this double strip was determined by the specific gravity method. It was then insulated with mica and covered with asbestos as shown in fig. 2.

The insulated thermometer-wire was next laid in the space

* This diminishes the tendency of the mica to split, and seems to do so by burning off the raw cut edges.

between the two iron strips, separated from them by slips of mica laid crosswise, and covered over by a plain mica strip (fig. 3). The whole was then rolled up on a temporary wooden form into a ring of four or five turns (fig. 4).

Thus, though asbestos was used in the core as insulation, it was the mica alone that was relied upon.

The ends of the iron specimen were then bent so as to leave the ring at right angles, and form a "stem" about 10 cms. in length. A portion of the same iron specimen resembling

Fig. 2.—Preliminary Stages. Half full size.

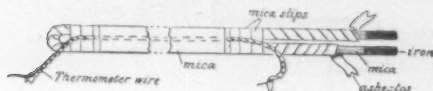


Fig. 3.—Section. Full size.

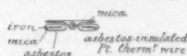


Fig. 4.—Section of completed Ring. Scale $\frac{1}{2}$ full size.

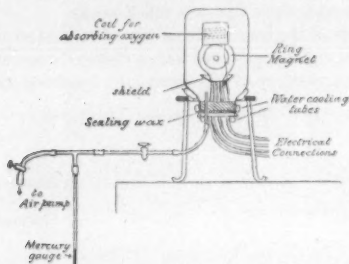


this stem both in form and insulation, but only leading just up to the ring and back (see fig. 6), was bound on beside the stem that compensation for its resistance might afterwards be made. The ends of the platinum thermometer-wire, together with a similar compensating resistance formed of a piece of the same platinum wire, were also led up the stem, which was in all parts most carefully insulated with mica.

The next step was to overwind the ring with three layers of platinum wire to serve respectively as magnetizing coil or primary, as heating coil, and as secondary. These wires, each of which, like the thermometer-wire, was doubly-insulated with asbestos in the way previously described, were

respectively 110, 150, and 100 cm. in length, and '6, '45, and '3 mm. in diameter. Mica insulation separated the several windings from each other and from the core.

Fig. 5.—Showing Ring in Glass Vessel.



The heating wire, wound back on itself to avoid magnetizing effect, was passed once or twice round the lower part of the stem to prevent that local abstraction of heat to which the stem would otherwise have given rise.

The secondary winding was wound last of all; it being possible in this way to secure better insulation, particularly from the primary, from which even a small leakage into the secondary was much more serious than any leakage from the heating coil; for in the former case every ballistic throw due to a change in the primary current would be associated with a change of zero, while in the latter case no such change would occur.

The ring was afterwards successively covered with a layer of mica, a layer of asbestos, a covering of iron-wire gauze, and, finally, with a jacket consisting of three coverings of soft asbestos millboard. The iron gauze, which became red hot when the ring was being used at high temperatures, was placed there to intercept oxygen on its way in.

The oxygen-absorbing coil was, however, quite distinct from this. It consisted of two bare iron wires, connected in parallel, wound side by side on a cylindrical asbestos frame, and surrounded by a loosely-fitting asbestos mantle. Before

the experiments they were heated by an electric current to a temperature at which they would readily absorb oxygen.

The coverings of the ring-magnet formed a mass of asbestos about 6 cm. in diameter; a small hole through them enabled one to see what sort of temperature had been reached in the middle of the ring.

The coil for absorbing oxygen was then made fast to the ring-magnet, and the leading wires brought through a cork (previously boiled in linseed oil), which was then inserted into the glass jar and covered with melted sealing-wax (fig. 5). Finally, the terminal wires were soldered to those leading away to the rest of the apparatus.

The following particulars relate to the last two ring-magnets constructed :—

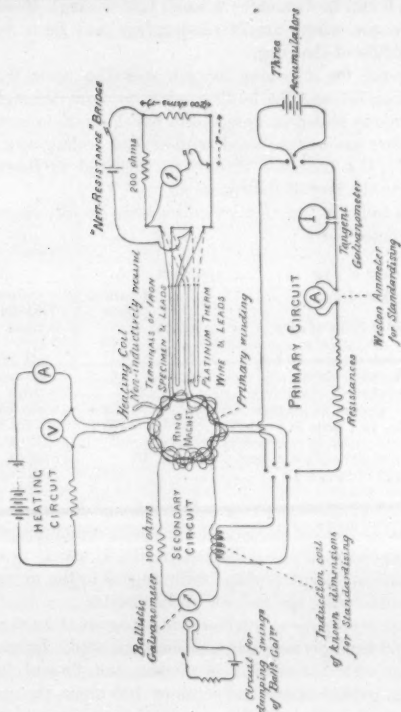
Core of ring specimen	A.	B.
Description of iron	Charcoal Iron.	Swedish Transformer Iron.
Number of turns of core	4	5 turns
Density of iron (after experiments)	7.775	7.461
Total sectional area of iron	.131	.143 cm. ²
Mean diameter of ring	2.35	2.23 cm.
Number of turns of primary	28	29 turns.
per cm. of magnetic circuit	3.792	4.143 turns per cm.
Field due to 1 amp. in primary	4.765	5.206 C.G.S. units.
Mean sectional area enclosed by primary	.72	.83 cm. ²
Number of turns of secondary	15	17 turns.
Resistance of iron at 18°	.0486	.0477 ohm.

Electrical Arrangements.—Reduction of Results to Absolute Measure.—The various electrical circuits communicating with the ring-magnet are indicated in fig. 6, which needs little explanation, except perhaps with regard to the arrangement for standardizing the ballistic galvanometer.

For this purpose an auxiliary ring-magnet of known dimensions and having a non-magnetic core was used. Its secondary, in series with that of the iron ring-magnet, formed, in circuit with the galvanometer and a coil of 100 ohms, the secondary circuit, as it remained throughout the experiments. On reversing a known current in the primary of the auxiliary coil, the resulting throw of the ballistic galvanometer was a measure of its sensitiveness to induction alterations in that particular secondary circuit, whether taking place in the core of the auxiliary coil, or in that of the iron ring-magnet. Such

standardizing throws were observed before the readings at each temperature, and in this way direct compensation was made for alterations in the resistance of the secondary circuit.

Fig. 6.—General Plan of Connexions.



The standardizing throw and the numerical data of the iron ring-magnet being known, it was possible to obtain from each ballistic observation the change of magnetic induction in the iron core to which it was due; correction being of course made for the induction which did not pass through the core.

The magnetizing current was measured with a tangent galvanometer, whose indications were compared with those of a standard Weston ammeter. The magnetizing force was assumed to be uniform in intensity throughout that part of the section of the ring occupied by the iron; which assumption is, in the present case, but approximately true, though the oblong cross-section favours the uniformity.

Description of the Experiments.

The iron cores in their initial condition may be regarded as unannealed; for the original annealing was destroyed during their preparation.

To study the effects of annealing, measurements were taken during the first heating of specimen A. With this exception, however, all the experiments recorded in this paper were made during the cooling from some definite temperature at which the iron had been thoroughly annealed, that all effects other than those due directly to change of temperatures might be eliminated. The duration of each of the long sets of experiments was about 20 hours.

The ring containing *specimen A* was first heated slowly up to 1050° , stopping at various stages on the way up for observations, and then allowed to cool, when another series of observations was made. It was afterwards heated to 920° to find whether the annealing was complete.

The ring was then taken to pieces, that the cross-section of its iron and the constants of the thermometer-wire might be re-determined.

The arrangements for absorbing the oxygen were with this ring not very perfect. The coil for this purpose had but one iron wire instead of two in parallel; and, becoming locally oxidized, the wire fused before it had absorbed all the oxygen. The cross-section of the iron core on removal from the ring and on detaching the oxide was found to have diminished by as much as 8 per cent. Since, however, no change of section occurred (as deduced from the constancy of the resistance and permeability) during the concluding heating to 920° , it may safely be assumed that the oxidation took place previous to the cooling from 1050° ; and that the section during subsequent experiments was the same as that measured at their close.

The ring-magnet, of which *specimen B* was the core, was, after preliminary magnetic measurements, heated at once to 840° , a little above the critical point; at this temperature it was kept for about two hours and then allowed to cool. The following day the ring was again heated to this temperature, it being intended to take a series of observations during cooling. The series had, however, to be postponed, owing to an unfortunate occurrence—the sealing-wax over the cork had slightly softened during the prolonged heating, and the excess of outside pressure began to drive the cork slowly into the jar, rendering it no longer air-tight. The heating current was immediately shut off; but, when cold, the iron core was found to have been partially oxidized by the inrush of air, reducing its section about 4 per cent. To prevent a repetition of this occurrence, an indiarubber tube containing flowing water was arranged to keep the neck of the jar cool.

During the second absorption of the oxygen, the iron-wire coil, which had already been used once for this purpose, burnt out. But it was thought that the absorption might be complete, and the experiments were proceeded with. After heating the ring, therefore, a third time to 840° , a series of measurements was made while the temperature was falling.

The ring was then still further annealed by heating to 1150° ; and, during cooling, a set of observations like the previous was taken. On their completion, this ring was also taken to pieces that the cross-section of the core might be re-determined and the constants of the thermometer-wire checked. The last absorption of the oxygen does not seem to have been at all complete, for during the last heating the section diminished by 6 per cent.

Thus, in these experiments, though the method of absorbing the oxygen was shown to be efficient, yet troubles due to oxidation were by no means absent. Irrespective of the diminution of the sectional area of the iron, the presence of oxide in the core might affect the results in a variety of ways:—(1) by the shunting effect due to its conductivity, or (2) by impairing the quality of the insulation, both tending to diminish the apparent resistance of the iron; and (3) by having an appreciable permeability, affecting the magnetic measurements.

(1) The shunting effect is quite negligible. (2) Though the inner surface of the mica was altered by contact with the oxide, the insulation was not seriously injured. (3) With regard to the third possibility :—In small fields, such as those used in the present experiments, the magnetic qualities of soft iron are at ordinary temperatures so very much more intense than even those of the magnetic oxide of iron, that no perceptible alteration of the ballistic galvanometer throws could arise from any oxide present. At temperatures above the critical point, the experiments of M. Curie, already cited, have shown that though the susceptibility of that oxide rapidly decreases with rise of temperature, yet it may be comparable with that of iron, which is also very feebly magnetic in the neighbourhood of 1000°C . The author's methods were, however, not adapted to the determination of the magnetic qualities of feebly magnetic bodies, and any determinations by them of the permeability of the iron core when $\mu < 10$ have in any case little or no worth; so that errors of such values need hardly be considered.

The only serious effect of the oxidation was to alter the sectional area of the iron. By directly measuring this section both before and after the experiments with each ring-magnet, uncertainties in the absolute values, both of the permeability and resistance measurements, were practically eliminated in all the series of observations, except that taken after annealing specimen B at 840° ; and even in this case the alteration of resistance which took place gave the means of determining with but little uncertainty that sectional area which was not directly measured. Whilst cooling from 1150° a pause of half-an-hour was made at 1071° . During this time the resistance of the iron did not increase, and it is therefore concluded that from this time onwards no further change of section due to oxidation took place.

Method of taking Observations at each Temperature.—The experiments with specimen B were as follows (with specimen A they were similar but not so full) :—

When the temperature had become quite stationary, and the "standardizing throw" had been observed, the iron core was *de-magnetized* by repeatedly reversing the magnetizing current whilst diminishing its strength.

Observations were then taken, by the method of reversals, of the magnetic induction in the core for each of eight values of the field; beginning with $H = 0.78$ and 1.53 to get the permeability under very small forces, and ending with $H = 9.20$. Next, the iron core was carried through a cyclic process of magnetization between definite limits of magnetic field ($H = \pm 6.83$), observations being taken by the "step-by-step" method—sixteen steps round the cycle. This process had, however, been previously gone through several times till the iron had reached a cyclic state. Finally, the iron was carried through a further cyclical magnetization, this time between definite limits of induction (e. g. $B = \pm 4550$ lines/cm²).

The last experiment was by far the most difficult; for the magnetizing currents had to be so adjusted that the sum of the ballistic throws during one half of the cycle might add up to a certain previously calculated amount depending on the magnitude of the standardizing throw. The difficulty was much reduced by diminishing the number of observations, though the number taken (just eight all round the cycle) barely suffices to determine the hysteresis loop.

The group of experiments at one temperature was concluded by a measurement of the resistance of the iron.

Throughout the whole of such sets of experiments, and particularly when working immediately below the critical point, where the magnetic properties vary so rapidly, the most scrupulous attention was paid to the temperature that it should not, if possible, be allowed to alter more than a fraction of a degree; and this attention was the more necessary when working with the larger magnetizing currents, for their heating effect in the primary circuit was quite perceptible.

The time-period of the ballistic galvanometer needle being about eleven seconds, magnetic "creeping" effects lasting much more than a second would not be recorded.

In reducing the observations for each curve of cyclic magnetization, the mean of the corresponding throws taken during the ascending and descending halves of the process has in every case been taken; though this eliminates any systematic unsymmetry which the curve might have possessed.

Discussion of Results.

The results of the experiments are given in the accompanying Tables and Curves (Plates V. & VI. and Woodcuts, pp. 170-177); the series being in each case arranged in the order in which they were taken.

I. *Curves of Magnetization by the Method of Reversals.*—The effect of annealing (at 1050°) on specimen A is strikingly shown in the curves marked 1, and in the corresponding permeability curves 9 and 10. Its effect on the *maximum* permeability and corresponding field is shown in Table IX. The ultimate result of the annealing was to increase the maximum permeability at ordinary temperatures three-fold ($\mu=1355$ to 4050) whilst reducing the corresponding field to little more than half its former value (3.72 to 2.02).

The first annealing of specimen B affected the permeability in a similar manner (compare curves No. 3). The experiments on this specimen, however, were carried out with a view to showing especially the *difference between annealing at a red heat and at a low white heat*. The curves of magnetization at different temperatures obtained after annealing respectively at 840° and 1150° are numbered 7 and 8; whilst the corresponding permeability-temperature curves for different strengths of magnetic field are shown in the curves 11, 12, and 13.

A reference to these will show that the difference between annealing at these two temperatures is very marked; the iron having by the last annealing been rendered much more susceptible to weak magnetic fields. Annealed at 840° , the maximum permeability at ordinary temperatures was about 4000 in a field of 1.84 C.G.S. units; annealed at 1150° this rose to the remarkably high value of 4680 in the much smaller field of 1.48. The effect of this difference in annealing is, at temperatures immediately below the critical point, even more striking, as is shown by a comparison of the μ - t curves (Nos. 11 & 12) for values of H below unity.

An interesting point with regard to specimen B is the *diminution of permeability* which it experiences *in the neighbourhood of 550° C.* This diminution can be traced on the μ - t curves for all but the highest values of H , though it is

especially noticeable when the magnetizing force is about 1 C.G.S. unit. The effect is more pronounced after the annealing at 1150° than after that at 840° . This peculiarity is hardly, if at all, possessed by specimen A, though it must be borne in mind that the temperature-interval between observations is here much greater, and the effect may thus be disguised. In some earlier experiments made on a ring-magnet having soft commercial iron wire for its core, the minimum of permeability, which occurred at about 500° , was still more marked than in specimen B; the depression being noticeable in the μ - l curves for fields as large as 5 C.G.S. units. It may be mentioned that, in a list of those temperatures at which the physical properties of iron undergo marked alteration, Tomlinson* gives 550° as that at which the internal friction of soft iron begins to rapidly increase, and at which the "specific heat of electricity" for that material changes sign.

The permeability under small magnetizing forces ($H < 5$ C.G.S. unit) rises at first slowly ($\cdot 2$ to $\cdot 3$ per cent. per degree) when the temperature is raised; then, in the neighbourhood of 300° , quite rapidly. It remains nearly constant between 400° and 550° , after which it rises with increasing rapidity to an enormously high value ($\mu = 12660$ for $H = 0.78$ is the highest observed) only to sink still more rapidly at the critical temperature to a value quite insignificant.

As the magnetic field gets smaller (curves 11 and 12) the maximum of permeability increases, and the temperature at which it occurs approaches the critical temperature, so that in very small fields the fall of the permeability is exceedingly sudden.

An examination of the B - H curves at various temperatures (Nos. 6, 7, and 8) shows that as the temperature rises, the initial slope of the curve increases and approaches more and more nearly to the maximum slope. And the critical temperature occurs just when they would coincide, *i.e.* when the B - H curve would start from the origin with its maximum slope, and the maximum permeability would occur in an infinitely small magnetic field.

* H. Tomlinson, Phil. Mag. vol. xxvi. July 1888, p. 21.

The *critical temperature* is as near as can be given 795° for specimen A, and 780° for specimen B. For the soft iron wire specimen it was about 770° .

Above this temperature the induction is proportional to the magnetizing force (curves 4 and 5) indicating the *same permeability for all forces*, a result in accordance with the experiments of M. Curie. The curve of variation of the permeability with temperature (No. 13, II.) is given with some diffidence as the observations were liable to considerable error (see first part of Tables IV. and V.), and as the results are too remarkable to pass without further corroboration. It indicates a maximum of about $\mu=100$ at 820° , and another of $\mu=17$ at about 1050° ; whilst between 920° and 980° the permeability is but 2.3.

II. *Curves of Cyclic Magnetization*.—The variations in the form of these curves due to changes of temperature can be studied in the series of curves Nos. 2, 4, and 5, in which the results recorded in Tables II., VI.-VIII. are plotted.

The process of annealing of specimen A is graphically depicted in the first of these series.

In series No. 5 it will be noticed how, as the temperature rises and approaches 550° , the inclination of the curves to the vertical axis increases, and how with further rise of temperature the curves become more erect and angular. Those taken at $764^{\circ}\cdot 5$ in this series are interesting as showing the magnetic condition of the iron almost immediately below the critical temperature. The coercive force required to remove the remanent magnetism was little more than $\frac{1}{10}$ of a C.G.S. unit. For the steep-sloping part of the curve the value of $\frac{dB}{dH}$ is about 15000; when H is much above 1 unit, its value is about 150. The transition from the steep part of the curve to the gently-sloping part is very sudden.

These values of $\frac{dB}{dH}$ are of the same order of magnitude in all the sets of curves of both series taken within 50° of the critical temperature, and they seem to represent an approximately constant property of the iron in this region.

The area enclosed by each hysteresis loop has been summed up and divided by 4π , giving the quantity $\frac{1}{4\pi} \int H dB$, the

work done in ergs per c.cm. of the iron by the magnetic forces during the performance of one cycle. The results are given beside the corresponding temperatures in the Tables numbered X. (*a* and *b*), and are plotted in curves 14-16, which show the way in which the hysteresis diminishes to zero as the iron is heated up to the critical temperature.

No evidence of hysteresis above this temperature could be experimentally obtained, a result which one might be led to expect from the fact that the permeability at these temperatures is the same in all magnetic fields.

A comparison of the two curves No. 15 shows the marked diminution of hysteresis at all temperatures which resulted from annealing at 1150° instead of at 840° . The curves expressing the hysteresis for cycles within definite limits of induction (Curves 16 I. & II.) are not comparable, as the limits during the later series were, owing to oxidation, greater than during the former. In order to render them comparable a third curve has been dotted in, for which the ordinates of I. were multiplied by a constant 1.11 (the assumed ratio of the hysteresis when $B = \pm 4260$ to that when $B = \pm 4550$, which, according to results of many observers, cannot at ordinary temperatures be far wrong). Comparing then curve I. with curve III. an idea is obtained of the extent to which the hysteresis can be diminished by annealing at the high temperature 1150° , instead of at the lower one 840° . The comparison is given on Table X. c.

Table X. d. shows concisely the various stages of annealing of specimen B. The hysteresis at ordinary temperatures for $B = \pm 4550$ was originally 1480 ergs per cub. centim. per cycle. Prolonged annealing at 840° reduced this to about 800 ergs; whilst annealing at 1150° brought the hysteresis down to 612 ergs.

In a paper read before the Institution of Civil Engineers about a year ago*, Prof. Ewing described some tests which he made on a sample of transformer plate of similar thickness and obtained from the same firm as the author's were. The maximum permeability was about 4500, while the hysteresis

* Ewing, "Magnetic Testing of Iron and Steel," Journ. Inst. C. E. May 1896.

varied from 740 down to 580 ergs for $B = \pm 4000$; and attention is drawn to these values as representing the best iron both as regards high permeability and low hysteresis which he had tested. It is therefore of interest to note that specimen B had after annealing at 1150° a maximum permeability of over 4600 at ordinary temperatures, and a hysteresis as calculated for $B = \pm 4000$ of just under 500 ergs, corresponding to a hysteresis loss of but $\cdot 295$ watts per lb. at a frequency of 100 $\sim$. The following values are taken from the magnetization curve at $24^\circ \text{C}.$ —

B.	H.	μ .
2000	$\cdot 74$	2700
4000	1 $\cdot$ 01	3960
6000	1 $\cdot$ 33	4520
8000	1 $\cdot$ 82	4400
10000	2 $\cdot$ 84	3520
12000	5 $\cdot$ 63	2130

Between 0° and 200° the *diminution of hysteresis with rise of temperature is $\cdot 08$ per cent. per degree*, when the limits of induction are kept constant.

W. Kunz * concludes from his experiments that for soft iron the hysteresis is expressible as a linear function of the temperature. This is not in accordance with the author's results. The curves expressing this relation (Nos. 16) for specimen B are far from straight, and show a decided dip at about 500° corresponding to the minima which occur near that temperature on the permeability-curves.

III. *Resistance-Temperature Curves.*—The variations of specific resistance with temperature are shown graphically on curves 17 and 18.

By taking a sufficient number of observations (see Tables XI. and XII.) the author has been able to show that the resistance-temperature curve undergoes *no abrupt change of direction* at or near the critical temperature. (Previous observers have given curves all of which were *broken* at this temperature; an examination of the figures given by Hop-

* W. Kunz, *Elektrotechnische Zeitschrift*, 1894, p. 196.

kinson and Le Chatelier in the papers already cited shows that in each case they would be more strictly satisfied by a *continuous curve* like those here given.)

The annealing of specimen A has reduced its specific resistance considerably.

With specimen B, the further annealing at 1150° has but slightly altered it from the condition as regards specific resistance in which it was after the annealing at 840° .

The temperature-coefficients and specific resistance at 0° and 1000° are given at the foot of Tables XI. and XII.

As regards the resistance-changes both of specimens A and B, and of the soft commercial iron previously tested :— The resistance rises with rising temperature at a rate which increases fairly uniformly till a specific resistance of about 100,000 C.G.S. units is reached. Here the maximum slope of the curve occurs at a point *but few degrees below the critical temperature*. The rate of increase of resistance then falls off rapidly, so that at 1000° the temperature-coefficient is about $\cdot 0024$ and the specific resistance 118,000.

With specimen B, which, judging from the low value of its specific resistance at 0° (10,050 C.G.S. units) and from the high value ($\cdot 0057$ at 0°) of its temperature-coefficient, should be a sample of very pure iron, the *maximum coefficient* after annealing at 1150° was $\cdot 0204$ at about 765° . The number of observations on this resistance-temperature curve (18 II.) make it perhaps permissible to note its irregularities a little more closely. A flat part of the curve will be noticed between 830° and 900° , and the temperature-coefficient, which remains tolerably constant over this range, has at 865° the value $\cdot 0068$. (Dr. Hopkinson gave $\cdot 0067$ as the coefficient of iron above the critical temperature.) The points of maximum curvature 800° and 920° seem to be in agreement with the minima of the permeability curve (No. 13) obtained at the same time.

The above experiments were carried out in the laboratories of the Physical Institute of the Polytechnicum, Zürich ; and the author takes this opportunity of expressing his gratitude to the director, Prof. H. F. Weber, for encouragement received throughout the progress of the work.

Zürich, March 1897.

TABLE I.—SPECIMEN A.

*Induction and Permeability in various Magnetic Fields
taken at different temperatures.*

(a) Before annealing. (Curves 1 and 9.)

H in C.G.S. units.	20°.		220°.		365°.		532°.		660°.	
	B	μ	B	μ	B	μ	B	μ	B	μ
·917	401	438	463	505	506	552	933	1017	1864	2033
2·24	2495	1114	2868	1281	2990	1335	4180	1867	5285	2360
3·94	5325	1352	5345	1357	5145	1306	6380	1620	7215	1830
6·10	7465	1225	7270	1192	6850	1107	8000	1312	8685	1440
8·26	8805	1066								

(b) After annealing at 1050° C. (Curves 1, 6, and 10.)

H in C.G.S. units.	774°.		722°.		636°.		506°.	
	B	μ	B	μ	B	μ	B	μ
·070			292	4170	109	1560	64	915
·140			583	4170	272	1950	166	1190
·343			2475	7220	1188	3465	617	1800
·917	5980	6520	6008	6550	5395	5880	4790	5220
2·24	7795	3480	8360	3730	8630	3850	8820	3940
3·94	8290	2105	9475	2405	9870	2480	10320	2620
6·10	8320	1366	10180	1670	10705	1750	11310	1855

H in C.G.S. units.	324°.		188°.		21°·5.	
	B	μ	B	μ	B	μ
·070	·48	685	29	414	21?	300?
·140	104	743	68	485	50	357
·343	366	1067	248	723	181	527
·917	3355	3660	1890	2060	1208	1318
2·24	9105	4060	9175	4065	8845	3950
3·94	10925	2775	11420	2900	11435	2904
6·10	11960	1960	12550	2060	12770	2093

(c) After re-heating to 920°. (Curves not given.)

H in C.G.S. units.	770°.		759°.		680°.		630°.		22°.	
	B	μ	B	μ	B	μ	B	μ	B	μ
·070			158	2260	112	1600	69	985	24	343
·140			391	2800	261	1870	164	1170	53	379
·343			1908	5560	1298	3785	810	2360	168	490
·917	5810	6340	5850	6380	5745	6260	5020	5470	1430	1560
2·24	8045	3590	8120	3625	8560?	3820?	8390	3745	8485	3790
3·94	8940	2270	9465	2405	9945	2525	9825	2495	11380	2890
6·10	9110	1494	10180	1670	10840	1778	10790	1770	12790	2095

TABLE II.—SPECIMEN A.

*Data for Cyclic B-H Curves at various Temperatures.*Limits of Cycle: $H = \pm 6.10$.

(a) Before annealing. (Curves 2.)

H	20°.	224°.	367°.	534°.	665°.
	B	B	B	B	B
6.10	7370	7145	6830	7980	8610
3.94	6985	6645	6300	7365	7715
2.24	6380	5945	5530	6500	6570
.917	5620	5090	4660	5410	5240
0	4800	4232	3805	4265	3750
-.917	3513	2793	2285	1750	- 526
-2.24	-1642	-2365	-2600	-3935	-5148
-3.94	-5410	-5345	-5170	-6380	-7140
-6.10	-7370	-7145	-6830	-7980	-8610
Hysteresis in ergs per cm. ³ }	3900	3210	2800	2670	1850

(b) After annealing at 1050° C. (Curves 2 and 14.)

H	765°.	708°.	625°.	506°.	325°.	175°.	21°-5.
	B	B	B	B	B	B	B
6.10	9100	10220	10640	11185	11850	12320	12480
3.94	8790	9630	10085	10615	11400	12000	12190
2.24	8075	8720	9220	9880	10815	11520	11760
.917	6720	7205	7730	8565	9925	10835	11180
0	2985	3930	4780	6190	8170	9565	10210
-.917	-5830	-5845	-5290	-4290	-2005	1389	4170
-2.24	-7880	-8275	-8400	-8620	-9090	-9145	-8990
-3.94	-8780	-9440	-9750	-10190	-10810	-11210	-11340
-6.10	-9100	-10220	-10640	-11185	-11840	-12320	-12480
Hysteresis in ergs per cm. ³ }	500	920	1345	2060	3090	4110	4570

(c) After re-heating to 920°. (Curves not given.)

H	738°.	679°.	630°.	22°.
	B	B	B	B
6.10	10370	10840	10660	12520
3.94	9565	10115	10020	12210
2.24	8515	9120	9085	11790
.917	6880	7510	7505	11185
0	3535	4397	4500	9695
-.917	- 5610	- 5240	- 4435	2766
-2.24	- 8100	- 8365	- 8080	- 8420
-3.94	- 9450	- 9820	- 9670	-11210
-6.10	-10370	-10840	-10660	-12520
Hysteresis in ergs per cm. ³ }	780	1280	1510	4720

TABLE III.—SPECIMEN B.

Preliminary Measurements at 24° C. (Curves No. 3.)

Induction and Permeability in various Magnetic Fields.

H	Initial condition.		After 2 hours at 840°.		After 5 hours at 840°.	
	B	μ	B	μ	B	μ
0.78	14	178	32	410	35	450
1.53	37	242	79	516	84	549
3.78	102	270	280	741	307	813
10.17	560	550	2656	2608	2995	2942
2.49	4352	1746	10295	4135	10525	4230
4.40	7265	1652	12395	2816	12405	2820
6.83	9220	1349	13420	1963	13340	1953
9.20	10420	1133	14005	1523	13920	1515

Cyclic **B-H** Curves between Limits **H** = ± 6.83 .

H	Initial condition.	After 2 hours at 840°.	After 5 hours at 840°.
	B	B	B
6.83	9120	13100	13120
4.40	8670	12800	12820
2.49	8005	12425	12430
1.017	7135	11905	11885
0	6165	10890	10740
-1.017	4265	1420	778
-2.49	-3882	-10105	-10380
-4.40	-7270	-12140	-12210
-6.83	-9120	-13100	-13120
Hysteresis in ergs per c. cm.	4740	5020	4625

Cyclic **B-H** Curves at 20° C.Limits of Cycle : **B** = ± 4020 .Limits **B** = ± 4140 .

Initial condition.		After 2 hours at 840°.		After 5 hours at 840°.	
H	B	H	B	H	B
2.47	4025	1.15	4015	1.170	4140
1.007	3520	0.646	3760	0.660	3810
0	2763	0	2887	0	2900
-1.007	870	-0.646	63	-0.660	242
-2.47	-4025	-1.15	-4015	-1.170	-4140
Hysteresis per cycle	$= 1218 \frac{\text{ergs}}{\text{cm.}^2}$		Hysteresis = 664 $\frac{\text{ergs}}{\text{cm.}^2}$	Hysteresis = 669 $\frac{\text{ergs}}{\text{cm.}^2}$	

TABLE IV.—SPECIMEN B.

Induction and Permeability in various Magnetic Fields
taken successively during cooling from 840° C.

Above critical Temperature. (Curves 4, 7, and 13.)

H	820°.		816°.		799°.	
	B	μ	B	μ	B	μ
1.017			19	19	40	39
2.49	29	12	35	14	97	39
4.40			78	18	163	37
6.83	84	12	101	15	262	38
9.20			124	13	363	39
Mean Permeability	}		12	...	16	...
					...	38

H	797°.		788°.		784°·5.		782°·5.	
	B	μ	B	μ	B	μ	B	μ
1.017					124	122		
2.49					273	110		
4.40					518	118		
6.83	349	51	409	60	787	115	825?	121?
9.20					1035	113		
Mean Permeability	}		...	...	...	116	...	...

Below critical Temperature. (Curves 4, 7, and 11.)

H	770°.		764°.		746°.	
	B	μ	B	μ	B	μ
.078			298	3820	198	2540
.153			663	4330	490	3200
.378			2780	7360	2248	5950
1.017			5675	5575	5830	5730
2.49			6720	2700	8020	3220
4.40	5960	1355	7070	1606	8565	1950
6.83	6500	952	7410	1084	8905	1304
9.20			7870	856	9165	996

H	710°.		648°·5.		567°.		467°.	
	B	μ	B	μ	B	μ	B	μ
.078	131	1680	90	1150	61	780	57	730
.153	322	2100	230	1500	141	920	143	935
.378	1722	4560	1041	2755	645	1710	511	1354
1.017	5955	5850	5685	5585	5110	5020	5195	5100
2.49	8630	3470	9030	3625	8935	3610	9475	3800
4.40	9840	2260	10440	2375	10695	2430	11030	2510
6.83	10580	1550	11350	1662	11770	1725	12090	1770
9.20	10955	1191	11870	1290	12315	1340	12660	1377

Table IV. (continued).

H	360°-5.		251°-5.		136°.		24°.	
	B	μ	B	μ	B	μ	B	μ
·078	50	640	31	400	30	380	20	250
·153	126	820	70	450	68	440	40	260
·378	571	1510	294	778	318	840	178	470
1·017	5125	5040	4360	4280	3253	3200	1934	1900
2·49	9800	3935	9940	3990	10005	4020	9420	3780
4·40	11580	2630	11980	2720	12140	2760	11835	2690
6·83	12705	1862	12890	1888	13260	1942	12980	1902
9·20	13320	1449	13525	1471	13860	1507	13600	1480

TABLE V.—SPECIMEN B.

Induction and Permeability in various Magnetic Fields taken successively during cooling from 1150° C.

Above critical Temperature. (Curves 5, 8, 12, and 13.)

H	1145°.		1047°.		980°.		975°.		960°.	
	B	μ	B	μ	B	μ	B	μ	B	μ
1·017										
2·49			42	17	15	6				
4·40										
6·83	52?	8					17	2·5	17	2·5
9·20										
Mean Permeability ...	...	8	...	17	...	6	...	2·5	...	2·5

H	922°.		805°-5.		851°.		829°-5		816°.		787°-5.	
	B	μ	B	μ	B	μ	B	μ	B	μ	B	μ
1·017												
2·49			129	52	173	69	260	104	260	104	120	48
4·40							434	99	434	99	215	49
6·83	16	2·3	388	57	475	70	650	96	781	114	336	49
9·20											518?	56?
Mean Permeability	} 2·3 ...		54 ...	69·5 ...	100 ...	106 ...	51					

Below critical Temperature. (Curves 5, 8, and 12.)

H	776°-5.		775°.		764°-5.		748°.	
	B	μ	B	μ	B	μ	B	μ
·078					631	8090	485	6210
·153					1935	12660	1585	10360
·378					4210	11140	4465	11820
1·017	925	908	1353	1330	5020	4930	6720	6600
2·49	1788	718	1964	789	5780	2320	7980	3200
4·40			2442	556	6305	1434	8290	1885
6·83			2798	410	6440	943	8555	1252
9·20			3006	327	6550	712	8645	940

Table V. (*continued*).

H	730°.		695°.		634°.		554°.	
	B	μ	B	μ	B	μ	B	μ
0.78	372	4770	243	3120	126	1620	73	935
1.53	1161	7600	713	4060	332	2170	172	1120
3.78	4165	11020	3565	9430	2128	5630	996	2640
1.017	6850	6730	6845	6720	5780	5680	4275	4200
2.49	8415	3380	8985	3610	8800	3530	7565	3040
4.40	9160	2085	9985	2270	10160	2280	9630	2190
6.83	9495	1390	10670	1562	11050	1620	10940	1603
9.20	9730	1024	11110	1209	11670	1269	11690	1271

H	457°.		352°.		249°.		137° 5.		24°.	
	B	μ	B	μ	B	μ	B	μ	B	μ
0.78	78	1060	77	980	48	615	43	550	35	450
1.53	172	1125	167	1090	113	740	99	650	85	555
3.78	989	2620	879	2325	457?	1210?	471	1246	356	942
1.017	5200	5110	5115	5020	4730	4630	4410	4330	4010	3940
2.49	8710	3495	9230	3705	9385	3770	9740	3910	9480	3810
4.40	10460	2380	10930	2485	11145	2535	11560	2625	11400	2590
6.83	11475	1680	11930	1748	12170	1783	12495	1830	12440	1823
9.20	12150	1321	12460	1355	12760	1388	13090	1423	13040	1418

TABLE VI.—SPECIMEN B.

Data for *Cyclic B-H Curves* taken successively at diminishing temperatures after annealing at 840° C.

Limits of Cycle: $H = \pm 6.83$. (Curves 4 and 15.)

H	764°.	746°.	710°.	648° 5.	567°.
	B	B	B	B	B
6.83	7370	8870	10350	11130	11495
4.40	7060	8545	9730	10500	10935
2.49			8870	9665	10090
1.017	6005	6425	7380	8295	8620
0		3246	4330	5515	5770
-1.017	-5560	-5820	-6000	-5555	-4535
-2.49			-8555	-8900	-8860
-4.40	-7060	-8480	-9700	-10250	-10480
-6.83	-7370	-8870	-10350	-11130	-11495
Hysteresis in ergs per c. cm.	426	573	1010	1650	2180

Table VI. (continued).

H	467°.	360°·5.	261°·5.	136°.	24°.
	B	B	B	B	B
6·83	11910	12540	12770	13040	12685
4·40	11400	12110	12390	12720	12360
2·49	10695	11495	11805	12230	11895
1·017	9520	10355	10810	11380	11110
0	7240	8245	8925	9770	9680
-1·017	-4940	-4010	-2608	-429	1395
-2·49	-9340	-9715	-9820	-9910	-9395
-4·40	-10830	-11495	-11735	-11980	-11610
-6·83	-11910	-12540	-12770	-13040	-12685
Hysteresis in ergs per c. cm. }	2550	3195	3700	4360	4700

TABLE VII.—SPECIMEN B.

Data for Cyclic **B-H** Curves taken successively at diminishing temperatures after annealing at 1150° C.

Limits of Cycle: **H** = $\pm 6\cdot83$. (Curves 5 and 15.)

H	764°·5.	748°.	730°.	695°.	634°.
	B	B	B	B	B
6·83	6145	8240	9420	10615	10760
4·40	5815	8030	9100	10035	10060
2·49	5490	7670	8495	9205	9080
1·017	5075	6800	7185	7820	7180
0	1389	2455	2856	3480	3215
-1·017	-5060	-6540	-6835	-6920	-5490
-2·49	-5485	-7575	-8320	-8870	-8430
-4·40	-5795	-7975	-9040	-9850	-9860
-6·83	-6145	-8240	-9420	-10615	-10760
Hysteresis in ergs per c. cm. per cycle }	120	328	426	797	1010

H	551°.	457°.	352°.	249°.	137°·5.	24°.
	B	B	B	B	B	B
6·83	10825	11360	11735	12110	12280	12150
4·40	10025	10810	11230	11730	11950	11810
2·49	8650	9050	10435	11125	11420	11300
1·017	6377	8340	8875	9980	10510	10400
0	3322	5395	6050	7520	8525	8560
-1·017	-4140	-4930	-3960	-3881	-3196	-2327
-2·49	-7600	-8710	-8650	-9295	-9670	-9290
-4·40	-9575	-10365	-10620	-11100	-11330	-11150
-6·83	-10825	-11360	-11735	-12110	-12280	-12150
Hysteresis in ergs per c. cm. per cycle }	1345	2025	2565	3130	3500	3660

TABLE VIII.—SPECIMEN B.

Data for Cyclic B-H Curves between constant limits of Induction taken successively at diminishing temperatures.

(a) After annealing at 840° C.

Limits of Cycle : $B = \pm 4260$. (Curves 4 and 16.)

764°.		746°.		710°.		648°·5.		567°.	
H	B	H	B	H	B	H	B	H	B
·60	4250	·60	4255	·73	4265	·81	4260	·93	4250
·31	3520	·32	3547	·34	3668	·36	3725	·56	3970
0	2092	0	2261	0	2611	0	2860	0	2870
−·31	−1762	−·32	−1359	−·34	−440	−·36	−189	−·56	−2263
−·60	−4250	−·60	−4255	−·73	−4265	−·81	−4260	−·93	−4250
Hysteresis } = 187 $\frac{\text{ergs}}{\text{cm.}^3}$ per cycle		216 $\frac{\text{ergs}}{\text{cm.}^3}$		318 $\frac{\text{ergs}}{\text{cm.}^3}$		357 $\frac{\text{ergs}}{\text{cm.}^3}$		398 $\frac{\text{ergs}}{\text{cm.}^3}$	

467°.		360°·5.		251°·5.		136°.		18°.	
H	B	H	B	H	B	H	B	H	B
·88	4265	·93	4265	1·05	4265	1·20	4265	1·318	4265
·38	3760	·40	3743	·42	3732	·43	3700	·426	3725
0	3010	0	2930	0	3000	0	3063	0	3115
−·38	−733	−·40	924	−·42	1378	−·43	1630	−·426	1910
−·88	−4265	−·93	−4265	−1·05	−4265	−1·20	−4265	−1·318	−4265
Hysteresis } = 396 $\frac{\text{ergs}}{\text{cm.}^3}$ per cycle		495 $\frac{\text{ergs}}{\text{cm.}^3}$		590 $\frac{\text{ergs}}{\text{cm.}^3}$		656 $\frac{\text{ergs}}{\text{cm.}^3}$		744 $\frac{\text{ergs}}{\text{cm.}^3}$	

(b) After annealing at 1150° C.

Limits of Cycle : $B = \pm 4550$. (Curves 5 and 16.)

764°·5.		748°.		730°.		695°.	
H	B	H	B	H	B	H	B
·640	4543	·403	4540	·443	4547	·516	4540
·210	3690	·174	3662	·184	3722	·197	3712
0	1660	0	2002	0	2272	0	2564
−·210	−2665	−·174	−1921	−·184	−1468	−·197	−897
−·640	−4543	−·403	−4540	−·443	−4547	−·516	−4540
Hysteresis } = 81 $\frac{\text{ergs}}{\text{cm.}^3}$ per cycle		109 $\frac{\text{ergs}}{\text{cm.}^3}$		128 $\frac{\text{ergs}}{\text{cm.}^3}$		178 $\frac{\text{ergs}}{\text{cm.}^3}$	

Table VIII. (continued).

634°.		554°.		457°.		352°.	
H	B	H	B	H	B	H	B
730	4540	935	4550	898	4553	881	4542
336	3928	373	3770	371	3840	361	4002
0	2720	0	2520	0	2827	0	3060
-336	-1886	-373	-1074	-371	-488	-361	239
-730	-4540	-935	-4550	-898	-4553	-881	-4542
Hysteresis } = 264 $\frac{\text{ergs}}{\text{cm.}^3}$ per cycle		335 $\frac{\text{ergs}}{\text{cm.}^3}$		379 $\frac{\text{ergs}}{\text{cm.}^3}$		475 $\frac{\text{ergs}}{\text{cm.}^3}$	

249°.		137°·5.		18°.	
H	B	H	B	H	B
1001	4550	1032	4545	1114	4545
382	3952	388	3980	398	3972
0	3043	0	3188	0	3184
-382	712	-388	1163	-398	1386
-1001	-4550	-1032	-4545	-1114	-4545
Hysteresis } = 508 $\frac{\text{ergs}}{\text{cm.}^3}$ per cycle		555 $\frac{\text{ergs}}{\text{cm.}^3}$		613 $\frac{\text{ergs}}{\text{cm.}^3}$ per cycle.	

TABLE IX.

Relation of Maximum Permeability and of the Field in which it occurs to Temperature.

(The values are estimated from the curves, and are only approximate.)

SPECIMEN A.

(Curves 9 and 10.)

Before annealing.

After annealing at 1050°.

Temperature.	Maximum Permeability.	Corresponding Magnetic Field.	Temperature.	Maximum Permeability.	Corresponding Magnetic Field.
°C.	μ	H	°C.	μ	H
20	1355	370	722	8350	44
220	1370	325	636	5950	92
365	1360	280	506	5450	105
532	1880	220	324	4860	146
660	2650	150	188	4370	170
			215	4050	202

Table IX. (*continued*).

SPECIMEN B.

(Curves 11 and 12.)

After annealing at 840°.

After annealing at 1150°.

Temperature.	Maximum Permeability.	Corresponding Field.
°C.	μ	H
764	7800	50
746	7090	58
710	6070	72
648.5	5610	93
567	4900	1.09
467	5190	1.18
360.5	5280	1.13
251.5	4940	1.38
136	4250	1.69
24	3980	1.84

Temperature.	Maximum Permeability.	Corresponding Field.
°C.	μ	H
764.5	14600	20
748	13400	25
730	11700	33
695	9430	43
634	6040	58
554	4240	99
457	5120	94
352	5050	96
249	4820	1.19
137.5	5020	1.36
24	4680	1.48

TABLE X.

Variation of Magnetic Hysteresis with Temperature.

(See Curves 14, 15, and 16.)

(a) Cycle between constant Limits of Magnetizing Force.

SPECIMEN A.

Limits: $H = \pm 6.10$.
After annealing at 1050°.

Temperature.	Hysteresis in ergs per c.cm. per cycle.
°C.	
765	500
708	920
625	1345
506	2060
325	3090
175	4110
21.5	4570

SPECIMEN B.

Limits: $H = \pm 6.83$.
Annealed at 840°.

Annealed at 1150°.

Temperature.	Hysteresis in ergs per c.cm. per cycle.
°C.	
764	426
746	573
710	1010
648.5	1650
567	2180
467	2550
360.5	3195
251.5	3700
136	4360
24	4700

Temperature.	Hysteresis in ergs per c.cm. per cycle.
°C.	
764.5	120
748	328
730	426
695	797
634	1010
554	1345
457	2025
352	2565
249	3130
137.5	3500
24	3660

Table X. (*continued*).

(b) Cycle between constant Limits of Magnetic Induction.

SPECIMEN B.

Annealed at 840°. Limits:
B = ± 4260 .*Annealed at 1150°.* Limits:
B = ± 4550 .

Temperature.	Hysteresis in ergs per c.cm. per cycle.
°C.	
764	187
746	216
710	318
648.5	357
567	398
467	396
360.5	495
251.5	590
136	656
18	744

Temperature.	Hysteresis in ergs per c.cm. per cycle.
°C.	
764.5	81
748	109
730	128
695	178
634	264
554	335
457	379
352	475
249	508
137.5	555
18	613

(c) SPECIMEN B.

Comparison of Hysteresis
after annealing at 840° and
1150°.(Reduced to a uniform Induction
Limit of **B** = ± 4550 .)

Temperature.	Annealed at 840°.	Annealed at 1150°.
°C.		
20	822	612
200	686	530
400	504	437
600	439	302
700	358	176
750	236	106

(d) Hysteresis of SPECIMEN B

at 20° and Induction Limits ± 4550 ,
showing various stages of annealing.

	Hysteresis in ergs per c.cm. per cycle.
Initial condition	1480
After 2 hrs. at 840°	810
" 5 hrs. "	780
" 6 hrs. " and very gradual cooling.	822
After heating to 1150° and very gradual cooling.	612

TABLE XI.

IRON SPECIMEN A.

Electrical Resistance at Different Temperatures.

(a) Before and during annealing. (Curve 17, I.)

Temperature. ° C.	Specific Resistance in C.G.S. units.	Temperature. ° C.	Specific Resistance in C.G.S. units.
18	14610	530.5	61550
127	22150	535.5	61550
189	26300	643	77400
210.5	27680	674	83400
213	27880	752	96700
354.5	41300	763	99700
373	43450		

(b) After annealing at 1050°. (Curve 17, II.)

[Critical Temp.: 795°.]

Temperature. ° C.	Specific Resistance in C.G.S. units.	Temperature. ° C.	Specific Resistance in C.G.S. units.
1050	115800 ?	330.5	37000
1026.5	115800 ?	201.5	25130
950.5	114200	171.5	23480
936	113100	18	13820
870	109400	16	13580
881	110000		
808.5	101100	After reheating to 920°. (Also plotted for Curve II.)	
805	101100		
786	97200	885.5	110600
761	92600	916	112800
724.5	86600	736	89000
706	84400	686.5	81300
653	75600	681	79750
620	71350	632.5	72400
504.5	55550	630	72900
506	55700	20	13940
327	36760	16.8	13510
321	36280		

	Before annealing.	After annealing. at 1050°.
Temperature-Coefficient	{ at 0° 0.044 { maximum value at about 780° 0.125 { at 1000°	0.045 0.144 0.0236
Specific Resistance.....	{ at 0° 13600 { at 1000°	12500 115600 { C.G.S. Units.

TABLE XII.

IRON SPECIMEN B.

Electrical Resistance at Different Temperatures.

(a) After annealing at 840°. (Curve 18, I.)

[Critical Temp. = 782°.]

Temp. ° C.	Sp. Resist. in C.G.S. Units.	Temp. ° C.	Sp. Resist. in C.G.S. Units.	Temp. ° C.	Sp. Resist. in C.G.S. Units.
826	108300	679	83750	358.5	39800
800	105100	649	78900	314.5	34920
786.5	103000	605.5	71950	252.5	28660
772	100000	555	64400	190.5	23280
764	98600	561	65400	128.5	18440
751	96200	517	59250	101	16410
739.5	94200	466.5	52700	52	13210
714.5	89300	413.5	45900	18	11110

(b) After annealing at 1150°. (Curve 18, II.)

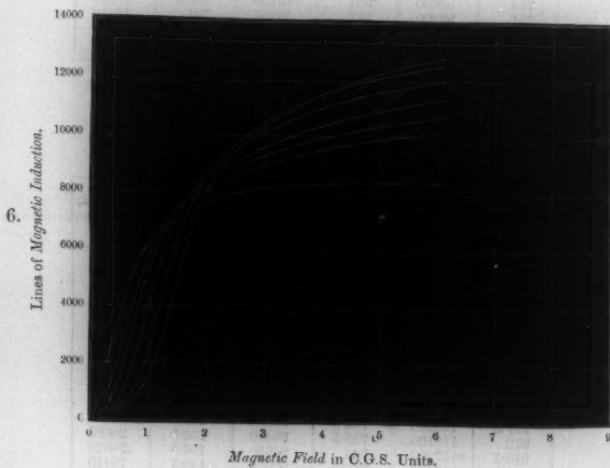
[Critical Temp. = 778°.]

Temp. ° C.	Sp. Resist. in C.G.S. Units.	Temp. ° C.	Sp. Resist. in C.G.S. Units.	Temp. ° C.	Sp. Resist. in C.G.S. Units.
1150	120600 ?	871	113400	630.5	76400
1142	121000 ?	858	112500	637	77400
1115.5	120850	844	111400	594	70850
1095.5	120600	824.5	109600	553.5	64850
1071.5	120350	813.5	108400	554	64900
1058.5	120050	792.5	106600	503.5	57850
1036	119650	787	104600	454.5	51770
1014.5	119100	781	103400	406	45720
989.5	118550	770	100900	386	43400
983	118350	765.5	100000	361	40580
977.5	118200	764.5	99850	295.5	34020
972	118100	745.5	96150	252.5	29080
946.5	117200	744	95650	196.5	24020
926.5	116600	725.5	92050	140.5	19410
916.5	116200	697.5	87200	84	15390
891.5	114900	658	80850	18	11140

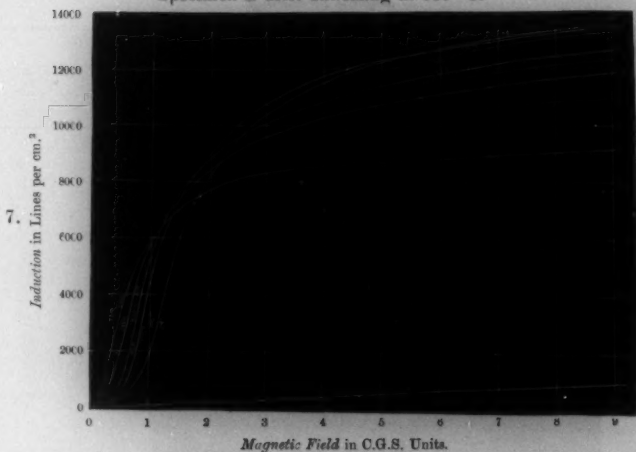
		After annealing at 840°.	After annealing at 1150°.	
Temperature-Coefficient	{ at 0°.....	·0057	·0057	
	{ maximum value at about 765°	·0189	·0204	
	{ at 1000°		·00244	
Specific Resistance	{ at 0°.....	10050	10050	} C.G.S. Units.
	{ at 1000°		118500	

B-H Curves at various Temperatures.

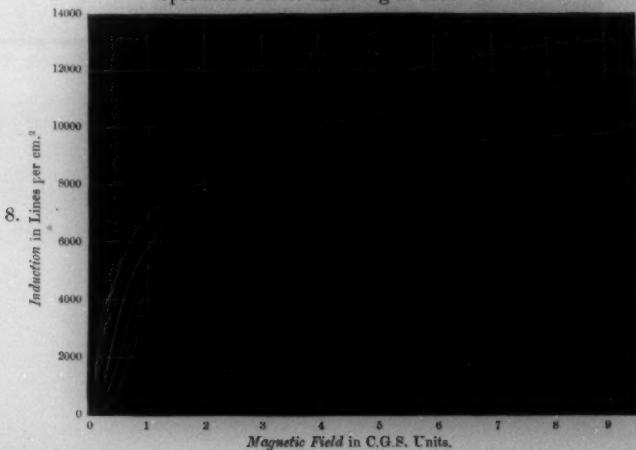
Specimen A after annealing at 1050° C.

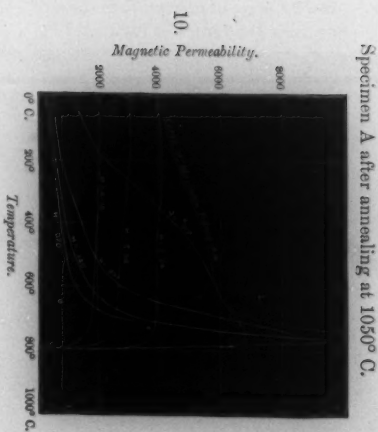
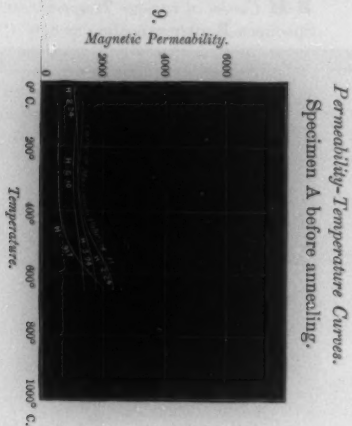


B-H Curves at various Temperatures.
Specimen B after annealing at 840° C.



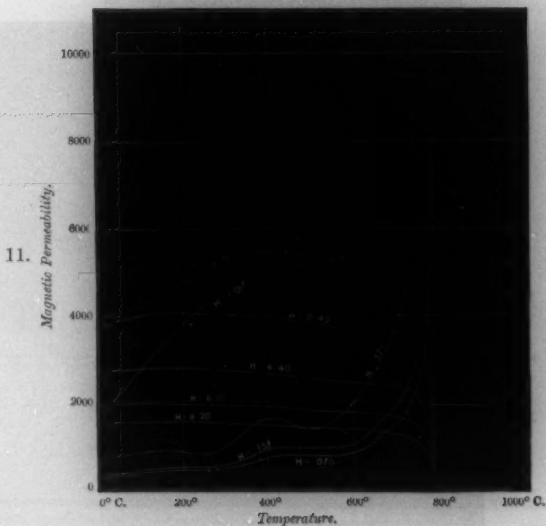
B-H Curves at various Temperatures.
Specimen B after annealing at 1150° C.



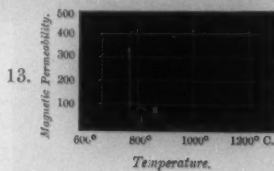


Permeability-Temperature Curves.

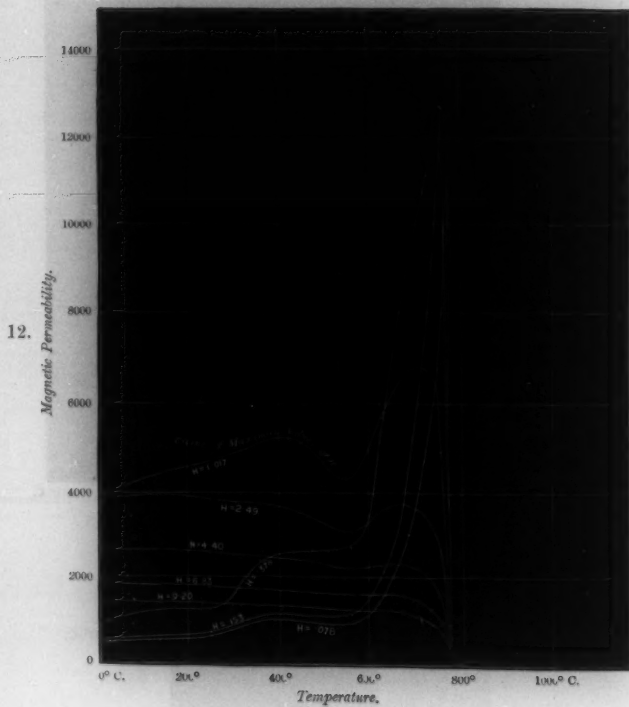
Specimen B after annealing at 840° C.

Magnetic Permeability of Iron specimen B
at Temperatures above its Critical Point.

Curve I. After annealing at 840° C. Curve II. After annealing at 1150° C.

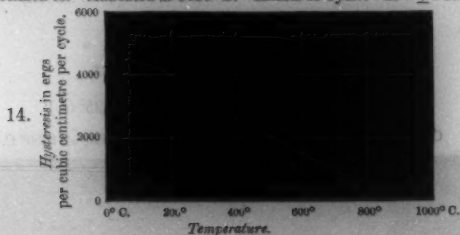


Specimen B after annealing at 1150° C.



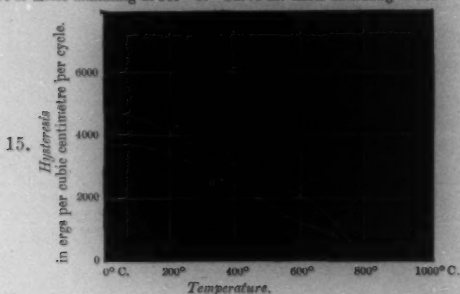
Hysteresis-Temperature Curves.

Specimen A. Annealed at 1050° C. Limits of Cycle. $H = \pm 6.10$.



Specimen B. Limits of Cycle. $H = \pm 6.83$ C.G.S. Units:

Curve I. After annealing at 840° C. Curve II. After annealing at 1150° C.



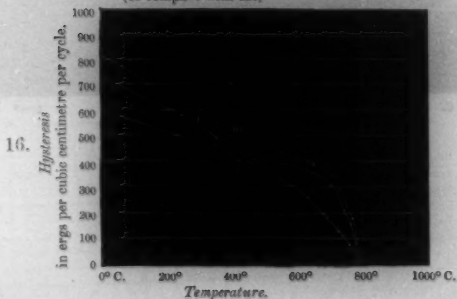
Specimen B.

Limits of Hysteresis Loop.

Curve I. After annealing at 840° C. $B = \pm 4260$.

" II. After annealing at 1150° C. $B = \pm 4550$.

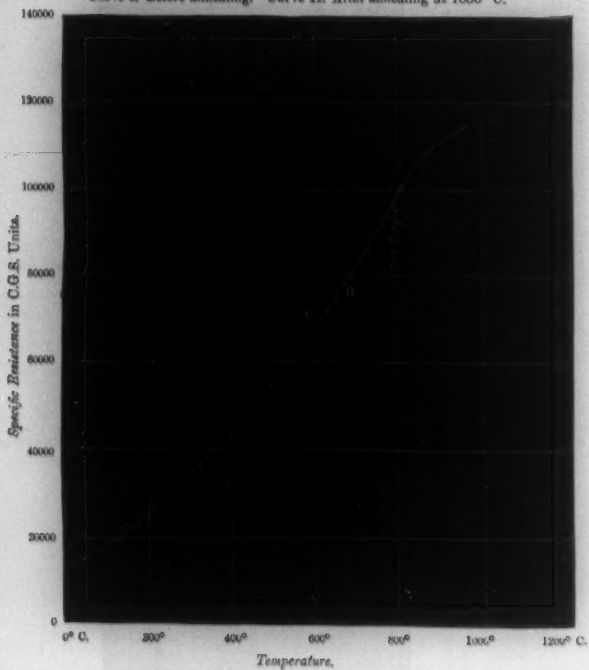
" III. Calculated from I. for the Limits $B = \pm 4550$.
(to compare with II.)



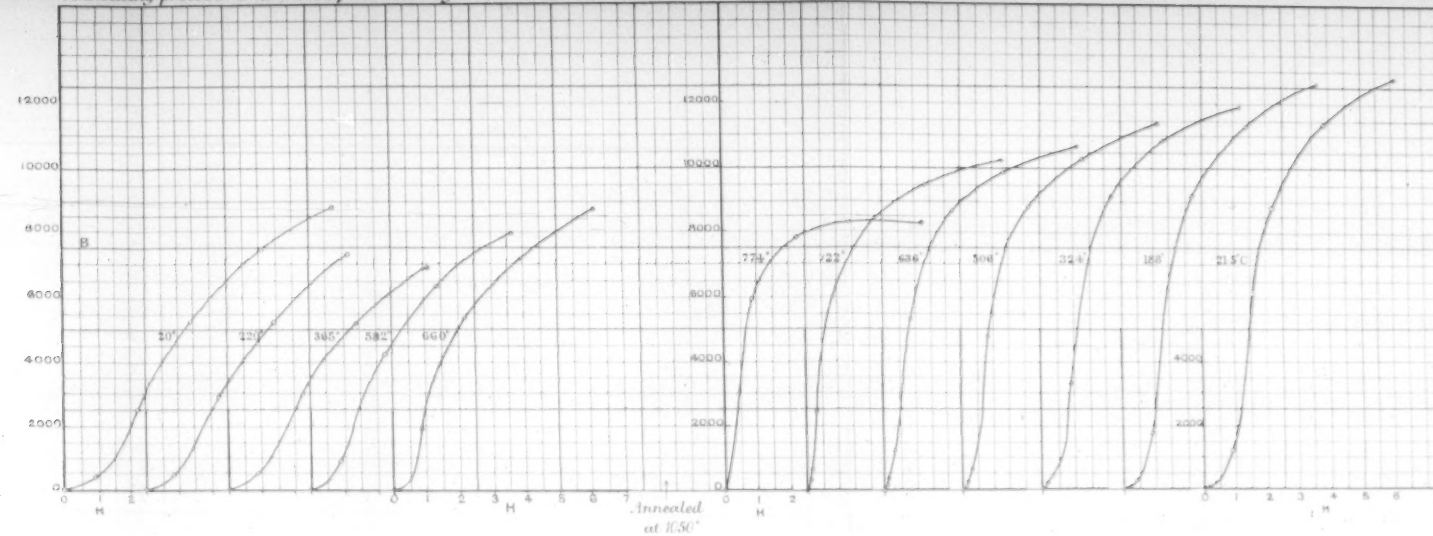
Resistance-Temperature Curves.

17. Iron specimen A. (Crit. temp. $795^{\circ}\text{C}.$)

Curve I. Before annealing. Curve II. After annealing at $1050^{\circ}\text{C}.$



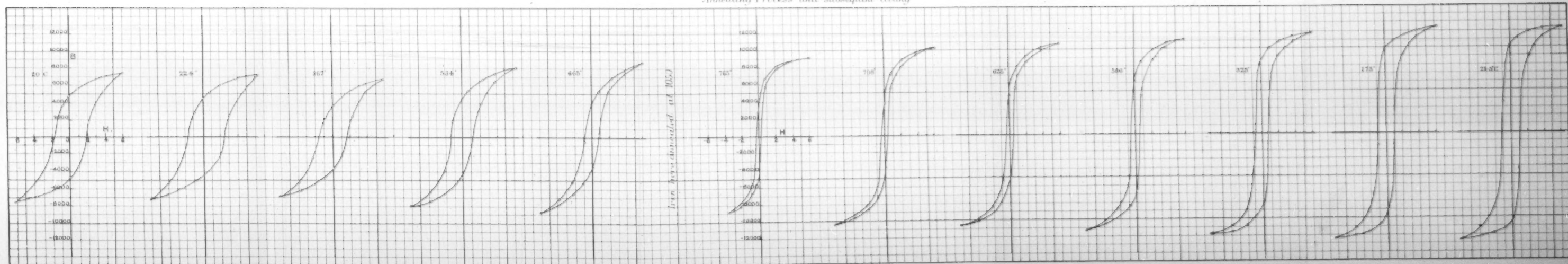
Curves No 1. B-H Curves by the method of reversals



MAGNETIC OBSERVATIONS WITH IRON SPECIMEN A.

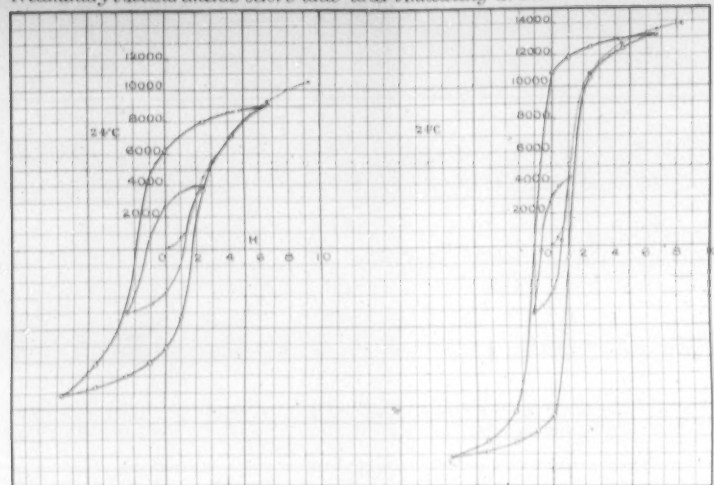
Crit. Temp. 795°

Curves No 2. Hysteresis Loops
Annealing Process and subsequent cooling

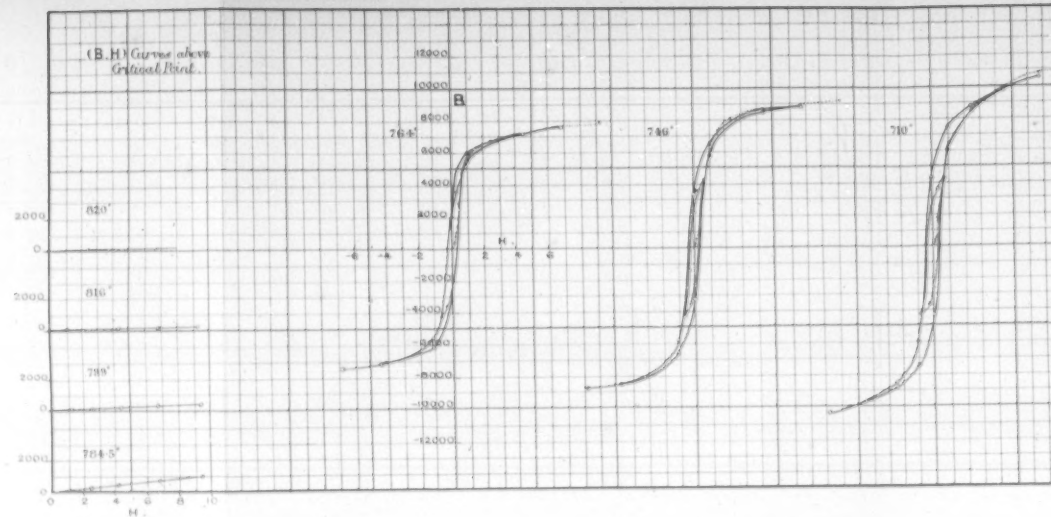


Curves N° 3.

Preliminary Measurements before and after Annealing at 840°.



Series taken after annealing at 840°.



Series of measurements taken after annealing at 1150°C.



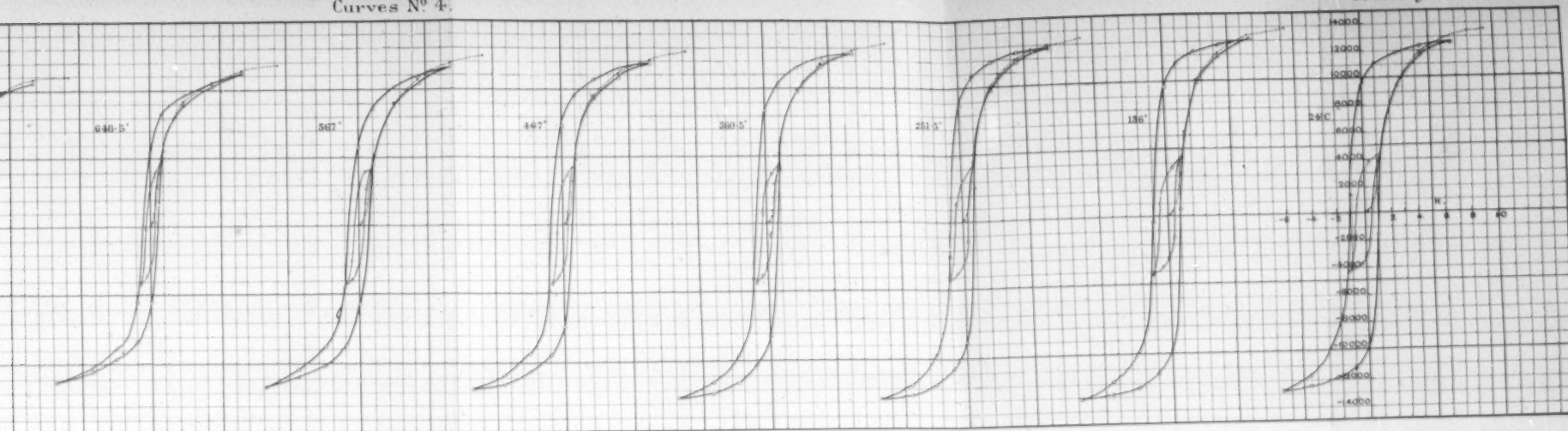
Explanation

The larger hysteresis loops are all taken exactly within the limits $H = \pm 6.83$ CGS Units.

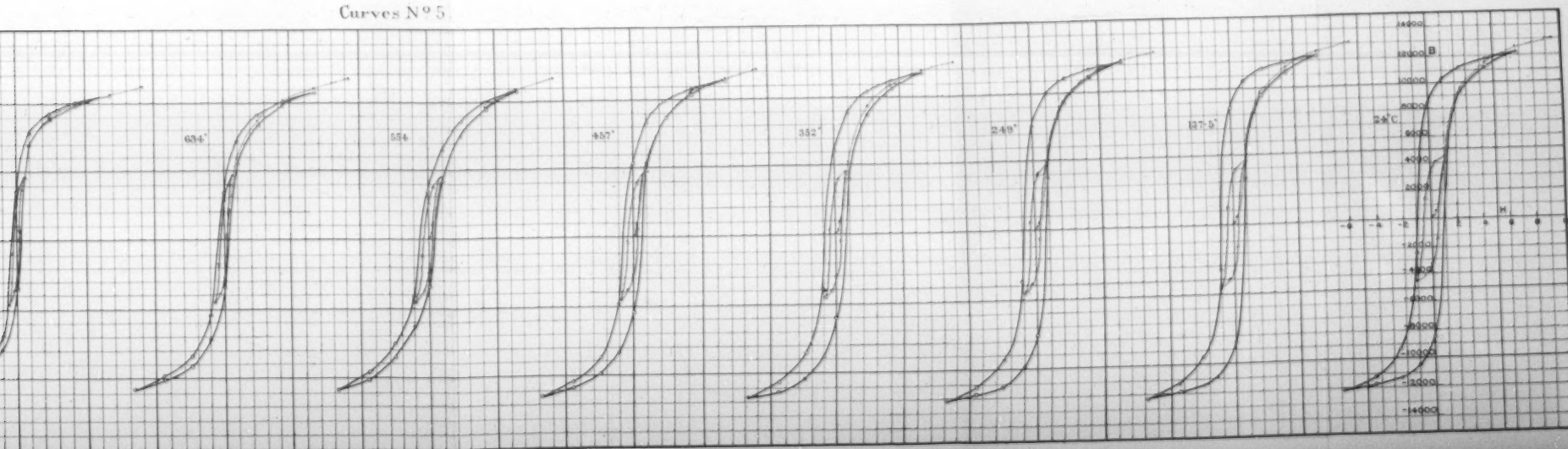
The smaller loops are within the limits of Induction $B = \pm 4260$ lines for the 1st series and $B = \pm 4550$ lines for the 2nd series

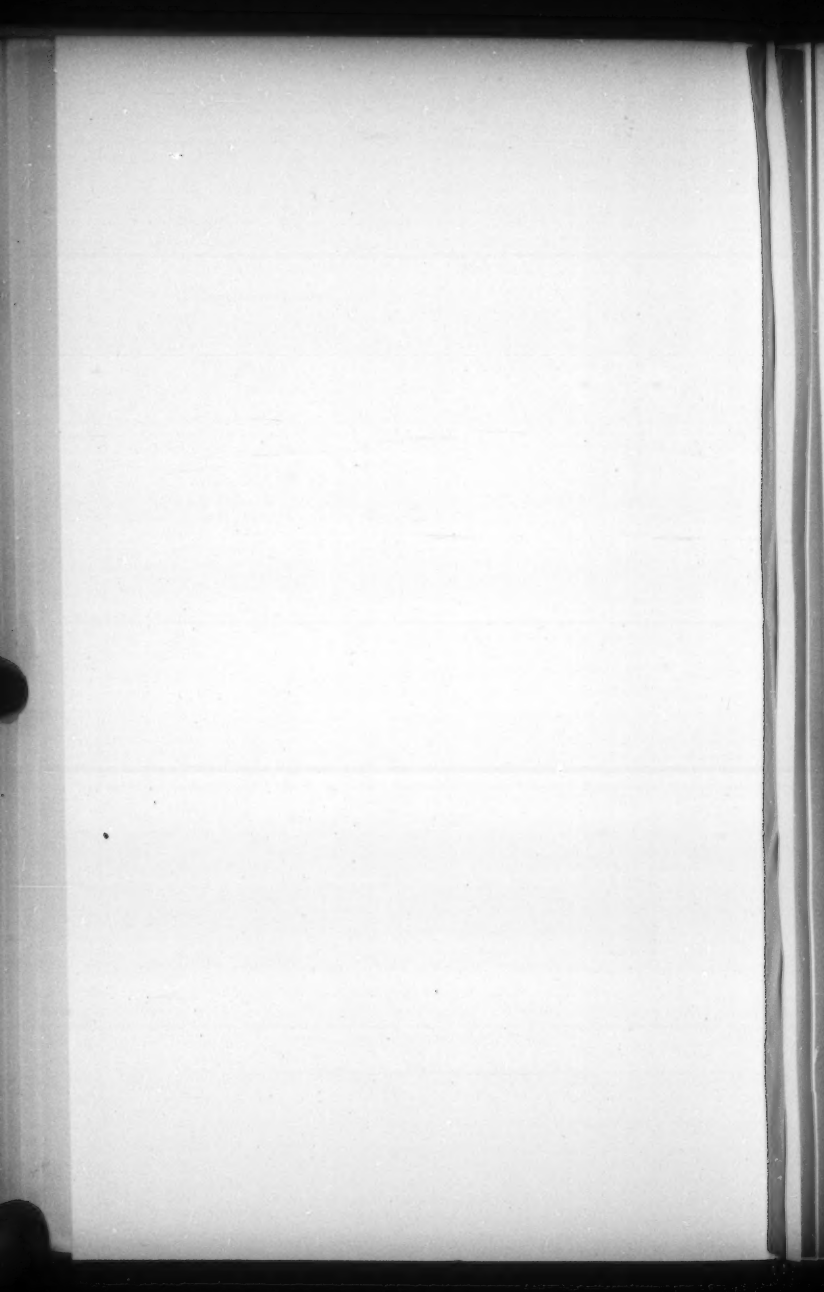
The dotted curves are the B, H curves taken by the Method of Reversals.

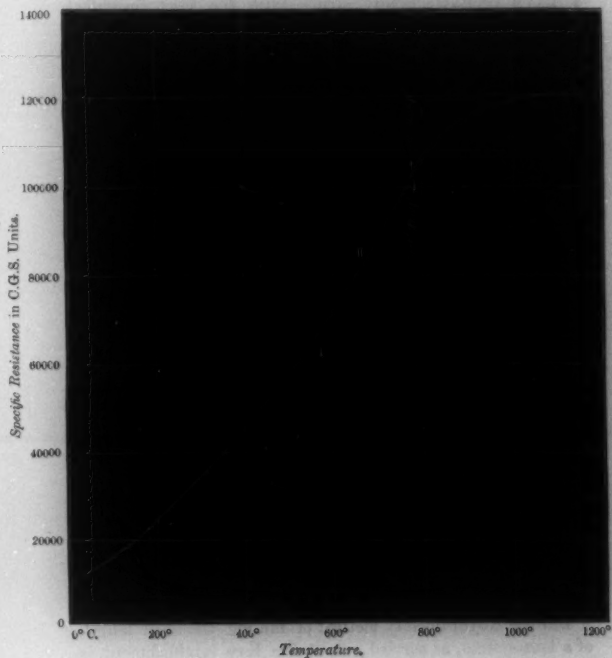
Curves N° 4.



Curves N° 5.





*Resistance-Temperature Curves.*18. Iron specimen B. (Crit. temp. $780^{\circ}\text{C}.$)Curve I. After annealing at $840^{\circ}\text{C}.$ Curve II. After annealing at $1150^{\circ}\text{C}.$ 

XV. *A new Definition of Focal Length, and an Instrument for determining it.* By T. H. BLAKESLEY, M.A.*

I HAVE been frequently struck with the very imperfect way in which the facts of focussing with lenses and lens-combinations are treated in works which purport to deal with such matters in a practical way. In many cases such works are by authors who in other respects give information and rules which are unimpeachable and the result of painstaking study, so that the defect in the particular to which I have alluded must be due either to the inherent inconvenience in the usual formulæ, so invariably and unnecessarily involving the inverse of distances of Object and Image from theoretically fixed but practically undetermined points, and the Focal Length; or to the imperfect way of quoting the rules in the theoretical text-books. If the latter be even partially the cause, and can be remedied, it will be quite unnecessary to inquire if the former is also operative. In addition to the troublesome formula of inverse distances, a difficulty is unnecessarily introduced by defining the focal length as the distance between two points, generally the principal focus and a focal point, or the lens itself if thin, instead of being an abstract length of straight line characteristic of the lens or lens-combination.

To illustrate my meaning, I may point out that the Coefficient of Self-Induction of a coiled conductor is expressed as a length which is fixed so long as the coil remains unaltered in geometrical conditions; but no one would ask between which points is this length to be measured. So the focal length of a lens-combination is simply an abstract length, and not necessarily the distance between two particular points.

But the Royal Society method of finding the focal length of a lens-system, as carried out at the Kew establishment, is based on the definition that the focal length is the distance between a focal centre and the principal focus; and these points having been separately found by processes not free from objection, the distance between them is indirectly measured

* Read June 11, 1897.

(*vide* Major Darwin's paper, 'Proceedings of the Royal Society,' 1892).

Various methods have been described in an admirably arranged bibliographical review of the subject by Prof. S. P. Thompson. In this paper the author also describes a method of his own with an apparatus for carrying it out, but that method depends on the same idea that the focal length is a distance between two points. The apparatus fixes those points and measures the distance between them. And the same may be said of all but one of the methods proposed by others, described in that paper.

I have to propose and explain a far more general definition of the focal length of any system of Coaxial Lenses, which leads naturally to more general methods of determining that length than any of those alluded to.

Now any function of the two distances of object and image from their appropriate focal centres and the focal length may be employed in conjunction with the usual formula to eliminate either of those distances; and if that function is at the same time an easy one to determine experimentally, and if the elimination leads to a simple formula for working purposes, a considerable gain in convenience and exactitude may be the result. Such a function is found in the magnification, which I define as the linear relation of Image to Object and which I symbolize under the letter m , and take as positive if the image is erect and is not inverted with regard to the object. Thus in the case of the object and image being at equal distances from, and on opposite sides of a thin double-convex lens of equal radii of curvature, I should describe the magnification as -1 . In this notation the negative sign will indicate reversal, the numeral the arithmetic magnification.

x
A

Let A be the position of one of a pair of conjugate foci on the axis of some coaxial lens-system, and let v be its distance from any fixed point on the axis measured positively in the direction the light is passing. Let m be the magnification, or the linear ratio of an image formed at this point to the object producing it.

Then it is easily proved that $\frac{dv}{dm} = f$ a constant; and this constant is in fact the focal length.

Hence $v - v_0 = f \cdot m$,
where v_0 is the value of v when the magnification is zero, *i. e.* at the principal focus.

It follows from the constancy of $\frac{dv}{dm}$ that dv and dm need not be infinitesimals; so that if any two positions of A be found for a clear image of an object moved in any way along the axis, the distance between these positions bears to the focal length a ratio which is simply the change in the magnification.

Similarly, if u is the position of the other focus but m retains its first signification,

$$\frac{du}{d\left(\frac{1}{m}\right)} \text{ is constant } = f',$$

and

$$u - u_0 = f' \cdot \frac{1}{m},$$

where u_0 is the value of u when m is infinite.

Incidentally,

$$\overline{u - u_0} \overline{v - v_0} = f \cdot f',$$

$$\frac{v - v_0}{u - u_0} = \frac{f}{f'} m^2.$$

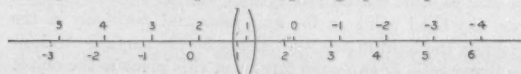
If the end media are alike, $f = f'$, which is the important case

$$\overline{u - u_0} \overline{v - v_0} = f^2,$$

$$\frac{v - v_0}{u - u_0} = m^2.$$

The former of these equations is a common one, but the latter is not so well recognized.

m^2 may be called the areal magnification. It clearly has an importance in the question of photographic exposure.



In the sketch let the straight line represent the axis of u

lens-system, and let the points marked off at equal distances (f) above the line be numbered in such a way that the number represents the relative size of the Object to the Image, when the Object is placed in any one of the points, *i. e.* for the focus of light before incidence, and let the points numbered below the line, and also at equal distances (f_1), bear a corresponding series of numbers representing the relative size of Image and Object when the Image is at any one of the points.

Then any point represented by p on one of these scales is conjugate to the point represented by $\frac{1}{p}$ on the other. The two principal foci must be marked 0 and the two focal centres must be marked 1.

If the media at the ends are the same, the common distance of the integral points will be the same in the two scales. This condition, as being the common one, I shall assume to exist unless the reverse is especially stated.

The common distance between any two consecutive integral points is the focal length; and this is one important characteristic of the system. Another characteristic is the aberration of the integral points of one scale from those of the other. A third is the order in which the numbers of the scales run with reference to the direction of the light.

It appears to me that two systems to deserve being called equivalent one to the other should have coincidence in all these three respects. If the top series has reference to light before encounter with the system, and light is supposed to come from right to left, then for a convex arrangement the upper series of numbers increases in the same direction, *i. e.* from right to left, and the lower series in the reverse direction; but for a concave arrangement these rules are reversed.

It will be found that one of these diagrams will not be misinterpreted by turning it upside down.

Hence in any combination it is necessary to find:—

- (1) The focal length.
- (2) Some point of known numerical denomination on one scale.
- (3) Ditto for the other scale.

With these three determinations all the facts connected

with image-formation as regards position and size may be determined for any position of the object, and *vice versâ*.

The optical bank with which the focal length is determined contains various sliding-pieces in a mahogany groove. Two of them carry pieces for the support of the cells containing the lens-combination, and two carry frames in which can be placed photographs on lantern-plate glasses of evenly-divided scales.

These photographs, being printed off the same negative, are exactly alike, and in use are turned with the film-side inwards towards the lenses.

If these photographs are made to occupy the positions of object and image, a comparison of the divisions actually on the plate examined with those of the image of the other scale received upon it will readily give the magnification.

Groups of exactly ten divisions, at ten divisions' interval, are also indicated by dark bars for the convenience of reading. An anatomical magnifier standing behind the scale serves to adjust to coincidence the planes of the actual and the image-scale with great accuracy; and it is also obvious that for any one position of the sliding-pieces the magnifier may be applied at both ends successively, each observation being thus checked by another.

To measure the movement of any sliding-piece a brass ruler is screwed to it at right angles to the motion; a slip of telegraph exchange-paper is fixed by drawing-pins to cork pieces at either end of the bank and the rulers pass immediately over this slip, so that lines can be drawn on the paper by a drawing-pen or pencil at any position, and the range of motion accurately measured.

To employ the instrument for convex lens-combinations, such as photographic lenses and projectors, several methods can be adopted. The one scale and the combination being fixed, search may be made for the image, which is adjusted accurately to the other scale, and a mark made on the paper at the edge at the corresponding ruler, and the magnification determined and checked, if necessary, at the other end.

The first scale may be then moved any distance, and the new position of the image found and adjusted as before by moving the second scale. A new mark is made, and the new magnification found and checked.

If m and n are the two magnifications and l the distance between the marks,

$$\frac{l}{m-n} = f.$$

Or the two scales may remain fixed and the combination may be moved until accurate adjustment is obtained in the two possible positions; the distance between those two positions being obtained by marks made on the paper along the edge of the ruler attached to the combination.

In this case the magnification which is the inverse in one of the positions to what it is in the other may be ascertained by *four* readings. Then obviously $n = \frac{1}{m}$ and $f = \frac{l}{m - \frac{1}{m}}$
 $= \frac{l \cdot m}{m + 1 \cdot m - 1}$, a very convenient form.

The slips of paper employed may be rolled up and stored, so that reference can be made to them should occasion require it.

The operation of finding the focal length of a concave lens or combination is less important perhaps, but the following general proposition may be of interest, and within limits may be made applicable to any case.

Suppose it possible to change the position of a portion of a combination, so as to recover as in the last operation the same pair of conjugate foci of that part of the combination. Then clearly the focussing of the whole combination will be restored to the same conjugate foci as before, though the magnification will be changed.

Let m be the factor of the total magnification due to the moving portion, and let M be the total magnification: in the first position M may be read off.

In the second position m changes to $\frac{1}{m}$, and M changes to N , say, which may also be determined.

If K be the magnification-factor due to the stationary portion of the combination,

$$Km = M, \quad \frac{K}{m} = N;$$

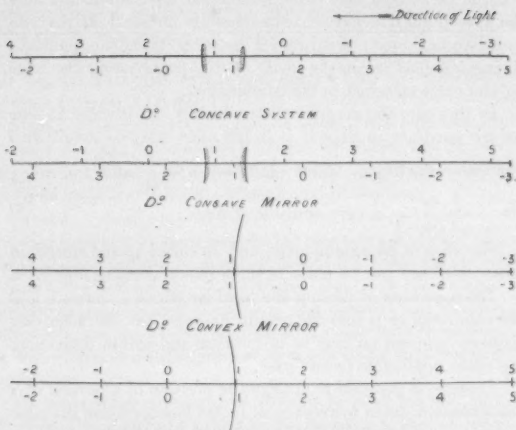
from which m and K can both be determined, viz.

$$m^2 = \frac{M}{N}, \quad K^2 = MN.$$

Hence the focal length of the moving part is

$$\frac{lm}{m^2-1} \quad \text{or} \quad \frac{l\sqrt{MN}}{M-N}.$$

TYPE OF A CONVEX SYSTEM



Note.—In all these diagrams light is supposed to come in from the right hand; the upper scale applies to the focus for the light before it encounters the system, the under scale to the focus for the light which has emerged from the system. The diagrams hold good if the sheet is turned upside down. Any erecting system of lenses applied to a given combination changes the character of the latter from concave to convex and vice versa.

I have dealt with the finding of focal lengths before showing how any particular focus may be found, with a view of accentuating how completely independent the two matters are; but it is clear that if any position of a pair of conjugate foci is taken and the magnification is measured, one has only to measure the distance of the scales from some fixed planes of the lens-system (say the edges of the brass cells) in order to allocate not merely the points for the particular magnification, but for all magnifications both above and below the line.

Such measurements could easily be made by a sliding rule, accurately to the tenth part of a millimetre; but as the end of the rule would of necessity abut against the photographic film, repeated measurements would be prejudicial to the latter. However, if the negative from which the scales are made is preserved, such scales can be easily and cheaply reproduced.

A very accurate way of fixing the principal foci is to place the combination at rest vertically on a plane mirror. Immediately above it hold a sharply-cut point of stiff white paper in a universal holder and adjust its position in the vertical line until its image formed by light passed twice through the lens-system coincides with itself. This adjustment may be most accurately made by a short focal-length hand-lens.

DISCUSSION.

Dr. S. P. THOMPSON said the paper was the most important contribution to geometrical optics that had appeared for many years. The introduction of the magnification function was a most useful device, leading to exceedingly simple results. The important thing to measure was not so much the focal length as the reciprocal of that quantity.

Dr. CHREE pointed out that in employing the Kew method it was not of great importance that the axis round which the lens-system is moved should pass through the nodal point. A small error in this adjustment imported none into the final result. He wished to refer to Hondaille's book on Lens Testing. A magnification method is given in that work.

Mr. BLAKESLEY, in replying, called attention to the use of his strip diagrams of magnification, for enlarging purposes in photography. When the magnification diagram was known, the focussing-cloth might be dispensed with.

XVI. *Note on Mr. Blakesley's paper, "A new Definition of Focal Length, &c." By Prof. A. GRAY*.*

I HAVE read Mr. Blakesley's paper on "A new Definition of Focal Length, &c.," with much interest. It is true, as Mr. Blakesley states, that the treatment of lenses and combinations of lenses in ordinary text-books is frequently faulty, and that much advantage would be gained by a consideration of the question from other points of view than that generally adopted in treatises on geometrical optics. Something of this kind has been achieved by the admirable series of papers on points in physical optics which have appeared from time to time, during the last fifteen years, from the pen of Lord Rayleigh, and the lucid and elementary discussion of the propagation of waves through lenses, and their reflexion from mirrors, published by Dr. S. P. Thompson, in Oct. 1889.

I think further clearness in the presentment of the action of lens systems in important cases would be obtained if more use were made of the notion of the apparent distance of an object seen through a system of lenses. This idea, which is at least as old as Smith's 'Compleat System of Opticks' (Camb. 1738), seems to have been strangely neglected until attention was called to it again by Lord Rayleigh in the *Phil. Mag.* for June 1886. Yet the formula for the apparent distance of an object situated on or near the common axis of a system of lenses, and viewed along that line, yields at once from its mere form many most valuable theorems: for example, that the interchange of position of the image and object, without change of position of the lens-system, does not affect the magnification; that the magnification of an object, seen through such a lens-system, is equal to the ratio of the real distance of the object from the object-glass to the apparent distance of the object from the eye, or the ratio of the breadth of the pencil at the object-glass to the breadth of the pencil at the eye.

In this mode of discussion attention is, as in Mr. Blakesley's

* Read June 11, 1897.

paper, focussed on the magnification produced by the lens-system. Hence the method of determining the focal length of a combination by comparisons of the magnifications for (1) two positions of the object at a measured distance apart, or for (2) two positions of the image at a measured distance apart. If l denote the distance in either case, m, n the magnifications, the focal length is $l \frac{mn}{m-n}$ in (1), and $\frac{l}{m-n}$ in the other; or if, as Mr. Blakesley suggests, the two possible positions of the combination for fixed positions of the object and image be taken, and l be the distance through which the combination is moved, the focal length is $\frac{lm}{m^2-1}$.

But this method of determining focal lengths has already been used for several years by Abbe for his optical combinations, and apparatus for the purpose is to be found described by S. Czapski (*Zeitschrift für Instrumentenkunde*, vol. xii. 1892). A discussion of the method and some account of the apparatus will be found in the very valuable treatise on "Optik," by Czapski, lately published as part of the *Handbuch der Physik*, just completed under the editorship of Dr. A. Winkelmann (see vol. ii. pp. 289, 290, *et seq.*).

Though the experimental method of Mr. Blakesley's paper has thus been in the main anticipated, his paper is valuable as a fresh and instructive view of the subject, and as directing attention to methods of focometry as yet apparently not very generally known.

Bangor, June 17, 1897

XVII. *The Effect of a Spherical Conducting Shell on the Induction at a Point in the Dielectric outside due to an Alternating Current in a Circular Circuit in the Dielectric inside, the Axis of the Conductor passing through the Centre of the Shell.* By C. S. WHITEHEAD, M.A.*

(1) It is well known that if there be an alternating current in a primary circuit sounds will be heard in a telephone in a secondary circuit, even if the latter be at a considerable distance from the former. If the sounds are preconcerted, signals may be, and in fact have been, transmitted in this manner on land through distances of four or five miles.

It has been suggested that the same plan could be employed to communicate with lightships. The primary is laid on the sea-bottom round the area over which the lightship swings, the ends of the cable being brought to land; the secondary is coiled round the lightship. I have been informed by Mr. S. Evershed that successful experiments were carried out by these means last summer in Dover harbour, the depth of the sea being about 1000 centims. To be able to judge whether a similar experiment will succeed in a greater depth of water, we must calculate the induction through the secondary circuit; sea-water being a conductor the investigation is somewhat complex, the final result, however, comes out in a simple form.

(2) The two following cases are considered:—

CASE I.

A circular circuit carrying an alternating current is placed in the dielectric inside a spherical conducting shell, the axis of the circuit passing through the centre of the shell: to find the normal magnetic induction at any point in the dielectric outside the shell.

CASE II.

The circular circuit is placed on one side of an infinite conducting plate, the plane of the circuit being parallel to

* Read June 11, 1897.

the plate: to find the normal magnetic induction at any point on the other side of the plate.

In both cases the following result is arrived at:—

$$\frac{v_0}{u_0} = e^{-q\eta},$$

where v_0 is the maximum value of the normal magnetic induction at any point outside, u_0 the maximum value of the normal magnetic induction due to the current in the primary, supposing the conducting shell or plate absent, at the same point, η is the thickness of the shell or plate,

$$q = \left(\frac{2\pi\mu p}{\sigma} \right)^{\frac{1}{2}},$$

μ is the permeability of the conducting shell or plate, σ its specific resistance, $p = 2\pi$ times the frequency.

Let the frequency be 300, which makes $p = 1885$; σ for sea-water $= 2.10^{10}$ C.G.S. units, $\mu = 1$.

If $\eta = 2000$ centims., which is about the depth of the sea by the North Sand Head Lightship off the Goodwin Sands, then

$$\frac{v_0}{u_0} = .21, \text{ and } 79 \text{ per cent. is lost.}$$

$$\text{If } \eta = 1000, \quad p = 1885,$$

$$\frac{v_0}{u_0} = .46, \text{ and } 54 \text{ per cent. is lost.}$$

The above value of σ is from experiments on a sample of sea-water from the North Sea made by Mr. S. Evershed.

(3) CASE I.

Take O the centre of the shell for origin, and axis of the circuit for axis of z .

Let D be the centre of the circuit, C any point in it.

OA = external radius of the shell $= a$.

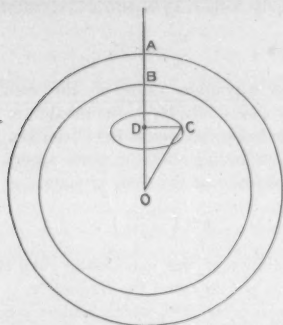
OB = internal radius of the shell $= b$.

OC $= c$, $\angle DOC = \alpha$.

$a - b = \eta$.

Let P, Q, R be the components of the electromotive intensity parallel to the axes of x , y , z respectively, σ the specific resistance of the shell, μ the permeability of the shell, μ' that of the dielectric, K' the specific inductive capacity of

the dielectric, r, θ, ϕ the polar coordinates of any point, θ being the colatitude, ϕ the longitude.



Let the current in the circuit be represented by the real part of γe^{ipt} , $p = 2\pi \times$ frequency, $i = (-1)^{\frac{1}{2}}$.

By symmetry the induced currents in the shell will flow in circles parallel to the plane of the circuit.

Let Φ be the electromotive intensity tangential to one of these circles;

$$\therefore P = -\sin \phi \cdot \Phi, \quad Q = \cos \phi \cdot \Phi, \quad R = 0. \quad (1)$$

Φ is by symmetry independent of ϕ .

Now P satisfies the equation

$$\nabla^2 P = \frac{4\pi\mu}{\sigma} \frac{dP}{dt} \text{ in the conductor,}$$

and

$$\nabla^2 P = \mu' K \frac{d^2 P}{dt^2} \text{ in the dielectric.}$$

Assume

$$P \text{ varies as } e^{ipt},$$

and let

$$\lambda^2 = -\frac{4\pi\mu ip}{\sigma},$$

$$\lambda'^2 = \mu' K' p^2.$$

$$\therefore \nabla^2 P + \lambda^2 P = 0; \quad (2)$$

in the dielectric we must write λ' for λ .

Q and R satisfy equations of the same form.

Assume

$$\left. \begin{aligned} P &= \psi_n(\lambda r) \left(y \frac{d}{dz} - z \frac{d}{dy} \right) \chi_n e^{i p t} \\ Q &= \psi_n(\lambda r) \left(z \frac{d}{dx} - x \frac{d}{dz} \right) \chi_n e^{i p t} \\ R &= \psi_n(\lambda r) \left(x \frac{d}{dy} - y \frac{d}{dx} \right) \chi_n e^{i p t} \end{aligned} \right\} \dots \dots (3)$$

These equations make $\frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz} = 0$, as should be the case.

χ_n is an arbitrary solid spherical harmonic of degree n .

Substituting in (2) we find

$$\frac{d^2 \psi_n}{dr^2} + \frac{2(n+1)}{r} \frac{d\psi_n}{dr} + \lambda^2 \psi_n = 0. \dots \dots (4)$$

Since $R=0$, the last equation of (3) gives

$$\frac{d\chi_n}{d\phi} = 0.$$

$\therefore$ from the first of (3),

$$\begin{aligned} P &= -\sin \phi \cdot \psi_n(\lambda r) \frac{d\chi_n}{d\theta} e^{i p t}, \\ \therefore \Phi &= \psi_n(\lambda r) \frac{d\chi_n}{d\theta} e^{i p t}. \dots \dots (5) \end{aligned}$$

Let ω = magnetic induction tangential to a meridian,

v = magnetic induction along a radius.

Then from the theorem that the line integral of the electromotive intensity round a circuit is equal to the rate of decrease of the magnetic induction through the circuit, we obtain

$$\begin{aligned} \frac{d\omega}{dt} &= -\frac{1}{r} \frac{d(\Phi r)}{dr}, \\ \frac{dv}{dt} &= \frac{1}{r \sin \theta} \frac{d(\Phi \sin \theta)}{d\theta}. \end{aligned}$$

But Φ varies as $e^{i p t}$,

$$\begin{aligned} \therefore \omega &= -\frac{1}{i p r} \frac{d(\Phi r)}{dr}, \\ v &= \frac{1}{i p r} \frac{1}{\sin \theta} \frac{d(\Phi \sin \theta)}{d\theta}. \end{aligned}$$

Since $\frac{d\chi_n}{d\phi} = 0$, χ_n satisfies

$$\frac{d}{dr} \left(r^2 \frac{d\chi_n}{dr} \right) + \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d\chi_n}{d\theta} \right) = 0,$$

$\therefore$ from (5)

$$\omega = -\frac{1}{i p r} \frac{d^2}{d\theta \cdot dr} \left\{ r \psi_n(\lambda r) \chi_n \right\} e^{i p t},$$

$$v = -\frac{1}{i p r} \psi_n(\lambda r) \frac{d}{dr} \left(r^2 \frac{d\chi_n}{dr} \right) e^{i p t}.$$

But

$$r \frac{d\chi_n}{dr} = n \chi_n,$$

$$\therefore \frac{d}{dr} \left(r^2 \frac{d\chi_n}{dr} \right) = n(n+1) \chi_n,$$

and

$$\begin{aligned} & \frac{d}{dr} \left\{ r \psi_n(\lambda r) \chi_n \right\} \\ &= \left\{ \frac{d \cdot r \psi_n(\lambda r)}{dr} + n \psi_n(\lambda r) \right\} \chi_n, \\ &= \phi_n(\lambda r) \chi_n, \end{aligned}$$

where

$$\phi_n(\lambda r) = \frac{d \cdot r \psi_n(\lambda r)}{dr} + n \psi_n(\lambda r), \quad (6)$$

$$\begin{aligned} \therefore \quad \omega &= -\frac{1}{i p r} \phi_n(\lambda r) \frac{d\chi_n}{d\theta} e^{i p t} \Bigg\} \\ v &= -\frac{n(n+1)}{i p r} \psi_n(\lambda r) \chi_n e^{i p t} \Bigg\} \dots \dots (7) \end{aligned}$$

Again, $\mu' K'$ is very small (it is of the order 10^{-21}), hence, unless p is very large, we may neglect λ'^2 . Hence in the dielectric

$$\nabla^2 P = 0,$$

we may therefore assume

$$P = \left(y \frac{d}{dz} - z \frac{d}{dy} \right) X_n e^{i p t},$$

where X_n is an arbitrary solid spherical harmonic, so that in the dielectric we must put $\psi_n(\lambda' r) = 1$, and, consequently, $\phi_n(\lambda' r) = n + 1$.

Therefore in the dielectric

$$\left. \begin{aligned} v &= -\frac{n(n+1)}{4\pi r} X_n e^{pt} \\ \omega &= -\frac{n+1}{4\pi r} \frac{dX_n}{d\theta} e^{pt} \end{aligned} \right\} \dots \dots (8)$$

The inducing system is in the dielectric inside the shell.

Hence in the dielectric inside the shell

$$\left. \begin{aligned} v &= -\frac{n(n+1)}{4\pi r} (X_n + X_{-n-1}) e^{pt} \\ \omega &= -\frac{1}{4\pi r} \left\{ (n+1) \frac{dX_n}{d\theta} - n \frac{dX_{-n-1}}{d\theta} \right\} e^{pt} \end{aligned} \right\} \dots \dots (9)$$

In the dielectric outside the shell

$$\left. \begin{aligned} v &= -\frac{n(n+1)}{4\pi r} Z_{-n-1} e^{pt} \\ \omega &= \frac{n}{4\pi r} \frac{dZ_{-n-1}}{d\theta} e^{pt} \end{aligned} \right\}, \dots \dots (10)$$

where Z_{-n-1} is an arbitrary solid spherical harmonic of degree $-n-1$.

In the conducting shell

$$\left. \begin{aligned} v &= -\frac{n(n+1)}{4\pi r} \psi_n(\lambda r) \chi_n e^{pt} \\ \omega &= -\frac{1}{4\pi r} \phi_n(\lambda r) \frac{d\chi_n}{d\theta} e^{pt} \end{aligned} \right\} \dots \dots (11)$$

The boundary conditions are (1) that the normal magnetic induction, (2) that the tangential magnetic force is to be continuous. The tangential magnetic force $= \frac{\omega}{\mu}$ in the conductor, and $= \omega$ in the dielectric.

$$\left. \begin{aligned} \psi_n(\lambda b) \chi_n &= X_n + X_{-n-1} \\ \psi_n(\lambda a) \chi_n &= Z_{-n-1} \end{aligned} \right\} \dots \dots (12)$$

$$\left. \begin{aligned} \frac{1}{\mu} \phi_n(\lambda b) \frac{d\chi_n}{d\theta} &= (n+1) \frac{dX_n}{d\theta} - n \frac{dX_{-n-1}}{d\theta} \\ \frac{1}{\mu} \phi_n(\lambda a) \frac{d\chi_n}{d\theta} &= -n \frac{dZ_{-n-1}}{d\theta} \end{aligned} \right\} \dots \dots (13)$$

From these equations we obtain

$$\begin{aligned}\frac{X_n + X_{-n-1}}{Z_{-n-1}} &= \frac{b^n \psi_n(\lambda b)}{a^n \psi_n(\lambda a)}, \\ \frac{(n+1)X_n - nX_{-n-1}}{-nZ_{-n-1}} &= \frac{b^n \phi_n(\lambda b)}{a^n \phi_n(\lambda a)}; \\ \therefore b^n \{(n+1)\phi_n(\lambda a)\psi_n(\lambda b) + n\phi_n(\lambda b)\psi_n(\lambda a)\} Z_{-n-1} \\ &= a^n (2n+1)\phi_n(\lambda a)\psi_n(\lambda a)X_{-n-1}. \quad (14)\end{aligned}$$

Let λa and λb be so large that $\frac{1}{\lambda a}$, $\frac{1}{\lambda b}$ may be neglected

$\psi_n(\lambda r)$ satisfies

$$\frac{d^2 \psi_n}{dr^2} + \frac{2(n+1)}{r} \frac{d\psi_n}{dr} + \lambda^2 \psi_n = 0.$$

Assume

$$\psi_n = \frac{e^{mr}}{r^{n+1}} u,$$

and let

$$\begin{aligned}\lambda^2 + m^2 &= 0, \\ \therefore \frac{d^2 u}{dr^2} + 2m \frac{du}{dr} - \frac{n(n+1)}{r^2} u &= 0.\end{aligned}$$

Let

$$u = a_0 + \frac{a_1}{r} + \frac{a_2}{r^2} + \dots$$

Substituting, we find in the usual manner,

$$u = 1 - \frac{n(n+1)}{2mr} + \frac{(n-1)n(n+1)(n+2)}{1 \cdot 2 \cdot (2mr)^2} - \dots$$

where

$$m = \pm i\lambda.$$

Hence, when $\frac{1}{\lambda r}$ is small,

$$\psi_n(\lambda r) = \frac{e^{i\lambda r}}{r^{n+1}} \text{ or } \frac{e^{-i\lambda r}}{r^{n+1}}.$$

Now Φ is zero at infinity and must decrease as r increases,

$$\therefore \psi_n(\lambda r) = \frac{e^{-i\lambda r}}{r^{n+1}},$$

$$\therefore \phi_n(\lambda r) = -i\lambda \frac{e^{-i\lambda r}}{r^n}.$$

Substituting in (14)

$$\begin{aligned} Z_{-n-1} &= \frac{(2n+1)be^{-\epsilon\lambda\eta}}{(n+1)a+nb} X_{-n-1} \\ &= e^{-\epsilon\lambda\eta} X_{-n-1} \dots \dots \dots (15) \end{aligned}$$

if both a and b be large compared with their difference.

Let Ω denote the solid angle subtended by the circuit at any point ;

V the potential due to the circuit ;

U the normal magnetic force.

$$\Omega = 2\pi \sin^2 \alpha \sum_1^{\infty} \frac{1}{n+1} \left(\frac{c}{r}\right)^{n+1} P_n'(\alpha) P_n(\theta),$$

if $r > c$. P_n is the n th zonal harmonic.

$$V = \gamma e^{\rho t} \Omega,$$

$$U = -\frac{dV}{dr},$$

$$= 2\pi\gamma \sin^2 \alpha e^{\rho t} \frac{1}{r} \sum_1^{\infty} \left(\frac{c}{r}\right)^{n+1} P_n'(\alpha) P_n(\theta). \dots \dots (16)$$

Now in the dielectric inside the shell,

$$v = -\frac{n(n+1)}{\epsilon\rho r} (X_n + X_{-n-1}) e^{\rho t},$$

$$\therefore -\frac{n(n+1)}{\epsilon\rho} X_{-n-1} = 2\pi\gamma \sin^2 \alpha \left(\frac{c}{r}\right)^{n+1} P_n'(\alpha) P_n(\theta),$$

$$\therefore X_{-n-1} = -\frac{2\pi\gamma \sin^2 \alpha \epsilon\rho}{n(n+1)} \left(\frac{c}{r}\right)^{n+1} P_n'(\alpha) P_n(\theta), \dots (17)$$

$\therefore$ from (15)

$$Z_{-n-1} = -e^{-\epsilon\lambda\eta} \frac{2\pi\gamma \sin^2 \alpha \cdot \epsilon\rho}{n(n+1)} \left(\frac{c}{r}\right)^{n+1} P_n'(\alpha) P_n(\theta). \dots (18)$$

But in the dielectric outside the shell

$$v = -\frac{n(n+1)}{\epsilon\rho r} Z_{-n-1} e^{\rho t},$$

$$\therefore v = e^{-\epsilon\lambda\eta} 2\pi\gamma \sin^2 \alpha e^{\rho t} \frac{1}{r} \left(\frac{c}{r}\right)^{n+1} P_n'(\alpha) P_n(\theta).$$

Now

$$\lambda^2 = -\frac{4\pi\mu\epsilon\rho}{\sigma} = (1-\epsilon)^2 \rho^2,$$

where

$$q^2 = \frac{2\pi\mu\rho}{\sigma},$$

$$\therefore \text{real part of } e^{-i\lambda\eta} e^{pt} = \text{real part of } e^{-q\eta} e^{i(pt-q\eta)} \\ = e^{-q\eta} \cos(pt-q\eta),$$

$$\therefore v = e^{-q\eta} 2\pi\gamma \sin^2 \alpha \frac{1}{r} \left(\frac{c}{r}\right)^{n+1} P'_n(\alpha) P_n(\theta) \cos(pt-q\eta). \quad (19)$$

Let v_0 be the maximum value of v , U_0 that of U ;

$\therefore$ from (16) and (19)

$$\frac{v_0}{U_0} = e^{-q\eta}. \quad (20)$$

The method employed in this investigation is taken from a paper by Professor H. Lamb, Phil. Trans. Part ii. 1883.

(4) CASE II.

The result for this case may be deduced from the preceding by using a particular case of a transformation due to Professor C. Niven (Phil. Trans. Part ii. 1883).

He shows that if P_n denote a zonal harmonic of the n th degree, $s = \sin \theta$, $n = ka$, $s = \frac{\rho}{a}$; then when n and a become infinite, k and ρ remaining finite,

$$P_n = J_0(k\rho),$$

$$P'_n = \frac{ka^2}{\rho} J_1(k\rho),$$

J_0 and J_1 being Bessel functions.

To find the value of Ω in terms of Bessel's functions. Let P be any point, draw PM perpendicular to the axis of z .

Let CD , the radius of the circuit, $= f$,

$PM = \rho$, $\angle POM = \theta$, $\angle COD = \alpha$, $OC = c$, $DM = z$, $OP = r$.

$$\Omega = 2\pi \sin^2 \alpha \sum_{n=1}^{\infty} \frac{1}{n+1} \left(\frac{c}{r}\right)^{n+1} P'_n(\alpha) \cdot P_n(\theta), \quad r > c.$$

Let

$$n = kr = k_1 c,$$

$$\sin \theta = \frac{\rho}{r}, \quad \sin \alpha = \frac{f}{c}.$$

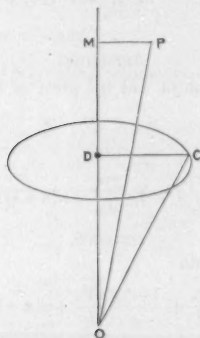
Let n, r, c become infinite, k, ρ, f remaining finite;
 $\therefore$ ultimately $k = k_1$,

$$P_n(\theta) = J_0(k\rho), \quad P_n'(x) = \frac{ke^2}{f} J_1(kf),$$

$$c = OD = OM - DM$$

$$= r - z \text{ ultimately ;}$$

$$\therefore \left(\frac{c}{r}\right)^n = \left(1 - \frac{z}{r}\right)^{kr} = e^{-kz} \text{ in the limit.}$$



The successive values of n are $1, 2, 3 \dots$; let $k + dk$ be the successive values of k ;

$$\therefore n + 1 = (k + dk)r;$$

$$\therefore rdk = 1;$$

$$\therefore \frac{k}{n+1} \left(\frac{c}{r}\right) = \frac{n}{n+1} dk = dk;$$

$$\therefore \Omega = 2\pi f \int_0^\infty e^{-kz} J_0(k\rho) J_1(kf) dk.$$

This result can also be deduced from the equation

$$\frac{d^2 \Omega}{dr^2} + \frac{1}{r} \frac{d\Omega}{dr} + \frac{d^2 \Omega}{dz^2} = 0,$$

remembering that

when $z=0$, $\Omega=2\pi$ from $r=0$ to $r=a$.

$\Omega=0$ from $r=a$ to $r=\infty$.

We thus find

$$v=2\pi\gamma e^{-q\eta}f\cos(pt-q\eta)\int_0^\infty ke^{-kz}J_0(k\rho)J_1(kf)dk,$$

$$U=2\pi\gamma f\cos pt\int_0^\infty ke^{-kz}J_0(k\rho)J_1(kf)dk;$$

$$\therefore \frac{v_0}{U_0} = e^{-q\eta}$$

as before.

APPENDIX.

It may be useful to add the proof of the transformation used in Case II.

If $\mu=\cos\theta$,

P_n satisfies

$$(1-\mu^2)\frac{d^2P_n}{d\mu^2}-2\mu\frac{dP_n}{d\mu}+n(n+1)P_n=0. \quad (1)$$

Let $s=\sin\theta$.

(1) transforms into

$$(1-s^2)\frac{d^2P_n}{ds^2}+\frac{1-2s^2}{s}\frac{dP_n}{ds}+n(n+1)P_n=0.$$

Assuming

$$P_n=a_0+a_1s+a_2s^2+\dots$$

we find in the usual manner

$$P_n=a_0\left\{1-\frac{n(n+1)}{2^2}s^2+\frac{(n-2)n(n+1)(n+3)}{2^2\cdot 4^2}s^4-\dots\right\}.$$

But $P_n=1$ when $s=0$;

$$\therefore a_0=1.$$

Let $n=ka, \quad s=\frac{\rho}{a},$

and let n and a become infinite, k and ρ remaining finite.

$$\frac{n(n+1)}{2^2}s^2=\frac{k^2\rho^2}{2^2},$$

ultimately

$$\frac{(n-2)n(n+1)(n+3)}{2^2 \cdot 4^2} s^4 = \frac{k^4 \rho^4}{2^2 \cdot 4^2},$$

$$\&c. \quad = \quad \&c.$$

$\therefore$ ultimately

$$P_n = 1 - \frac{k^2 \rho^2}{2^2} + \frac{k^4 \rho^4}{2^2 \cdot 4^2} - \frac{k^6 \rho^6}{2^2 \cdot 4^2 \cdot 6^2} + \dots$$

$$= J_0(k\rho).$$

Again

$$s^2 + \mu^2 = 1;$$

$$\therefore s ds = -\mu d\mu,$$

$$\mu^2 = 1 - s^2 = 1 \text{ ultimately,}$$

$$s = \frac{\rho}{a}; \quad \therefore ds = \frac{d\rho}{a};$$

$$\therefore d\mu = -s \frac{d\rho}{a} = -\frac{\rho d\rho}{a^2};$$

$$\therefore P'_n = \frac{dP_n}{d\mu} = -\frac{a^2}{\rho} \frac{dJ_0(k\rho)}{d\rho}$$

$$= \frac{k a^2}{\rho} J_1(k\rho).$$

DISCUSSION.

Mr. EVERSHED referred to some experiments of his own, from which he concluded that the author's formula gave too low an estimate of the attenuation; the discrepancy indicated that some term had been neglected.

Mr. YULE doubted whether the equations given by the author were quite applicable to sea-water. There was need, apparently, of a term involving the polarisation of the medium.

Mr. HEAVISIDE communicated a criticism of the paper. It was not necessary to investigate the problem for any particular form of circuit from which the waves proceed. The attenuating factor for plane waves, due to Maxwell, was sufficient. Taking the best-known value for the con-

ductivity of sea-water, there was no reason why the conductivity should interfere with signalling. A considerably greater conductivity must be proved for sea-water before it could be accepted that the failure of experiments on telegraphic communication with light-ships from the sea-bottom was due to that factor. It was unlikely theoretically, and Mr. Stevenson had contradicted it from a practical standpoint. For some reason, the account of the light-ship experiments had not been published, so that there was no means of finding the real cause of failure.

XVIII. *On the Decomposition of Silver Salts by Pressure.*

*By J. E. MYERS, M.Sc., Ph.D., late 1851 Exhibition Science Scholar, and F. BRAUN, Ph.D., Professor of Physics in the University of Strassburg i.E.**

As was demonstrated by Carey Lea† some years ago, silver salts and others may be decomposed by the application of pressure. By pounding a portion of the salt in a mortar one may readily effect decomposition.

A convincing proof is obtained by compressing a halogen salt of silver in the absence of sunlight and then subjecting the compressed mass to the developing and fixing processes of photography. A black residue of finely divided silver is the result. Exposure of the salt to sunlight, before compression, makes of course this test useless: the difficulty may be overcome by employing a mixture of AgNO_3 and KR ($\text{R}=\text{Cl}$, Br , or I) in the first instance.

The decomposition is most marked in the case of bromide of silver.

Two different metals immersed in liquid bromine constitute a galvanic element‡. The couple Ag , Br , Pt has an E.M.F. of about 0.95 volt. It appeared therefore of interest to

* Read June 11, 1897.

† *Phil. Mag.* xxxi. p. 323 (1891); xxxiv. p. 46 (1892); xxxvi. p. 351. (1893); xxxvii. pp. 31 and 470 (1894).

‡ F. Braun, *Wied. Ann.* xvii. p. 610 (1862).

determine the E.M.F. developed when AgBr is compressed between electrodes of platinum and silver.

The salt was contained in a so-called diamond mortar, which was placed under a press and subjected to a pressure of 3000–5000 atmospheres. The apparatus did not admit of a direct measurement of pressure, but for purposes of comparison, by using the same quantity of salt and turning the lever to the same position the same pressure (approximately) could be applied. The pestle was insulated from the mortar by means of a thin sheet of mica bent into a cylindrical form. A piece of platinum foil beneath the salt, and a short cylinder of silver immediately beneath the pestle, formed the electrodes, which were connected to the quadrants of an electrometer.

The following serves to give a general idea of the run of an experiment:—

(One Daniell element gave a deflexion of 132·1 scale-divisions.)

Ag Br.	Scale-divisions.
On applying pressure, immediate deflexion . . .	1·2
after 2 minutes . . .	2·8
" 10 " . . .	6·8
" 15 " . . .	7·2
" 40 " . . .	9·2
" 55 " . . .	9·0
" 85 " . . .	8·7
" 95 " . . .	8·2
" 105 " . . .	7·0
Pressure removed . . .	5·0

after which the deflexion rapidly sank to zero.

Thus the maximum E.M.F. generated was about 0·07 Daniell. The direction of current was from Ag to Pt through the compressed substance.

A higher applied pressure produced an E.M.F. of 0·09 Daniell.

The silver bromide was prepared from pure AgNO_3 by precipitation with pure KBr. Before each experiment the salt was carefully dried by heating for two hours or more at a temperature of 150°C ., and then allowed to cool in a

desiccator. Pure AgNO_3 showed no appreciable E.M.F. on compression, neither did KBr : a mixture of the two in equivalent quantities compressed between electrodes of Pt and Ag had an E.M.F. of 0.08 Daniell.

With AgCl and AgI the E.M.F.'s were smaller.

$\text{Ag, AgCl, Pt} = 0.03 \text{ Daniell.}$

$\text{Ag, AgI, Pt} = 0.04 \quad ,,$

In both cases a marked darkening in colour was noticeable.

Somewhat remarkable is the fact that under high pressure AgBr becomes gradually transparent even when 3-4 millim. thick. At the same time the electrical resistance increases considerably. For instance, in one case the resistance increased gradually from $3.65 \cdot 10^3$ ohms to $14.62 \cdot 10^3$ ohms (measured with alternating current and telephone) in the course of 48 hours, pressure being continuously applied.

Strassburg, Physik. Institut.

XIX. *A Method of determining Magnetic Hysteresis loss in Straight Iron Strips.* By J. A. FLEMING, M.A., D.Sc., F.R.S., Professor of Electrical Engineering in University College, London*.

[Plate VII.]

THE exact determination of the magnetic hysteresis loss in sheet iron as used for alternating current transformer construction has become of commercial importance of late years. Owing to the obvious necessity for reducing the core loss in transformers to the lowest attainable limit, it is now the custom to carefully test the iron to be used and to express the determined hysteresis loss in the iron sheet, in watts per pound per 100 frequency, at a maximum induction during the cycle of 2500 C.G.S. units. Two processes are in ordinary use for effecting this measurement. The most

* Read June 11, 1897.

accurate but most tedious is by the employment of a ballistic galvanometer and the graphical delineation of a number of (B, H) cycles. This method has the advantage that it can be conducted with either small or large samples of iron, but the reduction of the observations to give the required hysteresis loss is very tedious; and including the construction of the necessary ring-coil, the actual observations, and the reductions, a single determination of the hysteresis value of an iron can hardly be carried out, even by a couple of industrious workers, in less than a day.

The method employed by Professor Ewing and Mr. F. Holden of rotating a laminated specimen of the iron in a magnetic field, or rotating a field round the specimen and comparing the torque produced on the field-magnet or on the specimen with that produced by a similar sample of iron of known hysteretic value or else measuring it by a calibrated torsion spring, is very much more rapid. This last method is, however, open to the great objection that if used merely to compare a sample of iron with a standard sample, the user of the instrument is at the mercy of his instrument-maker. He receives with it one or more samples of iron which he is told have certain hysteretic values, but he cannot check the truth of the statement. Moreover, if he happens mechanically to injure the standard samples he may seriously change their hysteresis constant. The writer therefore sought for a method which should combine the advantages of both the above processes whilst eliminating their disadvantages, and which should be an absolute method, like the ballistic method, but at the same time a rapid method like the rotating-field method.

The following paper contains a description of a process which is based upon the use of the bifilar reflecting wattmeter and operates upon samples of iron, large or small, in the form of straight strips which are simply slipped into a long magnetizing coil. All the actual practical work and the numerous calculations involved in putting this process to test have been conducted under the author's direction by Mr. A. R. Peart and Mr. W. M. Park working in the Electrical Laboratory of University College, London; and to these gentlemen belongs the full credit for all this portion of the

research as well as for the labour required in much tedious numerical and graphical work.

The method is based upon an interesting discovery with regard to the distribution of the induction in a bundle of straight iron strips or iron or steel wires, when placed in a uniform magnetic field.

If a long, straight, magnetizing coil is prepared by winding insulated wire upon a paper or vulcanized fibre tube, and if the length of the solenoid is, say, ten times its mean diameter, then for a distance equal to at least three-quarters of the total length of the coil we have a practically uniform magnetic field in the interior of the bobbin, when a constant current is sent through the wire. If into this uniform magnetic field a bar of iron or straight bundle of iron strips or iron wire is introduced, the length of the iron being not greater than the uniform portion of the interior field of the coil when the iron is not there, this iron has magnetic induction created in it which varies in density from point to point in the bar, but is symmetrical in value about the central point of the bar.

The induction density (B) is greatest in the centre of the bar but falls off towards each end. If the magnetizing coil is traversed by an alternating current, and if the iron is laminated, the maximum value of the induction density in the bar can be easily determined at any point in its length by placing upon the bar a flat bobbin of fine insulated wire which is connected with an electrostatic voltmeter capable of reading from 1 to 20 volts (see diagram). The first portion of this investigation was concerned with the study of the mode in which this induction density varies from point to point in the length of a bundle of iron strips or iron wire or steel rods of various lengths and diameters, thus placed in the centre of a long magnetizing solenoid.

A magnetizing coil was prepared, wound on a paper tube, 5.7 centimetres in internal diameter and 99 centimetres long. On this tube was wound, in three layers, a coil of 1235 turns of cotton-covered copper wire, .064 inch diameter or of No. 16 size S.W.G. The resistance of this coil was 1.069 ohms at 15° C.

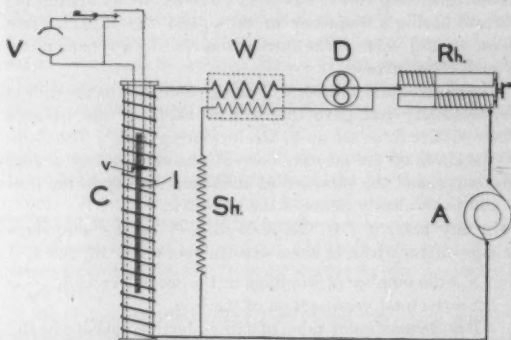
The number of turns per centimetre length of this coil was 12.47. Hence since the interior central field is $4\pi/10$ times

the ampere turns per unit of length of the coil, we have thus the value of the field H in the central regions of the interior of this bobbin given by the equation

$$H = 15.66 A,$$

where A is the ampere current through the coil.

The secondary or exploring coil consisted of a flat bobbin wound up of silk-covered copper wire .0024 inch in diameter



C. Magnetizing coil.

V. Electrostatic voltmeter.

Rh. Rheostat.

I. Iron strips.

W. Wattmeter.

A. Alternator.

v. Exploring coil.

D. Dynamometer.

and having 2000 turns; the thickness or length of the bobbin being 0.7 centimetre and the external diameter about 4.5 centimetres.

The exploring coil was wound on a circular ebonite former having a rectangular hole in it so as to closely fit the bundle of iron strips to be used, but at the same time it could be slid along the bundle.

The voltmeter used with it was one of Professor Ayrton's low-reading electrostatic instruments of the idiostatic type and read from 1 to 25 volts, the reading being taken by a mirror and scale. The voltmeter was calibrated when required by means of standard cells or a potentiometer.

The first experiments were conducted with strips of transformer iron, kindly furnished to us by Messrs. Sankey. This sheet iron was '0121 of an inch or '0307 of a centimetre in thickness and the strips were each one inch or 2.54 centimetres wide. The strips used were of various lengths: 60 centimetres, 30 centimetres, 15 centimetres, down to 5 centimetres. A bundle I (see diagram) of a certain number of strips was made and the secondary coil v slipped on to it. The magnetizing coil C was then traversed by an alternating current having a frequency of 80 $\sim$, and the R.M.S. (root mean square) value of the current was read by a dynamometer D or Kelvin balance.

The electrostatic voltmeter V being connected to the ends of the secondary coil gave the R.M.S. value of the induced electromotive force set up in the exploring coil. The form of the curve of the primary current was very nearly a true sine curve, and the variation of induction density in the iron at any point closely followed the same law.

In any case, if f is the *form factor** of the secondary voltage curve, which in these experiments was 1.13, and if

N = the number of windings in the secondary coil,

S = the total cross-section of the iron,

B = the maximum value of the induction density in the iron in C.G.S. units at that point in its length at which the secondary coil is placed, and

n = the frequency of the induction cycle; then if e_2 is the R.M.S. value of the secondary electromotive force in volts as measured by the voltmeter, we have always

$$10^8 e_2 = 4f'NSBn.$$

Hence the maximum value of the induction density B can be calculated from the known value of all the other quantities. In this manner a large number of measurements of the value of the induction density B were made at different points in the lengths of various bundles of iron strips of different lengths and total cross-sections.

* See 'The Alternate Current Transformer,' Fleming, vol. i., new. ed., p. 583, for a reference to this term.

These values were then laid down in a series of induction density curves so drawn that the ordinates represented the maximum value of the induction density in the bar at different transverse sections along the half-length of the bar, taken as corresponding abscissæ.

It has been shown by Steinmetz and fully confirmed by others that for most varieties of iron used for electrotechnical purposes the hysteresis loss in the iron, per cycle of induction, per cubic centimetre varies as a power of the maximum value of the induction density, which is very nearly 1.6, when that maximum value of the induction density has any value below about 9000 C.G.S. units. Hence if the ordinates of the above-mentioned induction curves are all raised to the 1.6th power and a new curve plotted showing the variation of $B^{1.6}$ along the half-length of the bar, we have another curve which represents the variation of the hysteresis loss per unit of volume along the half-length of the bar from point to point. A large series of such curves showing the variation of B and $B^{1.6}$ were drawn for bundles of iron strips and steel of different lengths and sections, and the results are collected and represented in the form of curves in the diagrams in Plate VII.

The upper curves in each case in the figs. 1, 2, and 3 in Plate VII. represent the variation of B along the half-length of the bar, and the lower curves the corresponding curves for the variation of $B^{1.6}$. The first pair of diagrams represent the result of varying the maximum induction density in the centre of a bundle of strips of iron of the same length and section.

The second pair of diagrams give the curves showing the same results for bundles of iron strips of varying lengths but the same section.

The third pair for various samples of iron strips of the same length but different section.

It is clear from these diagrams therefore that such a bar or bundle of iron strips placed in an originally uniform magnetic field has a non-uniform induction density produced in it at its various points, which is symmetrical about the centre; and a non-uniform hysteresis loss per unit of volume also symmetrical

about the centre. The maximum values of each being at the centre of the bar.

If we take the true mean value of the varying distributed hysteresis loss per unit of volume, it is obvious that at some point in the half-length of the bar there must exist an actual induction density B_1 such that the true mean hysteresis loss in the whole bar is proportional to $B_1^{1.6}$. Let this value of the induction density be called the *effective value*, and the point in the bar at which it occurs the *effective point*. Then the effective value B_1 must be equal to the 1.6th root of the mean of the 1.6th powers of the actual induction densities taken at equidistant points all along the bar, or

$$B_1 = \sqrt[1.6]{m \cdot B^{1.6}}$$

where $m \cdot B^{1.6}$ stands for the mean ordinate of the curve representing the varying values of $B^{1.6}$ all along the half-length of the bar.

If then this effective value B_1 of the induction density is calculated for each of the samples of iron strip used, and the position at which the actual induction has this effective value is marked off on the half-length of the bar (this position is represented by the *cross* on the curves given in Plate VII.), we have noticed the remarkable fact that *whatever the length or section of the iron strip may be, the point in the bar at which the actual induction density has a value equal to the effective value, always comes at the same proportional distance from the centre of the bar; and this distance is very nearly equal to .56 of the half-length of the bar from the centre or at .22 of the whole length of the bar from one end.*

The following Table I. embodies all the results taken with bundles of annealed transformer iron strip (Sankey's) of very different lengths and sections, and Table II. similar observations on rods of steel and hard iron; and it will be seen that the effective induction in the case of the iron strip is always found at a point in the bar or bundle which is very nearly 0.56 of the half-length of the bar from the centre. If therefore the secondary coil is placed at that spot and the secondary voltage then observed used to calculate the induction density, the value so obtained is that which corresponds to the true mean value of the varying hysteresis loss

per unit of volume all along the bar. In other words, if we suppose the bar magnetized by an alternating current uniformly and with a maximum induction density everywhere equal to that which it actually has under normal conditions at 0.56 of the half-length of the bar from the centre; then the hysteresis loss in such a bar would be equal to that found in the bar as it is. In all cases where induction density values are given, these are to be understood as being the maximum values during the cycle unless otherwise stated. The frequency of alternations used was 80.

We have therefore in this useful fact regarding the distribution of induction in iron strips in a uniform magnetic field, the foundation of a quick method of determining the hysteretic constant of a sample of iron.

If the iron sample, taken in strips, is placed as above in the interior of the long magnetizing coil, and if a sensitive wattmeter, suitable for use with alternating currents, is connected therewith, we can measure the total power taken up in the iron and in the copper of the coils when any known alternating current is passing through the magnetizing coil. If then the iron strips are withdrawn and another measurement of the same kind made with the current passing through the magnetizing coil adjusted to the same value as in the first case, this last reading gives us the wattmeter-reading corresponding to a known power taken up in the copper of the magnetizing coils. Hence the first wattmeter-reading gives us the iron loss, together with the copper loss, and the second reading gives us the copper loss alone. The iron loss gives us the true mean value of the hysteresis loss in the iron, assuming the iron is so well laminated that eddy-current loss is practically absent.

The above-mentioned law as to the distribution of the induction enables us, then, to assign the corresponding value of the maximum induction density.

Unless this can be done, the mere mean hysteresis loss measurement is no practical use*.

* Methods of measuring hysteresis loss by one observation of an instrument have been suggested by other observers; but the practical value of these methods has been very small, because the investigators have not shown how the corresponding maximum value of the effective induction is to be obtained. The rule above given supplies this deficiency.

TABLE I.
Summary of experiments made to ascertain the "Effective point" in bundles of annealed Transformer Iron Strips (Sankey's) when placed in a uniform alternating magnetic field.

Reference letter to diagram.	Length of specimen in centims.	Sectional area of specimen in sq. cms.	Induction at centre of bar, B, C.G.S. units.	Effective induction, B, C.G.S. units.	Ratio of $\frac{B_1}{B}$.	Distance of effective point from centre of bar.	Fraction of half-length of bar from centre at which effective induction exists.
A ₁	61	.936	564	420	.744	17.01	.558
A ₂	61	.936	1648	1240	.752	16.91	.554
A ₃	61	.936	2084	1563	.750	16.92	.555
A ₄	61	.936	2536	1900	.749	17.00	.557
A ₅	61	.936	3112	2335	.712	17.00	.557
A ₆	61	.936	3816	2870	.752	17.10	.561
B ₁	30	.936	1597	1192	.746	8.36	.557
B ₂	30	.936	4052	3012	.743	8.35	.557
B ₃	15	.936	1622	1265	.780	4.11	.548
B ₄	15	.936	3430	2660	.775	4.18	.557
B ₅	10	.936	1907	1554	.815	2.73	.546
B ₆	10	.936	1025	833	.813	2.76	.552
B ₇	5	.936	1622	1420	.875	1.39	.556
B ₈	5	.936	778	678	.871	1.49	.596
C ₁	61	.312	3485	2565	.736	16.84	.552
C ₂	61	.312	9673	7250	.749	16.81	.551
C ₃	61	.624	2125	1570	.739	17.07	.560
C ₄	61	.624	5834	4420	.758	17.60	.577

Mean of last column = .565.

TABLE II.
Summary of tests made as in Table I. upon Steel and Hard Iron Rods.

Reference letter to diagram.	Length of specimen in centims.	Sectional area of specimen in sq. cms.	Induction in centre of bar, B, C.G.S. units.	Effective induction, B _i , C.G.S. units.	Ratio of B _i to B	Distance of effective point from centre of bar in cms.	Fraction of half-length of bar from centre at which effective induction exists.	Nature of bar.
C ₆	61	·970	4499	3525	·783	6·90	·58	Round bar of silver steel $\frac{1}{16}$ inch in diameter.
D ₂	32	·178	5997	4784	·798	9·80	·61	Round rod of silver steel $\frac{1}{8}$ inch in diameter.
D ₃	32	·534	5555	4244	·764	9·31	·58	Three similar pieces of steel.
D ₄	20·45	·645	4746	3587	·756	6·67	·65	Bundle of 60 iron wires of hard iron of rectangular section.
D ₅	20·45	·645	2545	1912	·751	6·76	·66	

The operation of making the mean hysteresis measurement is effected by a sensitive mirror bifilar electro-dynamometer.

This was constructed in the following way :—

The fixed or series-coil consists of 90 turns of No. 16 S.W.G. cotton-covered copper wire wound in five layers on a rectangular block of wood. The movable coil, or shunt-coil, consists of a rectangular coil of 20 turns of No. 24 S.W.G. wire, the turns being fastened together with silk and shellac. This movable coil embraces, without touching, the fixed coil, as in a Siemens dynamometer. The movable coil is suspended by a bifilar suspension consisting of two silver wires $\cdot 002$ of an inch in diameter and 37 centim. long, fixed about 3 millim. apart. These wires are attached to the ends of the suspended coil, and serve to convey the shunt-current in and out of the coil. The movable coil carries a concave silver mirror by means of which a sharp image of an incandescent lamp-filament is formed upon a scale at a fixed distance of 100 centim. from the mirror. A torsion-head carries the upper ends of the bifilar suspension, and serves to adjust the zero.

In series with the shunt-coil is a high non-inductive resistance of 1000 ohms.

The resistance of the thick-wire fixed coil of the wattmeter is $\cdot 133$ of an ohm, and the resistance of the shunt-coil and suspension, without the associated inductionless resistance, is 11.7 ohms.

When such a bifilar wattmeter is employed to measure electrical power, it can be shown that the deflexions of the movable coil, when not large, are strictly proportional to the power passing through the wattmeter, so that each scale-division has an assignable power value, which in our case was almost exactly $\cdot 01$ of a watt per millimetre of scale. The arrangement of circuits was then as shown in the diagram on p. 205. The long magnetizing coil above described was connected in series with the fixed coil of the wattmeter, and also with a sensitive electro-dynamometer or ammeter suitable for measuring small alternating currents of about an ampere or less in value.

The shunt-coil of the wattmeter was joined across the ends

of the long coil. Suitable resistances beyond controlled the current.

The process of measurement was then as follows:—A bundle of straight iron strips was fitted with the secondary coil at the effective point, viz. $\cdot 56$ of the half-length of the bar from the centre. The bundle was slipped into the magnetizing coil, and an alternating current sent through the coil of such a magnitude (generally about $\cdot 2$ of an ampere) as to give an induction density of not more than about 6000 C. G. S. units at the effective point. The wattmeter deflexion was then read. To interpret this reading the iron strips were removed from the coil and another wattmeter measurement made on the copper coil alone. As the removal of the iron sends up the current strength by decreasing the inductance of the circuit, it is necessary to readjust the current to the same value as before, and to know that value in amperes. The resistance of the coil being known, and also the current passing through it, we can calculate the power being taken up in the copper coil alone for this standard current; and hence know the power in watts producing the observed wattmeter deflexion.

The wattmeter is therefore calibrated on the copper circuit of the magnetizing coil, and from the known copper loss in this coil and the proportionality of its deflexions we are enabled to assign at once the value in watts of its deflexion when used to measure both the iron loss and copper loss together when the iron strip is inserted in the coil.

If the induction is carried up as far as 4000 or 5000 C.G.S. units at the effective point, and if the strips have a thickness over $\cdot 01$ inch, then it becomes necessary to apply a small correction for the eddy-current loss set up in the iron. This can be at once done by using the following simple practical formula given by the author* for this loss in rectangular sectioned strips.

If t is the thickness of the strip in *mils* (1 mil = $\cdot 001$ inch), n is the frequency of the current, and B is the maximum

* See 'The Alternate Current Transformer,' Fleming, vol. ii. p. 490, for proof of this formula.

value of the induction density during the cycle at any point on the bar, then it can be shown that the eddy-current loss in watts per cubic centimetre at that point in the rectangular or sectional strip is closely given by the expression :—

$$\left(\frac{B}{10^8 nt} \right)^2,$$

and hence for a given frequency and thickness the eddy-current loss per pound of iron is very quickly found from a simple parabolic curve representing B^2 in terms of B .

In all the above observations with the iron strip used, which was 12 mils thick, this correction has been applied, and it is sufficient for this purpose to take as B in the above formula the value B_1 of the effective induction.

The eddy-current loss varies from point to point in the bar, being always proportional to the square of the maximum value of the induction density. Hence, properly speaking, the mean value of the eddy-current loss should be obtained by taking the mean square value of the induction all along the half-length of the bar, and using the mean-square value instead of the B^2 in the formula above.

We have, however, found that the point on the half-length of the bar at which the actual induction has a value equal to the root-mean-square value, or to the square root of the mean of the squares of the distributed induction, is always very close to the point at which the 1.6th root of the mean of the 1.6th powers of the distributed induction is found. In other words, the induction at the effective point is the induction corresponding to the mean eddy-current loss as well as to the mean hysteresis loss. This is shown by the curves given in figs. 4 and 5, Plate VII., in which the curve marked B is the curve of variation of the maximum value of the induction-density along the half-length of a bundle of iron strips, and the curves marked $B^{1.6}$ and B^2 represent (to different scales) the variation of these powers of B . In a bar 5 centim. long (see fig. 5, Plate VII.) the points where the actual induction had a value equal to $\sqrt{m \cdot B^2}$ and to $\sqrt[1.6]{m \cdot B^{1.6}}$ were only separated by a fraction of a millimetre. Hence by another curious coincidence we are enabled to apply the proper correction for the

eddy-current loss by taking the square of the effective induction B_1 and putting its value into the expression

$$\frac{B_1^2 n^2 t^2}{10^{16}},$$

where n is the frequency and t the thickness of the strip in mils.

In order to put the method to a test, a very careful determination was made by the ballistic method of the hysteresis loss of a sample of the transformer iron strip used. A ring-coil was prepared, and a series of (B-H) cycles of magnetization taken, and a hysteresis loop drawn, and from this loop it was found that for this iron the hysteresis loss was .25 watts per pound per 100 cycles per second, for a maximum value of the induction density equal to 2500 C.G.S. units.

Straight strips of exactly the same iron were then tested by the method here described, which for shortness will be called the "straight coil" method, and the results of these tests with the iron in different conditions of hardness are shown in the Tables III., IV., and V.

It will be seen that the results of the "straight coil" test with the wattmeter, as given in Table III., gave exactly the same mean value for the hysteresis loss of the strip when not bent and unbent, as did the calculations made from the ballistic test, viz.: .25 watts per pound per 100~ per maximum induction density of 2500. The difference in the time of taking the tests is, however, immense. A very few minutes suffices to take the wattmeter tests, and the necessary calculations can be made in a comparatively short time.

Many dozens of tests of irons can be made by the straight coil method in the time occupied by one ballistic test.

There is another method by which the bifilar wattmeter can be calibrated, but which is not so generally convenient as that above described. If the shunt and series coils of the wattmeter are joined up in series with each other, so as to form a dynamometer, and if a known current I is passed

through these coils, causing a deflexion θ in the movable coil; then when the instrument is used as a wattmeter with a total shunt-resistance R (including the added non-inductive resistance) the true power corresponding to a deflexion θ is I^2R watts.

Hence, from a curve showing the deflexion given by various currents, we can at once determine the constant of the instrument, when used as a wattmeter, with given shunt resistance.

If this standardization is taken with continuous currents, then it is necessary to eliminate the effect of the Earth's field by taking right and left deflexions. The curve connecting square of current strength and the deflexions of the movable coil is nearly a straight line, in the case of the bifilar wattmeter.

Standardizing our wattmeter in this way with a continuous current, we then made another set of hysteresis measurements, with 12 strips of perfectly new annealed transformer strips (Sankey's), with the results shown in Table VI. Two other sets of observations for hammered and hardened iron are given in Tables VII. and VIII.

As a matter of fact, in the above experiments in the observations given in Tables III., IV., V., VII., and VIII., the wattmeter was standardized by sending known and measured currents through the copper coil and obtaining a wattmeter constant by finding the deflexions corresponding to known copper losses in the copper coil. Thus are found the following figures, when using the wattmeter to measure power taken up in the magnetizing coil alone:—

Known copper loss (C^2R) calculated from observed current and coil resistance in watts.	Scale-deflexions of wattmeter in millimetres.	Watts per millimetre of scale-deflexion of wattmeter.
·398	39·1	·0102
·292	28·9	·0101
·197	19·0	·0103

TABLE III.
Hysteresis Test by "Straight Coil" method of annealed Transformer Iron Strip. Six strips used (new)
 61 cm. long, 2.54 cm. wide, and 12 mils thick.

Voltmeter scale-reading in mm.	Effective Induction density, B_v	Wattmeter scale- reading.		Total iron loss in watts per lb.	Eddy-current loss in watts per lb.	Hysteresis loss in watts per lb. at 100~ and 2500 B.	Mean hysteresis value.
		Iron in.	Iron out.				
303.0	6719	87.6	17.5	1.247	.245	.257	.25 watts per pound per 100 ~ per 2500 B.
225.3	5702	66.0	15.3	.9027	.176	.243	
131.8	4284	40.0	7.8	.5747	.099	.251	
95.1	3623	43.0	8.1	.4377	.071	.251	

TABLE IV.
Hysteresis Test by "Straight Coil" method of Transformer Iron Strip. Same strips used as in Table III., but once bent and unbent.

Voltmeter scale-reading in mm.	Effective Induction density, B_p .	Wattmeter scale-reading.		Total iron loss in watts per lb.	Eddy-current loss in watts per lb.	Hysteresis loss in watts per lb. at 80~.	Hysteresis loss in watts per lb. at 100~ and 2500 B.	Mean hysteresis value.
		Iron in Coil	Iron out of Coil					
56.9	2786	19.0	4.2	.303	.042	.261	.273	.28 watts per pound per 100 ~ per 2500 B.
128.8	4244	39.0	8.4	.635	.098	.537	.288	
204.0	5424	58.1	12.4	.948	.160	.788	.285	
273.0	6346	76.0	15.9	1.247	.219	1.028	.289	

TABLE V.
Hysteresis Test by "Straight Coil" method of Transformer Iron Strip, the same quality as in Tables III. and IV., but bent and unbent several times. Twelve strips used 30 cm. long, 12 mils thick, cut from the same pieces used for the ballistic galvanometer test.

Voltmeter scale-reading in mm.	Effective Induction density, B.	Wattmeter scale- reading.		Total iron loss in watts per lb.	Eddy-current loss in watts per lb.	Hysteresis loss in watts per lb. at 80~.	Hysteresis loss in watts per lb. at 800~ and 2500 B.	Mean hysteresis value.
		Iron in coil.	Iron out of coil.					
374.5	3778	135.7	105.1	.539	.078	.461	.296	.28 watts per lb. per 100~ per 2500 B.
281.4	3219	101.4	78.1	.415	.057	.358	.297	
199.5	2672	71.2	54.8	.277	.038	.239	.268	
111.6	1964	42.1	32.9	.160	.0215	.139	.252	
71.1	1625	57.0	43.1	.125	.017	.107	.282	
50.4	1314	39.2	29.9	.083	.011	.072	.261	

TABLE VI.

Hysteresis Test by "Straight Coil" method of Transformer Iron Strips. Twelve new annealed strips used, never bent. Wattmeter standardized by Potentiometer and Continuous Currents.

Voltmeter scale-reading in mm.	Effective Induction- density, B _l	Wattmeter scale- reading.		Total iron loss in watts per lb.	Eddy-current loss in watts per lb.	Hysteresis loss in watts per lb. at 100~ and 2500 B.	Hysteresis loss in watts per lb. at 80~.	Mean hysteresis value.
		Iron in.	Iron out.					
497.0	4742	90.6	26.0	.517	.118	.399	.178	.21 watts per pound per 100 ~ per 2500 B.
338.8	3576	63.6	17.1	.340	.069	.271	.190	
267.0	3136	50.7	14.7	.295	.053	.242	.210	
204.9	2715	39.8	11.7	.232	.038	.193	.210	
130.8	2138	51.6	14.6	.154	.025	.129	.208	
110.3	1957	44.0	11.9	.133	.022	.111	.205	
77.1	1628	31.0	8.7	.093	.016	.070	.191	
49.5	1295	21.1	6.1	.062	.010	.052	.187	
398.5	3913	71.0	17.5	.466	.083	.383	.234	
282.0	3223	100.9	25.1	.334	.056	.278	.230	
200.5	2679	73.2	18.1	.242	.039	.203	.226	
100.0	1950	42.9	11.0	.118	.021	.097	.181	
77.2	1627	61.4	16.1	.103	.016	.087	.217	
49.2	1302	41.2	11.7	.067	.010	.057	.203	

N.B. The method of standardizing the wattmeter by use as a dynamometer employing a continuous current as above does not give such consistent or accurate hysteresis determinations as when it is standardized on a known copper resistance with known alternating currents.

TABLE VII.
Hysteresis Test by "Straight Coil" method on Hammered Transformer Strip (unannealed). Six strips used 61 centim. long, 2.54 centim. wide, and 12 mils thick.

Voltmeter scale-reading in mm.	Effective induction-density, B_e .	Wattmeter scale-readings.		Total iron loss in watts per lb.	Eddy-current loss in watts per lb.	Hysteresis loss in watts per lb. at 80~.	Hysteresis loss in watts per lb. at 100~ and B 2500.	Mean hysteresis value.
		Iron in.	Iron out.					
114.0	3975	54.5	23.5	.562	.086	.476	.284	.290 watts per pound per 100 cycles at B 2500.
160.0	4737	72.9	30.2	.778	.123	.655	.297	
206.0	5438	90.5	37.5	.969	.161	.802	.290	

Calibration Table of Wattmeter.

C.R.	Wattmeter deflexion.	Watts per millim.
.1875	20.9	.00897
.3263	36.9	.00884
.1195	13.3	.00898

TABLE VIII.

Hysteresis Test by "Straight Coil" method of 18 strips of Iron of inferior quality. Length 61 cms., width .63 centim., thickness 26.5 mils.

Voltmeter scale-reading in mm.	Effective induction- density, B_v	Wattmeter scale-reading in mm.		Total iron loss in watts per lb.	Eddy loss in watts per lb.	Hysteresis loss in watts per lb. at 80~.	Hysteresis loss in watts per lb. at 100~ and B 2500.
		Iron in.	Iron out.				
83.4	2054	51.9	23.7	.319	.110	.209	.360
136.0	2644	79.7	34.1	.517	.183	.334	.360
176.0	3028	101.0	42.8	.659	.239	.420	.386

Calibration Table of Wattmeter.

C ^o R.	Wattmeter deflection in mm.	Watts per mm.
.272	30.0	.00908
.381	41.4	.00920

In the case of the observations given in Table VI., we employed the method of determining the wattmeter constant by passing known currents through its coils joined up in series and used as a dynamometer, and from the curve showing relation of (current)² to deflexions we could at any moment find the wattmeter constant.

The method described above has this great advantage that the test can be made on fairly large samples of iron, and it can be made with an alternating current having any desired and practical form factor. The iron is tested under conditions which are exactly similar to those under which it will be used in transformer manufacture, and, in fact, the straight magnetizing coil and iron sample constitute an open circuit transformer of very small power factor.

The above described method effects a great saving of time in comparison with the ballistic method, whilst at the same time it is an absolute method and does not depend upon comparisons with other iron samples of supposed known hysteretic value.

In converting hysteresis losses from one maximum induction-density B to another by the aid of Steinmetz's law, it is useful to have the arithmetic values, for different values of B , of the ratios of $B^{1.6}$ to $2500^{1.6}$; since $B = 2500$ C.G.S. units is generally taken as the standard induction-density in reference to hysteresis determinations. The following table (Table IX.)

TABLE IX.

Showing the Values of the Ratio of $B^{1.6}$ to $2500^{1.6}$.

B.	$\frac{B^{1.6}}{2500^{1.6}}$	B.	$\frac{B^{1.6}}{2500^{1.6}}$
1000	.231	5500	3.531
1500	.442	6000	4.058
2000	.700	6500	4.614
2500	1.000	7000	5.193
3000	1.338	7500	5.795
3500	1.713	8000	6.430
4000	2.118	8500	7.085
4500	2.563	9000	7.764
5000	3.031	9500	8.466
		10000	9.190

gives these ratios with sufficient accuracy for all practical work. Thus, for instance, if the value of the hysteresis loss in an iron has been determined at a maximum induction-density during the cycle of 5000 units, and we wish to know the value at 2500 units, the observed value has to be divided by 3.031 to obtain the required result.

DISCUSSION.

Mr. CARTER asked whether roots other than the 1.6th gave a similar constant value of the induction density.

Dr. FLEMING said it seemed to be the result of accident that the 1.6th root gave a constant value for iron.

XX. *Attenuation of Electric Waves along Wires and their Reflexion at the Oscillator.* By EDWIN H. BARTON, D.Sc., F.R.S.E., Senior Lecturer in Physics at University College, Nottingham*.

LAST year Mr. Bryan and the present author realized experimentally the absorption by a terminal bridge of the electric waves incident upon it†. The first application of this result which suggested itself was the use of such a bridge in determining the attenuation (or rate of decay) of the waves along their parallel leads. For, if one can at pleasure either (1) practically absorb the waves, or (2) reflect them practically undiminished, a suitable electrometer being inserted in the line, we have in the first case a single passage of the wave-train past the electrometer, and in the second a number of passages of the wave-train until it is practically quenched by its repeated losses. Thus, the electrometer in the second case is affected by the sum to infinity of two geometrical progressions, viz., those due to the forward and return waves respectively. Whereas in the first case, when the

* Read June 25, 1897.

† Proc. Phys. Soc. vol. xv. pp. 23-30, 1897; Phil. Mag. [5] xliii. pp. 39-45, 1897.

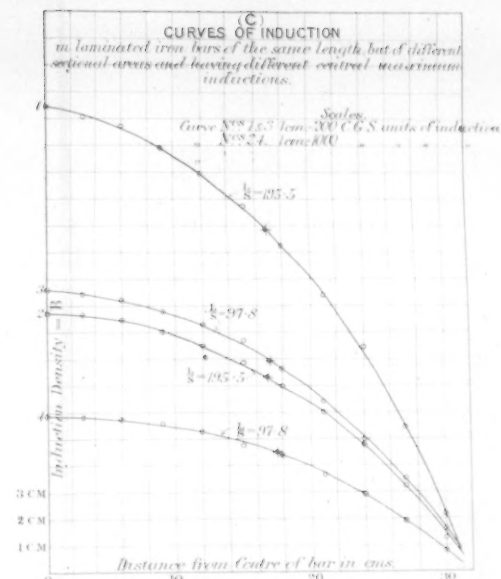
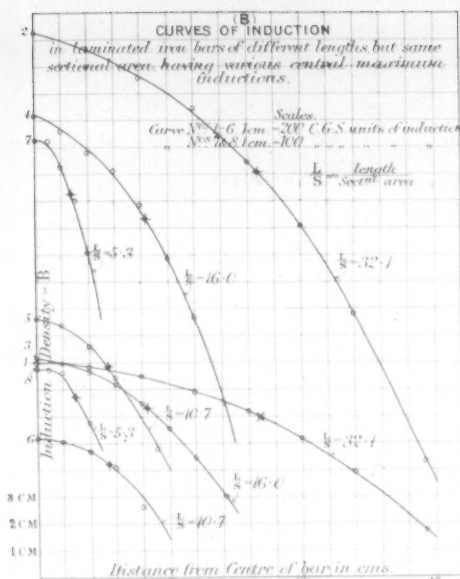
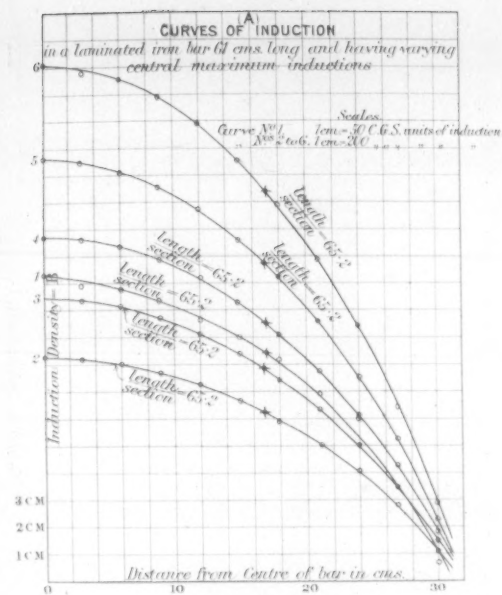


Fig. 4.

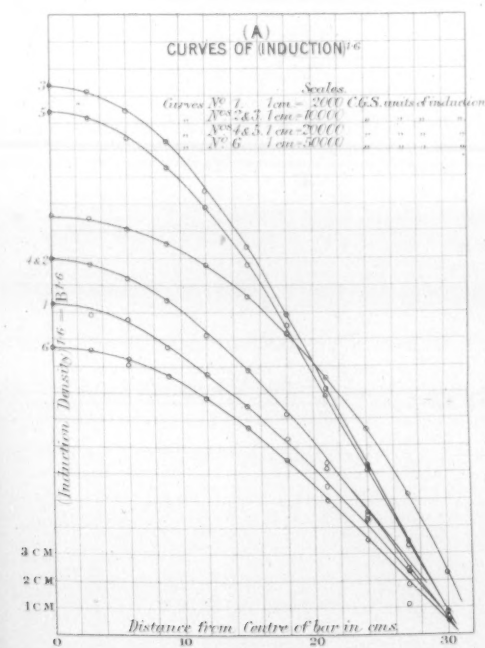
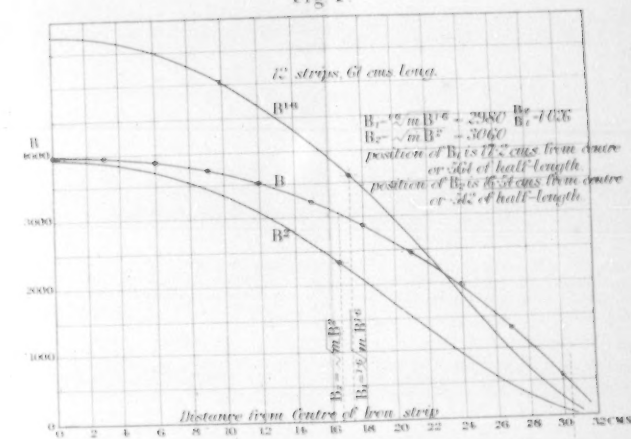


Fig. 1.

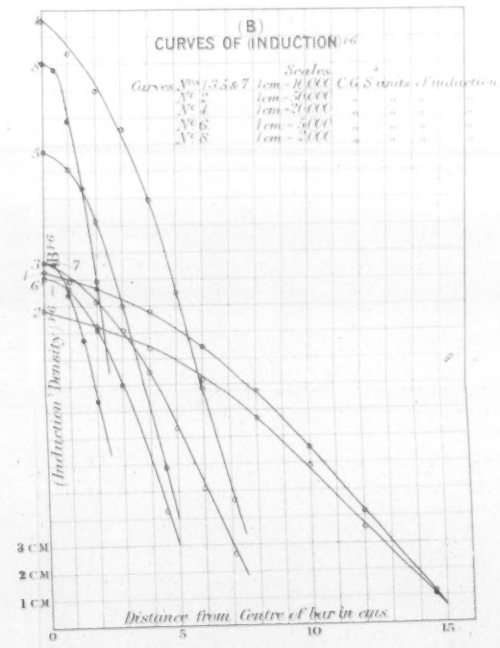


Fig. 2.

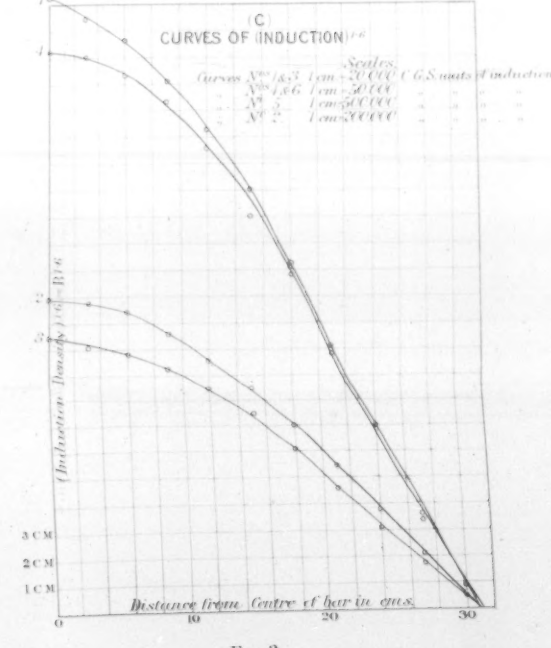


Fig. 3.

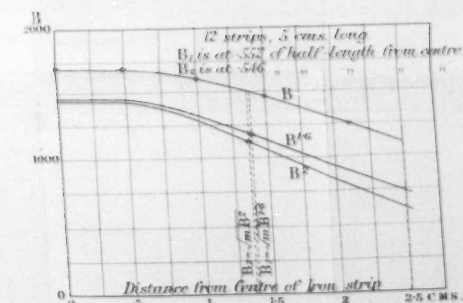


Fig. 5.

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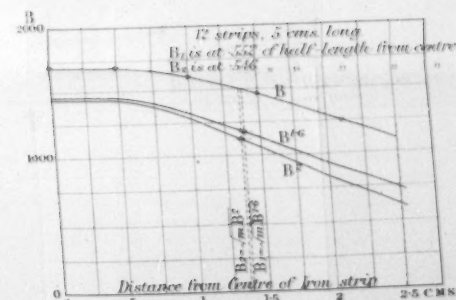
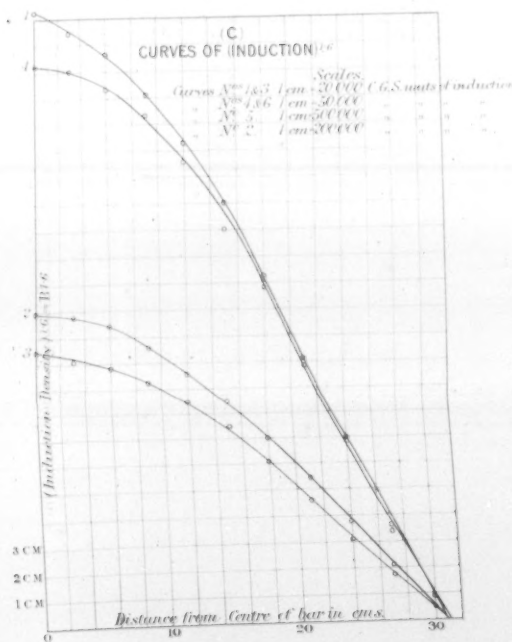
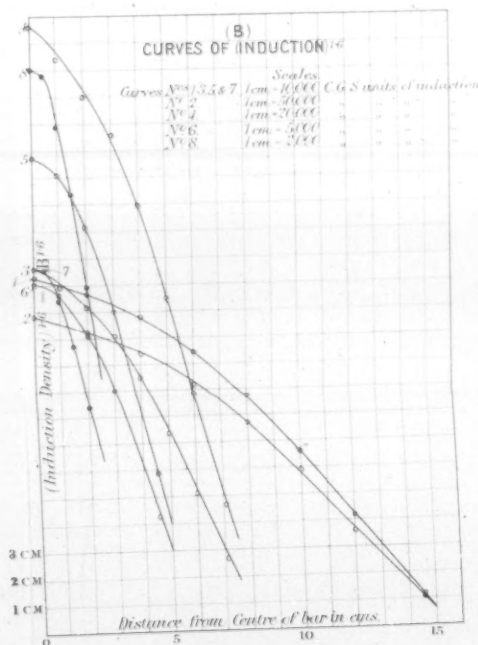
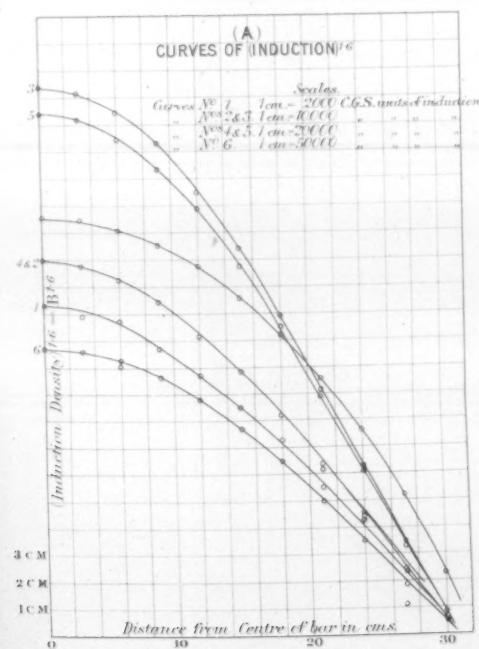
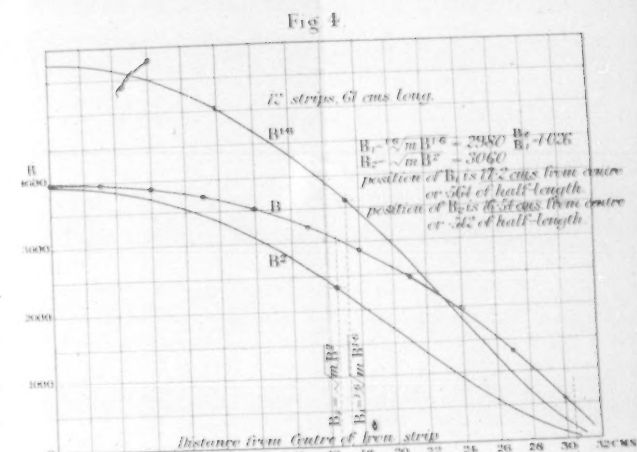
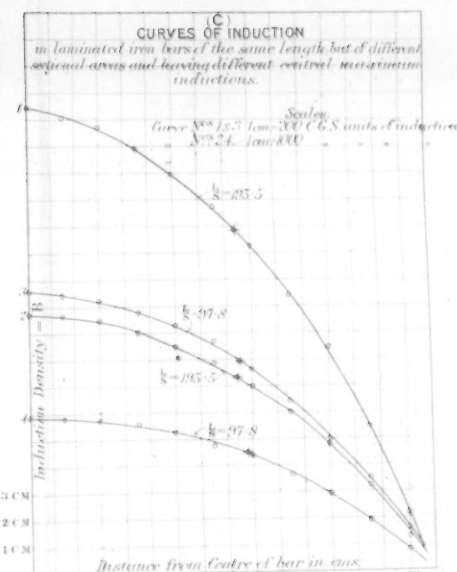
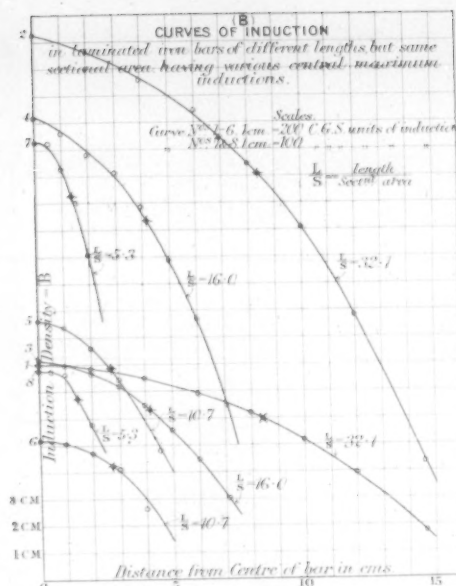
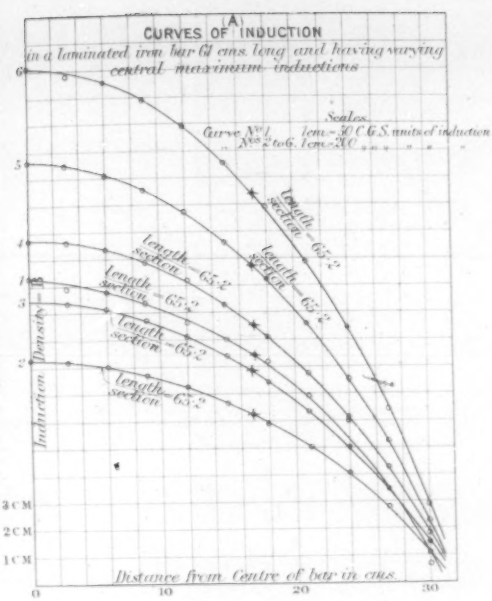
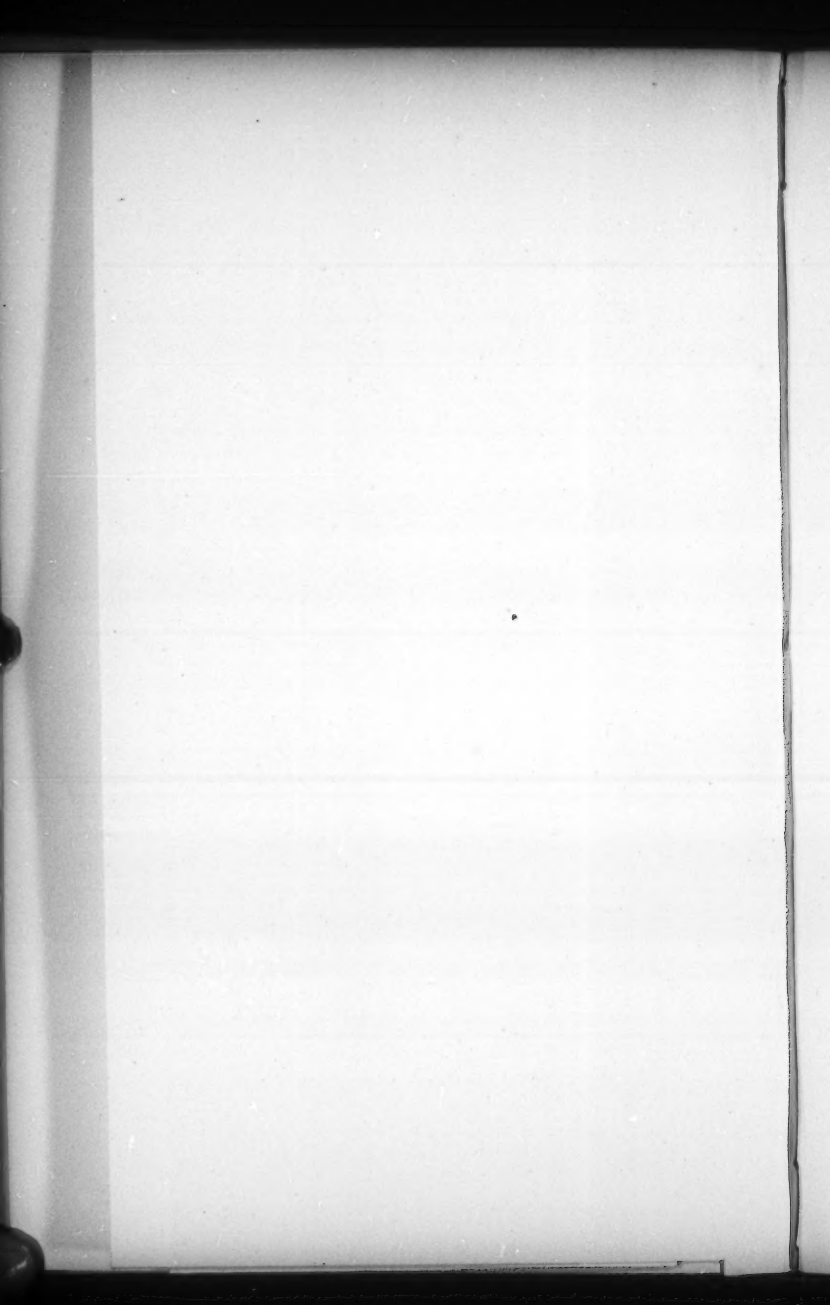


Fig. 1.

Fig. 2.

Fig. 3.

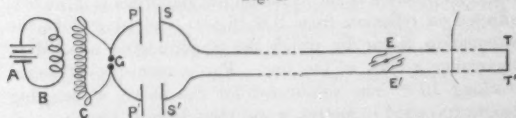
Fig. 5.



waves are absorbed at the end of the line, the electrometer is affected by the first term only of one of the above series. Hence the ratio of the electrometer-throws in the two cases is a function of the attenuation constant and the dimensions of the line, and would appear to be immediately calculable. But this is not all. For the above is on the assumption that the losses occur only along the wires themselves, whereas something is lost also on reflexion at the primary oscillator where the waves are generated. This fact soon became apparent on the repetition of the experiment with different lengths of line, and the value of this reflexion coefficient was substantially confirmed by further experiments and by the application of this new view of the case to experiments previously carried out in Bonn.

Experimental Arrangement.—The arrangement of the apparatus adopted in the determination is diagrammatically represented in fig. 1. In this figure, A denotes the battery

Fig. 1.



of two storage-cells, B and C are the primary and secondary coils respectively of the induction-coil, of which C has a resistance of 3000 ohms and an inductance of about 20 henries. G is the spark-gap, which was adjusted to 2 mm. The wire PGP', measured along the semicircle, was about 2 m. PP' are condenser-plates of zinc 40 cm. diam. placed opposite to and 30 cm. distant from the precisely similar plates SS'. The line is represented by SETT'E'S', and consists of two parallel copper wires 1.5 mm. diam., and kept 8 cm. apart by wood separators at intervals of about 2 m. EE' denotes the electrometer, which has a single plane needle, initially uncharged, and suspended by a fine quartz fibre between two disks attached to the line at E and E'. The needle is therefore electrified by induction whenever a wave passes EE', and its ends are consequently attracted to the

disks whatever the sign of their potential-difference. T T' signifies the several pieces of apparatus successively used at or near the end of the line, and will be described in detail for each case when dealt with. The lengths of the line before and after the electrometer will also be stated in connexion with each experiment. The waves generated by the primary oscillator were about 8.5 m. long.

Theory.—Let δ_1 be the electrometer-throw obtained when a completely absorbing bridge is used at T T' (fig. 1), and let δ_2 be the throw when a "no-resistance" bridge is used at T T', *i. e.*, a piece of short thick copper wire, which therefore reflects the waves without appreciable loss. Then it is required to express the ratio δ_2/δ_1 as a function of the attenuation and reflexion coefficients and then, from the experimental values of the above ratio, to solve the resulting equations for the coefficients sought.

Let the maximum potential-difference of the two wires of the line due to any wave be taken as the amplitude of that wave, let ρ be the factor by which the amplitude of a wave is affected on reflexion from S S' (fig. 1), and let $e^{-\sigma x}$ be the attenuation factor by which the amplitude is affected in traversing x cms. of the line. For convenience of actual working $10^{-\sigma x'}$ was substituted for the above, x' denoting length traversed in metres, σ was then deduced from s .

Now the throws of the electrometer are proportional to the time-integral of the square of the amplitude of the passing wave-train, *i. e.*, proportional to its energy: hence, if the lengths of the line before and after the electrometer are l_1 and l_2 respectively, and a wave-train of initial energy i leaves the oscillator, we have with the "absorber" at the end of the line

$$ie^{-2l_1\sigma} = k\delta_1, \dots \dots \dots (1)$$

where k is the electrometer constant.

Again, with the no-resistance bridge or short circuit at the end, we have two series of impulses at the electrometer, the energy of the forward waves being

$$ie^{-2l_1\sigma} [1 + \rho^2 e^{-4(l_1+l_2)\sigma} + \dots],$$

and that of the backward ones [$e^{-4l_2\sigma}$] times the above. Hence we obtain

$$\frac{ie^{-2l_1\sigma}(1+e^{-4l_2\sigma})}{1-\rho^2e^{-4(l_1+l_2)\sigma}}=k\delta_{21} \dots \dots (2)$$

whence

$$\delta_2/\delta_1 = \frac{1+e^{-4l_2\sigma}}{1-\rho^2e^{-4l_2\sigma}} = r \text{ say, } \dots \dots (3)$$

l being written for l_1+l_2 .

Two experiments with different values of l_1 and l_2 furnish two equations like (3); these are then cast in the form

$$\rho^2re^{-4l\sigma} = r-1-e^{-4l_2\sigma}, \dots \dots (4)$$

and ρ^2 eliminated between them. The result was an equation in σ of the form

$$A_1e^{-a_1\sigma} + A_2e^{-a_2\sigma} + \dots = 0,$$

or say

$$u=0.$$

Various trial values were then given to σ and plotted as abscissæ of a curve, the corresponding values of u being its ordinates. The value of σ satisfying the equation was thus found graphically as that at which the curve cut the axis of abscissæ.

Experiments.—In the experiments the bridge used as an absorber was placed absolutely at the end of the line, as it is only when in that position that it can absorb all. It consisted of pencil marks on a ground glass disk as previously described, and before use each time was carefully adjusted to the requisite resistance for absorption as mentioned in the paper already referred to. A doubt has been expressed as to the constancy of the value obtained for a resistance of this kind by the Post-Office box, some similar resistances having been said to measure differently according to the voltage used in the test. Probably this might be the case to a slight extent if extreme variations were used, but with the pencil-mark resistances employed in this work no change of value was observable, whether a single Leclanché cell was used or a battery of several storage-cells, so that the resistance was determined with all the accuracy needed for the present purpose.

The other bridge, intended to reflect undiminished the waves incident upon it, was placed a little short of the end of the line, and occupied alternately two positions a quarter of a wave-length apart. This precaution was adopted lest, far as the bridge was beyond the electrometer, the head of the wave-train reflected from the bridge might reach the electrometer before the tail had passed it. In this case a slight interference would arise at the electrometer, and its throws would not be of the values due to simple passages and re-passages of the wave-train. By using the above two positions for the bridge this source of error is obviated, for, if one gave additive interference at the electrometer, the other would give about equal subtractive interference, and their mean would be the result desired as though no interference had occurred.

The length l_2 is of course measured from the electrometer to the point midway between these two positions of the bridge.

Further, to eliminate errors due to the unavoidable irregularity of the sparking in the primary oscillator, a number of readings, about 20, with the absorber only at the end of the line were alternated with a like number of readings with the reflecting bridge interposed. The separate ratios of the electrometer-throws obtained under the two conditions were then taken, and the probable error of their arithmetic mean calculated by the theory of probabilities.

Observations and Results.—Four such sets of readings were taken with different values of l_1 and l_2 . One set is shown in Table I., and the data and results of the whole four summarized in Table II.

It may be interesting to regard, in the light of the results in the second table, two sets of observations taken by the writer in Bonn during the summer of 1893 while working under the late Professor Hertz*. In the course of those experiments (directed primarily to an object quite different from

* Proc. Roy. Soc. vol. lvii. pp. 72 & 75 (1895); D.Sc. Thesis for the University of London, 1894, pp. 20-22 and 28-29; Wied. Ann. Bd. liii. pp. 525-527 and 528-529 (1894).

TABLE II., summarizing Data and Results of Experiments.

Lengths of line		Ratio of Throws $=r$.	The unknown quantities for which the equations were actually solved.		Attenuation Constant. σ .	Reflexion Coefficient. ρ .	Remarks.
before Electrometer $=l_1$.	after Electrometer $=l_2$.		s .	ρ^2 .			
metres. 117.5	metres. 20	2.41 ± 0.04	0.000564 }	0.4776	0.0000130	0.69	As these were the best sets of observations the resulting equations were solved simultaneously.
65.0	20	2.744 ± 0.035		0.53	0.0000115	0.73	This being the worst set of readings (as seen by the large probable error) was treated alone. Any <i>num- ber</i> of pairs of values could, of course, be found that would satisfy the equation.
117.5	48	2.43 ± 0.05	0.00043 0.0005 }	0.4776	0.000010	0.69	This is the set given in Table I., and is seen to be slightly less trustworthy than those at the head of this table. Two solutions are given of this set, the first seems the more prob- able.
				0.555	0.0000115	0.745	

that now under discussion) the electrometer was placed almost at the end of the line. By an entirely different method from that now adopted, ratios were obtained for the electrometer-throws due (1) to the passage past it of the sum to infinity of the geometrical progressions formed by the outward and return waves, and that due (2) to the single passage of the initial wave-train. The lengths of the line to the electrometer in the two cases were 160 and 91.5 m. respectively, and the corresponding ratios were 1:0.34 and 1:0.31. Hence for this case equation (4) takes the form

$$\rho^2 r e^{-4l\sigma} = r - 2, \quad \dots \dots \dots (5)$$

since $l_2 = 0$.

Solving simultaneously the two equations thus derived from the experimental values of r , we obtain

$$\left. \begin{aligned} s &= 0.00027, & \rho^2 &= 0.476, \\ \sigma &= 0.0000062 & \text{and } \rho &= 0.69 \end{aligned} \right\} \dots \dots \dots (6)$$

In these experiments the primary oscillator as regards the parts SPGP'S' (fig. 1) was precisely the same as in the present paper, and it seems noteworthy that its reflexion coefficient is practically the same. The values of the resistance and inductance of the secondary circuit of the induction-coil used in Bonn cannot be stated, as they were not at the time regarded as of any material consequence and accordingly were not measured.

It will be seen that the attenuation constant in the 1893 case is about half that in the present work. This may possibly be due to the wood separators used along the line being much drier in a German summer than in an English winter and spring.

Comparisons with Heaviside's long-wave theory of reflexion.—After obtaining experimentally the unexpected result that of the wave energy incident upon the primary oscillator only about one-half or less was reflected along the line, it seemed incumbent on one to test this if possible by theory. And in this connexion I have pleasure in acknowledging my indebtedness to Mr. Oliver Heaviside for an explanation of his

theory applied to the case of reflexion from a terminal arrangement prior to its publication*.

On Heaviside's theory developed originally for long waves, say of telephonic frequency, we have for the reflexion coefficient

$$\rho = \frac{Z - Lv}{Z + Lv}, \quad \dots \dots \dots (7)$$

where Z is the resistance operator, L is the inductance per unit length of the line, and v is the speed of light.

Now the resistance operator in the case of condensers, secondary and primary coils is given by

$$Z = \frac{1}{Sp} + (R_2 + L_2p) - \frac{M^2,^2}{R_1 + L_1p}, \quad \dots \dots (8)$$

where S is the capacity of the condenser, the R 's are the resistances and the L 's the inductances of the coils, M their mutual inductance, and p the time differentiator.

Of the three terms on the right-hand side of (8), the first only is to be used if the arrangement is short-circuited at the spark-gap, G (fig. 1), the first and second only if the primary circuit of the induction-coil is open, but all three if the primary is closed.

Now in the actual experimental case the sparks at the primary oscillator are started by the break of the primary circuit of the induction-coil, the interruptor remaining open during the greater part of its period (say the $\frac{1}{30}$ of a second). But the time required for the wave-train to pass to the end of the longest line used and back again, say a dozen times (which practically extinguishes it), is only of the order 14 millionths of a second. Hence the form of (8) to be adopted as applicable to the present case is

$$Z = \frac{1}{Sp} + R_2 + L_2p, \quad \dots \dots \dots (9)$$

if indeed the long-wave theory holds at all for waves so short as 8 or 9 m.

* 'Electromagnetic Theory,' pp. 780-9; 'The Electrician,' April 9, 1897.

But, on working from equation (9) the value of ρ , we obtain a result differing from unity by less than 1 in 10^{10} .

It would appear therefore that the long-wave theory fails to apply to the present case of high-frequency waves and their reflexion from condensers and coils of high inductance, each of which involves of course the time-differentiator.

In the case of a resistance practically devoid of inductance, on the other hand, as was seen in the last paper on the absorbing bridge, the long-wave theory seems to hold within a close approximation.

Reflexion from Condenser only.—However, in order to test the matter experimentally a little further, a second set of plates like PP' SS' (fig. 1) was made and put up at the end of the circuit at TT', but this set had simply one wire in the form of a semicircle to connect the far plates instead of a spark-gap as at G. The induction-coil was also omitted entirely at this end of the line.

Hence the long-wave theory for reflexion for this arrangement gives $Z=1/Sp$ and

$$\rho = \frac{1/Sp - Lv}{1/Sp + Lv} \dots \dots \dots (10)$$

And this equation, applied to an incident wave of the type $\phi_1 \sin nt$, gives for the reflected wave $\phi' \sin (nt + \alpha)$, where

$$\left. \begin{aligned} \phi'/\phi &= \rho = 1, \\ \tan \alpha &= \frac{-2LvSn}{1 - (LvSn)^2} \end{aligned} \right\} \dots \dots \dots (11)$$

and

Here then is an opportunity of comparing simply the behaviour of the short waves with what may rightly be expected of the longer ones. For in this arrangement there is no spark-gap whose heated state in the former case leaves one in some doubt as to how far, when the waves return to it, it acts as a short circuit and so possibly cuts out the induction-coil entirely.

The experiment was now conducted by alternating electrometer readings (1) with the condenser at TT', fig. 1, and (2) with a no-resistance bridge placed across the wires just before the end of the line.

If then the condenser reflected all, as by equations (10) and (11) must be the case with long waves of the simple harmonic type whatever the capacity of the condenser, we should find the ratio of the electrometer-throws taken under the two conditions to be unity. Whereas the experiment gave the ratio of throws with no-resistance bridge to those with condenser only to be 1.26 ± 0.07 , the lengths of the line before and after the electrometer being 117.5 m. and 48 m. respectively. Applying now to this case the theory of the phenomena analogous to that developed in the early portion of this paper, and substituting for ρ^2 and s in the equation so derived the values 0.4776 and 0.000564 as shown in Table II., we obtain for the reflexion coefficient of the condensers alone

$$\rho' = 0.82 \dots \dots \dots (12)$$

This value, while differing but little from that of ρ for the condensers, spark-gap, and induction-coil at the oscillator, differs in the direction that one might anticipate. It also supports the value previously found for ρ , in that ρ' though greater than ρ is still distinctly less than unity. It will be seen from the end of the article in 'The Electrician,' April 9th, 1897, that Mr. Heaviside scarcely expected the long-wave theory to be valid for the high-frequency waves here used.

Conclusion.—The chief results arrived at may be thus summarized:—

1. The value of the attenuation constant σ for the line experimented upon and the waves in use is of the order 0.000013, where the amplitude of the waves is affected by the factor $e^{-\sigma x}$ in traversing x cm. of the line. The energy of the waves is thus reduced to half by traversing about 250 m. of the line.

2. The reflexion coefficient applicable to the primary oscillator and induction-coil as used to generate the waves is of the order 0.69, or rather more than half the energy is lost on reflexion. The reflexion coefficient of a precisely similar arrangement of condenser-plates but devoid of spark-gap and induction-coil is rather greater but still distinctly less than unity. On the long-wave theory the second of

these reflexion coefficients would be precisely equal to unity and the first of them practically so.

I have to thank Mr. H. A. Ratcliff for help at the beginning of this work, and especially Mr. R. S. Willows for his valuable assistance at the later experiments and their reduction.

University College, Nottingham,
May, 1897.

XXI. *Experiments on the Condensation and Critical Phenomena of some Substances and Mixtures.* By Dr. J. P. KUENEN*.

1. THE investigation contained in this paper is a continuation of a research on "The Condensation and the Critical Phenomena of Mixtures of Ethane and Nitrous Oxide," which was read before the Physical Society some time ago†. The results arrived at in the latter paper were stated as follows:—

(1) All mixtures of C_2H_6 and N_2O which contain more than 0.1 of C_2H_6 have critical temperatures lying beneath the critical temperature of C_2H_6 .

(2) The vapour-pressures of the mixtures lie partly above those of N_2O , and accordingly show a maximum situated near 0.2 of C_2H_6 .

(3) This maximum does not disappear with rise of temperature, but remains up to the critical region; the maximum-curve reaches the plaitpoint-curve.

(4) The mixtures ranging between 0.2 and 0.5 of C_2H_6 have retrograde condensation of the second kind, all the others retrograde condensation of the first kind.

I have now discovered two more instances of mixtures with similar properties, viz. mixtures of ethane and acetylene, and mixtures of ethane and carbonic acid. I will begin by

* Read June 25, 1897.

† Kuenen, Proc. Phys. Soc. vol. xiii. pp. 523-548.

relating some experiments which were made in connexion with the former paper, and afterwards give an account of the results obtained with those mixtures.

A. *On Ethane prepared by different methods.*

2. The ethane used in my former experiments had been prepared from sodium acetate by electrolysis. Its vapour-pressures and critical constants, especially the latter, were different from those given by other observers. The different values are the following, p being expressed in atmospheres :—

	Dewar*.	Olszewski†.	Haenlen‡.	Kuenen§.
t_c . . .	35°	34°	34·5°	32·05°
p_c . . .	45·2	50·2	50	48·8

Ethane from Ethyl Iodide.

3. In order to ascertain the probable reason of this divergence I prepared some ethane from ethyl iodide, the method used by Haenlen, and investigated its properties. The results have been communicated to the Royal Society of Edinburgh. I will therefore simply state my conclusions here so far as they bear upon the point in question.

Ethane from ethyl iodide contains a variable quantity which may go up to several per cent. of an admixture of higher critical temperature and higher density than ethane, probably butane. This impurity raises both the critical temperature and pressure, and lowers its vapour-pressures. Pure ethane is difficult if not impossible to obtain by this method.

This result accounts for Haenlen's critical values being higher than mine (and probably also for Olszewski's figures), and it shows at the same time the greater purity of my ethane. Even this was not absolutely pure, as shown by some change of the vapour-pressure with volume. But whatever the admixture may be that this was due to, it was probably too

* Phil. Mag. (5) xviii. p. 214.

† Bulletin Ac. des Sciences de Cracovie, 1889, p. 27.

‡ Lieb. Ann. cclxxxii. p. 245.

§ Proc. Phys. Soc. xiii. p. 533.

small to affect the critical temperature by more than a few tenths of a degree and the critical pressure by more than some tenths of an atmosphere.

Ethane from Sodium Acetate.

4. In order to confirm my results, and at the same time to obtain some more pure ethane for further experiments, I repeated the preparation of ethane from sodium acetate. The anode, consisting of two pieces of platinum foil (joint surface 150 cm.²), was placed inside a good-sized porous pot, care being taken to obtain a high current-density*. A current of about 15 amperes was used. Special precaution was required to prevent the indiarubber stopper which closed the porous pot from being burnt by the heat developed. A glass tube, passing through the stopper, was bent into a U-shape outside and contained some mercury. The contact with the platinum foil was made by platinum wires soldered to the foil.

The gas was washed in sodium hydrate, fuming sulphuric acid, and potash, and collected in a gasometer which was filled with a solution of potash. This gasometer consisted of two carboys, closed with indiarubber stoppers and connected by a wide glass tube. The yield of ethane was very high, very little oxygen being formed. The oxygen was moreover absorbed by introducing a piece of phosphorus into the gasometer.

5. In order to purify the gas further, it was compressed by means of a Natterer pump into a small copper cylinder fitted with high-pressure stopcocks†. In this process it was once more washed in potash and subsequently dried by calcium chloride and phosphorus pentoxide. Pump and cylinder were previously exhausted by a mercury air-pump. During the compression the copper cylinder was placed in solid carbonic acid. The pressure required to liquefy the gas was pretty high: this shows that it contained a considerable

* Murray, Journal Chem. Soc. lxi. pp. 10-36.

† Most of my high-pressure apparatus are manufactured by Giltay, Delft, Holland, and are of the same type as those used in the Physical Laboratory at the University of Leyden (Director, Prof. Kamerlingh Onnes).

quantity of some permanent gas; this may have been air which in some unknown manner had got into the gas. By thoroughly boiling the ethane, as much of this admixture was removed as possible. The results obtained with this ethane show that some admixture was left, of about the same amount as in the gas which I used in my former experiments. It is noteworthy that with other substances, such as carbonic acid and nitrous oxide, I always succeeded in removing practically all impurity by this process; for some reason the same thing seems more difficult with ethane. It is not impossible that ethane forms a mixture of constant boiling-point with the admixture, whatever it may be.

6. The apparatus which was used for filling the compression-tubes with the gases consisted almost of one piece of glass. The mercury air-pump was sealed to the double mercury gasometer which I used for preparing the mixtures (see below). This was connected at the other side with a tube leading to the compression-tube, and another tube through which the purified gas was supplied from the compression cylinder. The connexion with the cylinder was made by means of a copper tube, sealed with sealing-wax into a glass tube, the connexion with the compression tube being made by a ground joint. This was so adjusted, that the tube after being filled could be turned upright without being detached, by which manipulation the tube was shut off at the bottom by some mercury. There are two reasons why I prefer this manner of filling the tubes to sealing them off. In the first place many gases are more or less decomposed at the temperature of softened glass, and in the second place the sealing place is apt to end in a sharp point, while for the proper action of the little iron stirring-rod which I use inside the tubes, a rounded top is absolutely necessary. It need hardly be added that the apparatus was fitted with a certain number of safety-tubes dipping in mercury. I found that the amount of spring in the apparatus was quite sufficient, and that special glass springs were superfluous.

7. For the observations properly speaking, I used Ducretet's apparatus. The temperatures of air-manometer and gas-tube were kept constant with the running water of the supply. A constant water-pressure was obtained by means of an overflow

arrangement, and the current before passing round the gas-tube was heated by a long coil in warm water. A sufficiently constant temperature was obtained in this manner.

TABLE I.

Ethane from Sodium Acetate (April 1897).

t .	p_b	p_c	v_b	v_c
5.55	26.90	27.42	231.2	33.0
14.5	32.91	33.52	177.1	35.6
15.0	33.23	33.77	176.1	35.5
23.05	39.74	40.27	130.9	39.0
30.2	46.35	46.72	94.5	46.2
32.2	48.64 C		61.5	

8. Table I. gives the results obtained: p_b and v_b are the pressures and volumes when the condensation begins, p_c and v_c when the whole substance is liquefied; p is expressed in parts of an atmosphere, the v 's in this and in all the other tables are expressed in an arbitrary unit, but reduced to the same initial volume at 0° and 1 atmosphere. C indicates the critical point. The experimental determination of the p_c 's gives no difficulty if the stirring-rod is continually used. For p_b this is not always the case, because stirring is not so effective when the volume is much larger. In cases of doubt p_b and v_b were therefore determined graphically as the points of intersection in the p - v diagram of the two parts of the isothermal, the part where the substance (mixture) is homogeneous and the part where there are two phases. Another advantage in drawing the latter part of the isothermal is that from its shape and the direction of its curvature some idea may often be obtained as to the nature of the admixture*.

9. Tables II. and III. contain values obtained for two samples of ethane on a former occasion. The pressures as shown by a comparison of the p - t curves are this time slightly

* Compare a paper on ethane in the Proc. R. S. of Edinburgh, 1897. I thought it unnecessary to publish the p - v diagrams as they offer no special interest.

TABLE II.
Ethane. (April 1895.) 1st sample.

t .	p_b .	p_c .
5.85	27.38	27.92
10.65	30.45	31.11
15.4	33.84	34.48
22.4	39.73	40.25
29.35	45.94	46.25
31.0	47.64	47.95
31.95	48.81 C	

TABLE III.
Ethane. (April 1895.) 2nd sample.

t .	p_b .	p_c .
3.95	26.17	26.89
15.4	34.11	34.66
22.4	39.89	40.32
29.3	45.89	46.19
32.05	48.91 C	

lower and the critical temperature a little higher. Absolutely pure ethane would probably have pressures slightly lower even and a slightly higher critical temperature. The p - t curves in the diagrams for ethane and the other single substances are drawn with p_b ; as this value is probably nearer the true value of p than p_c .

B. Mixtures of Carbonic Acid and Acetylene.

10. In my former paper I gave a first instance of mixtures having critical temperatures below those for the component substances. The only instance of critical temperatures above those of the components seemed to be that of mixtures of carbonic acid and acetylene. According to an observation of Dewar's *, a mixture of $\frac{1}{4}$ CO₂ and $\frac{1}{2}$ C₂H₂ has a critical temperature of 41° C., those for carbonic acid and acetylene being 31° and 37° respectively. It seemed necessary to try the experiment, because experiments on mixtures have always

* Proc. R. S. of London, xxx. p. 542.

to be accepted with caution, unless it appears that exceptional precautions have been taken to avoid retardation and impurities*. The phenomenon, if confirmed, would have been worth completely investigating. The results which I have obtained contradict Dewar's experiment.

Preparation of Carbonic Acid.

11. Both carbonic acid and acetylene were prepared before I was in a position to apply the method of purification described for ethane by condensation and distillation. I had therefore to resort to chemical methods of purifying the gases. Carbonic acid was made from sodium bicarbonate and sulphuric acid. The acid was contained in a boiling-flask provided with a dropping-funnel with the sodium bicarbonate solution. All solutions used were previously boiled and cooled in a current of carbonic acid. The gas passed through sulphuric acid and phosphorus pentoxide. Before the solution was admitted the whole apparatus had been exhausted with a mercury air-pump.

12. The purity of the carbonic acid was first tested in the compression-apparatus and appeared to be quite satisfactory.

TABLE IV.
Carbonic Acid.

t .	p_b .	p_c .	Amagat.
14.95	50.02	50.24	50.0
20.5	57.16		57.0
25.85	64.92	65.06	64.5
31.1		73.26	C

Amagat's values for the critical point are $31^{\circ}35$, 72.9 atm. The agreement is very close, the deviations of my values

* Prof. Dewar, on learning that I had repeated this experiment of his (forming part of an essentially qualitative investigation carried out in 1880), told me that his acetylene contained C_2H_2Br . This accounts for the high critical temperature and low vapour-pressures observed by him.

being probably due to a slight admixture of permanent gas. This would, in fact, make the vapour-pressures and critical pressure higher and the critical temperature lower. Similar deviations will be noticed by comparing some of Andrews's values with Amagat's, the former having been reduced to atmospheres.

<i>t.</i>	Andrews.	Amagat.
13.09	48.05-49.50	47.6
21.46	58.94-61.35	58.2

Andrews's critical constants are 30°·9 and about 73 atmospheres. The deviations are in the same direction, only much more pronounced. Andrews's carbonic acid contained about $\frac{1}{500}$ of air.

Preparation of Acetylene.

13. This gas was prepared in the same apparatus as carbonic acid, from calcium carbide and water. The water had been previously saturated with carbonic acid. The gas was washed in silver nitrate to absorb the phosphoretted hydrogen and sulphuretted hydrogen which it contained. It was freed from carbonic acid by soda-lime and dried in sulphuric acid and phosphorus pentoxide. Two samples were obtained. The first probably contained some air and also some phosphoretted hydrogen. In preparing the gas for the second time, a second washbottle with silver nitrate was introduced, and this bottle was only just beginning to get blackened, when the acetylene was collected. The only possible impurity of this sample is therefore a trace of air.

Some of this sample was used in making the mixture with CO₂, the results for which are communicated below. Some mixtures of the first sample with CO₂ gave, however, similar results. The results obtained with the purer sample are contained in Table V.

14. Evidently the gas contained an admixture of some other substance. If this admixture is air, the vapour-pressures are slightly higher than for pure acetylene, and the critical

temperature a few tenths of a degree too low. In this case also the p_b 's were used in drawing the vapour-pressure curves in the diagrams.

TABLE V.

Acetylene.

t .	p_b .	p_c .
14.95	38.09	38.66
20.15	43.18	43.75
26.8	50.29	50.85
27.1	50.55	51.12
30.8	55.05	
35.25	61.02 C	

The pressures agree very well with those given by Villard *, who finds 37.9 at 15° C., and 42.8 at 20° 2. Ansdell's † values are a good deal different, his pressures being 32.77 at 13° 5, 39.76 at 20° 15, 48.99 at 27° 55, 56.20 at 31° 6. For the critical point he found 68.0 atmospheres and 36° 9 C. As to Pictet's ‡ values, they show no resemblance whatever with the others. His pressures were: 29.0 at 14° 1, 33.5 at 19° 5, 38.5 at 27° 6, and 48.0 at 36° 5 (critical point).

Mixture of Carbonic Acid and Acetylene.

15. The mixture was made in a mercury gasometer consisting of two parallel wide glass tubes of 300 c.c. each, provided at the bottom with a tap, an indiarubber tube, and a funnel with mercury sufficient to fill up the whole of the tubes. At the top the tubes ended in threeway stopcocks communicating together, the whole, as described before, connected on one side with the pump, and at the other side with the rest of the apparatus. The mixture contained $\frac{1}{2}$ C₂H₂ and $\frac{1}{2}$ CO₂.

* *Ann. de Chimie et Physique*, March 1897.

† *Proc. R. S. of London*, xxix. p. 211 (1879).

‡ *Zeitschrift Compr. und flüssige Gase*, i. p. 32 (1897).

16. The results obtained are the following :—

TABLE VI.
Mixture $0.5 \text{ C}_2\text{H}_2 - 0.5 \text{ CO}_2$.

t .	p_b .	p_c .
7.35	35.98	36.90
14.95	43.46	44.62
20.15	49.54	
24.75	55.18	56.10
28.15	57.10	57.95
31.55	64.79	65.17
32.55	66.52 C	

The differences between p_b and p_c (fig. 1), the pressures at the beginning of the condensation and at the end, are somewhat larger than for acetylene, as might be expected. It would be possible to make an allowance for the impurity of the component gases, and in that manner obtain approximate corrected values for the pressures. This would, however, be a rather uncertain calculation, and in drawing the border-curve for this mixture I have preferred to use the direct experimental values. Obviously, though the slight impurities of the CO_2 , and specially of the C_2H_2 , also affect the values for the mixture, the divergence from the true values can only be of the same order of magnitude as for the components.

17. The diagram (fig. 1) shows that the plaitpoint curve is a line with small curvature and that mixtures of carbonic acid and acetylene have critical temperatures between those for the pure gases. The border-curve for the mixture investigated is relatively narrow. This entails small differences in composition between the coexisting phases, the vapour containing only a little more carbonic acid than the liquid. It also follows that the critical phenomena will closely resemble those of a pure substance. This was found to be the case, the plaitpoint temperature and critical temperature nearly coinciding.

18. An instance of a critical temperature above those for the components has therefore not been discovered as yet. From the theory it may be deduced that this phenomenon

Fig. 1.
(Vide Tables IV., V., VI.)

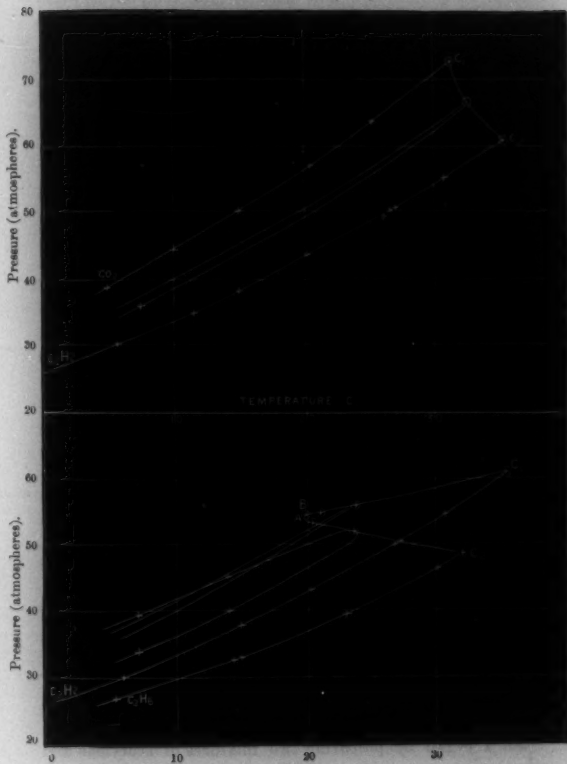


Fig. 2.
(Vide Tables I., V., VII.-XI.)

will probably occur for mixtures which have a minimum vapour-pressure at low temperature; in the same way as the critical temperatures below those for the components seem to occur for mixtures which have a maximum vapour-pressure (as for nitrous oxide and ethane).

The law of the connexion between the two phenomena will be again referred to in connexion with the application of Prof. van der Waals's theory.

C. Mixtures of Acetylene and Ethane.

19. The relative position of the vapour-pressure curves and critical points in the t - p diagram of these gases was such as to promise interesting results for mixtures of them: in fact the same reasoning holds for this combination which led me originally to the choice of nitrous oxide and ethane. If we consider the curves for the four gases the critical temperatures of which are between 30° and 40° — CO_2 , C_2H_6 , C_2H_2 , and N_2O —we find that besides with the combinations C_2H_6 - N_2O and C_2H_6 - C_2H_2 we may expect similar phenomena with C_2H_2 - N_2O , a combination which I have not been able to try. Results obtained with the couple C_2H_6 - CO_2 will be described later on. They agree with those of N_2O - C_2H_6 , and C_2H_2 - C_2H_6 . The case C_2H_2 - CO_2 was discussed above. The combination CO_2 - N_2O offers no interest *a priori*. Moreover, some time ago I investigated a mixture of these two gases, but did not obtain any remarkable results.

Mixtures of some of these four gases with hydrochloric acid (51°) or methyl fluoride ($44^\circ\cdot9$) also promise new results. Mixtures of hydrochloric and carbonic acids were investigated by Dewar* and Ansdell†. Their figures differ from each other, but the plaitpoint curve does not seem to offer any peculiarities. It is hardly necessary to add, that a number of combinations of substances at other temperatures might be mentioned which would probably display similar phenomena.

20. I began by preparing two mixtures of acetylene and ethane containing 0·68 and 0·27 of the latter gas respectively.

* Proc. R. S. of London, xxx. p. 543.

† *Ibid.* xxxiv. pp. 113-119.

After having got the constants for these two and not having any more acetylene available at the time, I made new mixtures by partly exhausting the compression-tube containing the 0.27 C_2H_6 mixture and introducing fresh quantities of ethane. In this manner four more mixtures were obtained. Though great care was taken not to introduce any traces of air into the tubes by this repeated process, it is not a safe method, and the results seem to show, that some impurity must have got into the mixture. Moreover, the calculation of the composition of each fresh mixture was again somewhat uncertain. The consequence is, that the last mixture prepared in this manner, which, according to the calculations, contained 0.76 C_2H_6 , showed higher changes of vapour-pressure ($p_c - p_b$) than the first mixture ($x=0.68$), and at the same time a slightly lower critical temperature and higher critical pressure. All this does not agree with the position and shape of the plaitpoint curve. The results for this mixture had therefore to be rejected. This also makes the values for the other three mixtures (especially the last one) doubtful. It must, however, be understood that this small uncertainty does not make the general result in the least doubtful, only the curves would be slightly different for the mixtures if pure.

TABLE VII.
Mixture 0.68 C_2H_6 -0.32 C_2H_2 .

t .	p_b .	p_c .	v_b .	v_c .
6.97		39.48		33.9
7.05	34.1		172.5	
13.45	40.35	45.25	134.3	37.2
22.2	49.17	51.83	88.2	46.5
23.05		52.35		50.2
23.15		52.40		50.7
23.35		52.41		54.3
23.4		52.32 P		57.4 P
23.45	51.31		72.9	
23.5	51.33		72.9	
23.55		52.29		59.1
23.6		52.22		60.2
23.65	51.72	52.16	67.2	60.3
23.75		52.06 C		65.1

TABLE VIII.
Mixture 0.61 C_2H_6 -0.39 C_2H_2 .

t .	p_b .	p_c .	v_b .	v_c .
8.85	39.6	44.04	136.6	34.3
13.97	44.6	48.42	112.9	37.3
20.15	52.10	53.68	74.6	48.4
20.55	52.61	53.68	70.6	52.4
20.75	53.06	53.47 P	67.5	59.5
20.77	53.4?	C	62.5	

TABLE IX.
Mixture 0.51 C_2H_6 -0.49 C_2H_2 .

t .	p_b .	p_c .	v_b .	v_c .
8.4	40.87	45.60	130.4	33.1
14.0	46.4	50.50	106.3	36.1
19.35	53.27	55.02	73.2	46.1
19.75	54.10	55.02	65.2	51.2
19.85		54.87 P		54.4
19.9	54.24		64.4	
19.95	54.57	54.72	60.7	57.7
19.96	54.65?	C	58.2	

TABLE X.
Mixture 0.41 C_2H_6 -0.59 C_2H_2 .

t .	p_b .	p_c .	v_b .	v_c .
7.95		42.70		31.4
8.0	39.95		136.2	
14.0	46.1	48.19	107.3	34.6
19.35	52.0	53.67	83.1	40.6
20.95	54.50	55.11	67.7	47.9
21.0	54.62	55.12		50.0
21.05		55.12		50.6
21.1		55.05 P		53.7
21.12		55.02		55.2
21.15	54.99	C	55.1	

TABLE XI.
Mixture 0.27 C_2H_6 -0.73 C_2H_2 .

t .	p_b .	p_c .	v_b .	v_c .
7.6	38.25	39.61	148.4	29.7
13.95	44.63	45.64	114.5	31.7
22.2	53.9	54.55	75.7	39.0
23.75	56.05	56.27	61.7	47.7
23.8		56.25		51.1
23.85	56.23	C	55.4	

21. The Tables VII.-XI. contain the results for the five mixtures; the diagram (fig. 2) shows the relative position of the vapour-pressure curves for the pure substances, the border-curves for the mixtures, and the plaitpoint curve which envelops the border-curves. Only two of the border-curves, for mixtures 0.68 and 0.27, have been drawn in order not to make the diagram too complicated.

The diagram is qualitatively the same as that for nitrous oxide and ethane: quantitatively there is a difference in the circumstance that the mixture of minimum critical temperature and the mixture of maximum vapour-pressure are much nearer to each other than in the other case. Mixture 0.51 was about the mixture of maximum pressure (point B), while the minimum critical temperature (point A) occurs for a mixture containing only a little more ethane ($x=0.53$ or 0.54). The same points B and A were much farther apart for $N_2O-C_2H_6$, where they belonged to mixtures $x=0.2$ and 0.5 respectively.

22. It was stated in my former paper that the critical phenomena of the mixtures of N_2O and C_2H_6 are little marked. Between C and A, and also between C and B, the condensation should be retrograde of the first kind between the two critical temperatures t_p and t_c ; between A and B the condensation should be retrograde of the second kind. The latter phenomenon has never yet been observed owing to disturbing circumstances such as gravitation and small impurities. For $C_2H_2-C_2H_6$ the chances of observing special critical phenomena between A, B are obviously even smaller than for $N_2O-C_2H_6$.

I succeeded this time in observing critical phenomena of the first kind with all five mixtures. This is chiefly owing to the temperatures being more constant than formerly. The phenomena were very marked for the mixtures 0.68, 0.61, and 0.51, much less marked with mixtures 0.41 and 0.27. Though the relative position of P (plaitpoint) and C (critical point) is the same along the curves C_1B and C_2A , this is not the case as regards the point of maximum pressure M. Between B and C_1 , M is between P and C; along AC_2 , P is between M and C. The results with mixtures 0.68, 0.61, and 0.27 are in accordance with this rule; mixture 0.41 shows M

to be near P but on the wrong side; mixture 0.51, which is about the mixture of maximum vapour-pressure, should have one critical point; evidently it contained some other admixture. Moreover, the differences between p_b and p_c are considerable for this mixture, while they should be very small, not larger than for the components used. Altogether the results show that at least some of the mixtures contained a small admixture of air. The figures are therefore less trustworthy than those for $N_2O-C_2H_6$ or those for $CO_2-C_2H_6$, the combination to be described presently.

The $v-x$ curves have the same character as those for $N_2O-C_2H_6$: it seems therefore unnecessary to publish them.

D. Mixtures of Carbonic Acid and Ethane.

23. The ethane used in these experiments was the same as for the mixtures with acetylene. The carbonic acid was taken from an ordinary carbonic-acid cylinder, after having been distilled and purified in a small high-pressure cylinder in the way as described for ethane (see § 5).

TABLE XII.
Mixture 0.50 C_2H_6 -0.50 CO_2 .

t .	p_b .	p_c .	v_b .	v_c .
8.8		47.47		32.2
8.85	45.74		113.8	
9.05	45.94		111.8	
9.1		47.74		32.4
14.95	52.94	54.10	82.6	37.1
17.55	56.63	57.18	62.4	44.8
17.75	56.90	57.24	60.8	51.4
17.85	56.99	57.15	59.9	55.9
17.88	57.10		57.4	

TABLE XIII.
Mixture 0.43 C_2H_6 -0.57 CO_2 .

t .	p_b .	p_c .	v_b .	v_c .
8.95	47.27	48.46	106.8	30.7
14.95	54.50	55.30	77.4	35.5
17.55		58.42		44.8
17.58	58.18		57.9	
17.62	58.37		52.7	

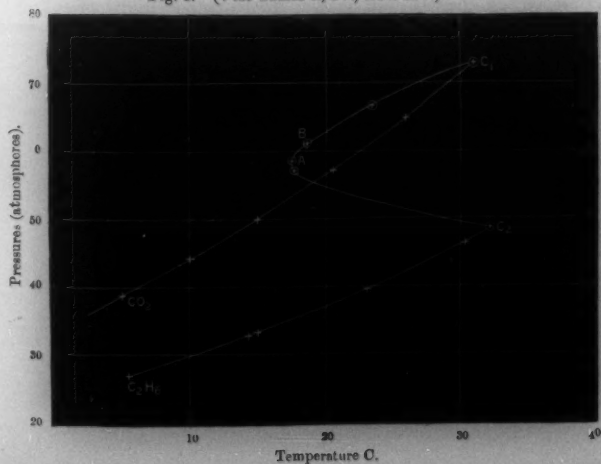
TABLE XIV.
Mixture 0.30 C_2H_6 -0.70 CO_2 .

t .	p_b .	p_c .	v_b .	v_c .
8.95	48.58	49.07	106.0	28.8
14.95	55.82	56.28	79.8	32.2
17.28	59.02	59.31	66.9	35.0
18.68		61.19		41.4
18.69	61.14		53.3	
18.73		61.28		42.8
18.75	61.21	61.31	52.5	46.1
18.77		61.29		
18.80		61.26		48.5

TABLE XV.
Mixture 0.15 C_2H_6 -0.85 CO_2 .

t .	p_b .	p_c .	v_b .	v_c .
10.35	48.84	49.46	111.2	25.4
16.0	55.72	56.32	90	27.2
23.15		66.32		37.7
23.2	66.15		52.9	
23.35	66.44	66.52	49.6	41.8
23.37		66.57		43.2
23.4		66.54		47.6

Fig. 3. (Vide Tables I., IV., XII.-XV.)



24. These mixtures (Tables XII.-XV., fig. 3) show again the phenomena of minimum critical temperature and maximum vapour-pressure which had been found with mixtures of ethane with N_2O and C_2H_2 . The minimum critical temperature occurs for a mixture $x=0.45$ about (A), the maximum vapour-pressure for $x=0.30$ approximately (B). These compositions are nearer together than for $N_2O-C_2H_6$, but not so near as for $C_2H_2-C_2H_6$.

25. Again, the critical phenomena characteristic of mixtures were very little marked. As just stated, mixtures containing ethane between the limits 0.30 and 0.45 ought theoretically to show the critical phenomena of the second kind. For mixture 0.43 points P and C were, however, so near together that I could not observe a difference. For mixture 0.50 P is between M and C; for mixture 0.30 M and P coincide, and very nearly also with C; for mixture 0.15 M seems to lie between P and C. All this agrees with the rules given above. The differences between p_b and p_c are very small, especially for the maximum mixture. This also is in accordance with theory. The results for this combination are more trustworthy than those for $C_2H_2-C_2H_6$, and as good or better than those for $N_2O-C_2H_6$.

The v - x diagram (not published) has the same character as for the two other combinations.

E. Application of Van der Waals's Theory*.

26. If taken as generally as possible, van der Waals's theory of mixtures depends on the assumption of the existence of a relation between p , v , and t for a mixture of the same character as for pure substances. The p - v curve must be supposed to have an unstable part below a certain temperature, and in changing the composition of the mixture gradual changes in the relation between p , v , and t should take place. The laws of the condensation, the critical phenomena, in general of the coexistence of phases are then found by an application of thermodynamics in one of its many forms: van der Waals

* *Archives Néerl.* xxiv. pp. 1-56; *Zeitschrift physik. Chemie*, v. pp. 133-173; *Kon. Akad. van Wet. Amsterdam*, Nov. 24, 1894, pp. 133-137; *ibid.* Mei 25, 1895, pp. 20-30; Juni 29, 1895, pp. 1-12.

used the ψ function ($\psi = \epsilon - T\eta$). (T = absolute temperature.) In fact wherever thermodynamics is used for purposes of this kind, the existence of a similar equation of condition must be presupposed, though the actual form of this equation can only be given in the simplest cases, and even then only qualitatively. It is well known that the theoretical equation for single substances given by van der Waals does not agree with the results of experiments, and the same is true for the analogous equation for mixtures of two substances which he uses in his theory. It is well, however, to bear in mind that a number of the results which he arrives at are independent of the form of the equation. Even those results which are not, still may have a resemblance to actual phenomena in the same manner as for single substances.

In the following account of some of the theoretical results I will only consider the case of the coexistence of vapour and liquid, *i. e.* a separation of the liquid into two will be excluded from the consideration.

I shall sometimes follow a method different from van der Waals's, chiefly for the sake of shortness and simplicity.

27. Let us suppose a set of isothermal curves (p - v diagram) to be drawn for the mixture as determined by observations of p , v , and t in homogeneous states, and completed by partly unstable interpolation-curves joining the curves in the homogeneous gaseous and liquid condition.

Suppose we forget for a moment that the substance is a mixture, and apply the Maxwell-Clausius criterion for finding the coexisting phases in the usual manner. We should then get the border or saturation-curve separating the homogeneous states from those where these are unstable or less stable than the separated states. The diagram thus obtained gives us the condensation-pressures, the vapour- and liquid-densities, and the critical point for the mixture, if supposed to behave like a pure substance, *i. e.* not dividing into phases which have a different composition. The real border-curve can only be found by a more complicated application of thermodynamics: still we may easily see one thing, that the real border-curve would lie entirely outside the hypothetical one just obtained. Instead of the horizontal line cutting off equal areas from the isothermal, we get a sloping line: this

line has again to cut off equal areas. Obviously this is only possible by making it start lower and finish higher than the straight line. This is true at all temperatures, and the conclusion is, as stated, that the border-curve will be entirely outside the hypothetical one. From this it follows, *e. g.*, that the real critical temperature is higher than the critical temperature for the undivided mixture; also that the critical isothermal does not touch the border-curve at the top M but at a point to the right C, where $\frac{dp}{dv} < 0$ instead of $= 0$ as for a single substance. The volume in the critical point is obviously larger and the pressure smaller than the same quantities at the top of the border-curve.

28. The last fact about the critical pressure (point C) not being the maximum pressure (point M) for the mixture on the border-curve was pointed out above with reference to the p - t diagram. In fact the loops for the mixtures are nothing but the real border-curves represented in a different manner. The third point P which we considered in that case (plait-point), the point of contact of the loop and the enveloping curve (plaitpoint curve), cannot be found by a simple geometrical construction in the p - v diagrams. The connexion between the latter diagrams for different mixtures is not so simple as the connexion between the loops (border-curves) in the p - t diagram. A point of intersection of two loops gives the pressure and temperature at which the two mixtures to which the loops belong will coexist as vapour and liquid*. The enveloping-curve contains the points of intersection of loops infinitely near each other; in other words, it gives the conditions of pressure and temperature of the coexistence of identical phases: at those points the mixtures are therefore in the critical condition, and will display the critical phenomenon. Evidently these points P do not in general coincide with either M or C. At the critical point C the mixtures do not show the critical phenomenon, the latter belonging to P, where the temperature is lower. This was fully explained on former occasions.

* This is not so when a mixture of maximum or minimum pressure is between the two mixtures.

29. The properties of the v - x diagram were sufficiently discussed in my previous paper. Also its meaning as projection on the v - x plane of curves on the ψ surface studied by van der Waals. It was found in the first place that the maximum mixture in its critical point behaves entirely as a single substance; this fact would appear in the p - v diagram by the critical isothermal touching the border-curve at the top, and by points M, P, and C coinciding. If the maximum mixture is the same at all temperatures (as is approximately realized by N_2O - C_2H_6 within the limits of temperatures used), the real border-curve coincides entirely with the hypothetical border-curve examined above. If the maximum mixture changes with temperature the two curves coincide at the top only.

30. The consequence of this coincidence is that the critical point of the maximum mixture may be determined in exactly the same way as for a single substance: the conditions being

$$\frac{dp}{dv} = 0, \quad \frac{d^2p}{dv^2} = 0.$$

The direction of the vapour-pressure curve of the maximum mixture at B (*i. e.* the direction of the maximum-curve at that point) is also the same as for a single substance. This direction may be easily proved to satisfy the equation

$$\left(\frac{dp}{dt}\right) = \left(\frac{\partial p}{\partial t}\right)^*,$$

* This equation is true for single substances at the critical point, and for the maximum mixture at B; for other mixtures it would hold at the hypothetical critical point for the homogeneous mixture. The hypothetical vapour-pressure curve for a mixture lies in between the two branches of the loop in the p - t diagram, and comes to an end at this hypothetical critical point at some distance from the plaitpoint-curve. The equation consequently does not hold for a mixture at its plaitpoint. It seems therefore impossible to identify the expression $\left(\frac{\partial p}{\partial t}\right)_{sx}$ with the $\frac{dp}{dt}$ of the hypothetical vapour-pressure curve at the plaitpoint of an arbitrary mixture, although this is done by van der Waals in his interpretation of the equation which gives the difference of the value of $\left(\frac{dp}{dt}\right)_p$ and $\left(\frac{\partial p}{\partial t}\right)_{sx}$ (*Kon. Ak. Amsterdam*, Mei 25, 1895, pp. 13, 30; Juni 29, 1895, p. 3, form. (5)).

the index indicating the quantity which is kept constant in the differentiation. Van der Waals* has discovered that for single substances $\frac{T}{p} \frac{dp}{dt}$ is approximately equal to 7 at the critical point. It is an interesting confirmation of the theory that my maximum-curves also give a value about equal to 7 at point B.

31. An important question in connexion with this is, What is the direction of the plaitpoint-curve at B? In my former paper this point was left undecided. In the meantime Prof. van der Waals, to whom I showed my results, has published two papers† in which the properties of both points B and A are completely discussed. I will deduce here some of his results, sometimes using his equations in a different manner.

32. For $\frac{dp}{dt}$ of the plaitpoint-curve the following may be written:—

$$\left(\frac{dp}{dt}\right)_P = \left(\frac{\partial p}{\partial t}\right)_{ex} + \left(\frac{\partial p}{\partial v}\right)_{tx} \left(\frac{dv}{dt}\right)_P + \left(\frac{\partial p}{\partial x}\right)_{et} \left(\frac{dx}{dt}\right)_P.$$

At B both $\frac{\partial p}{\partial v}$ and $\frac{\partial p}{\partial x}$ will be seen to be = 0. This follows immediately from the conditions of the v - x diagram, $\frac{\partial p}{\partial v}$ being = 0, because the line of equal pressure always touches the connodal curve at the plaitpoint, and is therefore parallel to the v -axis in this particular case; while $\frac{\partial p}{\partial x} = 0$, because the point has still the properties of a maximum pressure-point. Neither $\frac{dx}{dt}$ nor $\frac{dv}{dt}$ can be ∞ along the plaitpoint-curve at B, because that would mean that the temperature went through a maximum or minimum. The result is that at B

$$\left(\frac{dp}{dt}\right)_P = \left(\frac{\partial p}{\partial t}\right)_s = (\text{see above}) \left(\frac{dp}{dt}\right) \text{ along maximum curve,}$$

i. e. the maximum curve touches the plaitpoint-curve at B.

* "Continuity" &c., Supplementary Notes; *Kon. Ak. Amsterdam*, Mei 25, 1895, p. 27.

† *Kon. Ak. Amsterdam*, Mei 25, 1895, pp. 20-30; Juni 29, pp. 1-12.

The diagram in my former paper is therefore incorrect, as the plaitpoint-curve was drawn with a break at B.

33. As to point A, it may be proved that $\left(\frac{dp}{dt}\right)_P$ is infinite there. We must use for this the conditions which hold for the point where a plait is split up. A plaitpoint is a point of the connodal curve on the ψ surface, but at the same time of the spinodal curve. The connodal curve is the curve traced out by the double tangent-plane: the spinodal curve, which is inside the connodal curve, separates the part of the surface where the substance is stable from the unstable part. The equation of this curve is

$$A = \frac{\partial^2 \psi}{\partial v^2} \frac{\partial^2 \psi}{\partial x^2} - \left(\frac{\partial^2 \psi}{\partial x \partial v} \right)^2 = 0.$$

Inside the curve $A < 0$, outside $A > 0$.

Applying the condition that $A=0$ in the plaitpoint, we have

$$\left(\frac{dA}{dt}\right)_P = \left(\frac{\partial A}{\partial t}\right)_{xz} + \left(\frac{\partial A}{\partial x}\right)_{vt} \left(\frac{dx}{dt}\right)_P + \left(\frac{\partial A}{\partial v}\right)_{xt} \left(\frac{dv}{dt}\right)_P = 0.$$

Now at A $\frac{\partial A}{\partial t}$ is > 0 , because just before the division of the plait $A < 0$ in that point, and just after $A > 0$.*

Moreover we have at point A $\left(\frac{\partial A}{\partial x}\right)_{vt} = 0$, and $\left(\frac{\partial A}{\partial v}\right)_{xt} = 0$.

This will be seen to be true on considering that in the direction of the x -axis A passes from negative through 0 back to negative, in the direction of v from positive through 0 back to positive. The conclusion we come to is that either

$\frac{dx}{dt}$ or $\frac{dv}{dt}$, or both, are infinite at A. The last alternative is the true one; but either of the three is sufficient for our purpose. If we put in the general equation for $\left(\frac{dp}{dt}\right)_P$ in (32)

$\left(\frac{dx}{dt}\right)_P = \infty$, we find $\left(\frac{dp}{dt}\right)_P = \infty$, because at A neither $\frac{\partial p}{\partial x}$ nor $\frac{\partial p}{\partial v}$ is 0, the direction of the p line not coinciding with either the v or the x axis.

* A more rigorous proof may be given by working out $\frac{\partial A}{\partial t}$: compare van der Waals, *l. c.* Juni 29, p. 2.

34. From $\left(\frac{dp}{dt}\right)_p = \infty$ at A it follows, that the plaitpoint curve has a vertical tangent at A and the loop for the mixture of minimum critical temperature touches the plaitpoint curve at A in the ordinary manner. In my former paper I drew the plaitpoint curve with a break at A, as suggested by the experimental data, and drew some conclusions from this which now appear to have been erroneous. The two branches of the connodal curve at the moment of the division of the plait intersect as two straight lines. This was not sufficiently brought out in the diagram (fig. 11) *. Moreover the curves were drawn in such a manner, that a line parallel to the v -axis had, under certain circumstances, four points of intersection. This was an unnecessary assumption which, in cases of splitting up of the first plait, is not a probable one. Altogether the paragraphs 40-42 with the figures 7, 11, and 12 should be withdrawn.

35. All these results are entirely independent of the form of the equation between p , v , and T for the mixtures. We will now apply van der Waals's equation for some calculations regarding the mixtures investigated.

The equation between p , v , and T has the form

$$\left(p + \frac{a_x}{v^2}\right)(v - b_x) = RT,$$

in which a_x and b_x depend on the composition of the mixture x in the following manner:—

$$\begin{aligned} a_x &= a_1(1-x)^2 + 2a_{12}x(1-x) + a_2x^2, \\ b_x &= b_1(1-x)^2 + 2b_{12}x(1-x) + b_2x^2 \dagger; \end{aligned}$$

a_1, b_1 being the constants for the first and a_2, b_2 for the second substance; a_{12} is the attraction-constant between the two substances, b_{12} the volume-constant for the combination. According to the simple suppositions underlying the deduction of the formula, we should have:

$$\sqrt[3]{b_{12}} = \frac{1}{2} \{ \sqrt[3]{b_1} + \sqrt[3]{b_2} \}.$$

36. Now the three combinations investigated offer a good opportunity for calculating a_{12} and b_{12} . We saw that the

* Kuenen, Proc. Phys. Soc. xiii. p. 545.

† Lorentz, Wied. Ann. xii. p. 134.

same formulæ hold for the critical point of the max. mixture as for a single substance. From the T and p in the maximum point B we can therefore calculate a_x and b_x and deduce from these the a_{12} and b_{12} . The values of a_{12} will give some idea of the relative attraction of the two kinds of molecules, probably quite as accurate an idea as the ordinary a gives of the mutual attraction of the same molecules. The b_{12} obtained may be compared with the theoretical value.

37. The results of my calculations are the following :—

TABLE XVI.

	t_c	p_c	a	b
N_2O	309	71.9	0.00753	0.00197
C_2H_2	308.25	61.02	0.00881	0.00231
CO	304.1	73.26	0.00714	0.00190
C_2H_6	305	48.8	0.01078	0.00286
			a_{12}	b_{12}
$N_2O-C_2H_2$ ($x=0.18$) ...	302.8	65.3	0.00859	0.00239
$C_2H_2-C_2H_6$ ($x=0.51$) ...	292.96	54.7	0.00789	0.002295
$CO_2-C_2H_6$ ($x=0.30$) ...	291.8	61.3	0.008076	0.002357
$CO_2-C_2H_2$ ($x=0.50$) ...	305.55	66.5	0.00792	0.00210

I have added to the table calculated values of a_{12} and b_{12} for the combination $CO_2-C_2H_2$. The mixture of these two gases was proved to have a very narrow border-curve, and consequently the critical point observed cannot have been very much above the hypothetical critical point. The calculation of a_{12} and b_{12} from the critical constants, though not legitimate, must have given values for a_{12} and b_{12} not very far wrong.

The values of b_{12} calculated from the formula in § 35 coincide with the values in Table XVI. in the case of $N_2O-C_2H_6$ and of $CO_2-C_2H_6$: for $C_2H_2-C_2H_6$ b_{12} is even smaller than b for C_2H_2 . a_{12} is in all cases smaller than the mean of the a 's, and for $C_2H_2-C_2H_6$ even smaller than a for C_2H_2 . It is chiefly on this smallness of a_{12} that the existence of a minimum critical temperature depends.

38. Knowing a_{12} and b_{12} , we may calculate the critical

constants for the other mixtures, if supposed to remain undivided. It is with the temperature that we are chiefly concerned.

TABLE XVII.

$N_2O-C_2H_6$.		
x .	$273 \frac{8}{27} \frac{a}{b}$.	T_c
0.18	302.8	302.8
0.43	299.0	299.05
0.55	298.7	299.05
0.76	300.5	300.85
$C_2H_2-C_2H_6$.		
0.27	296.7	296.8
0.41	293.1	294.15
0.51	293.0	293.0
0.61	293.6	293.8
0.68	294.8	296.75
$CO_2-C_2H_6$.		
0.15	296.1	296.4
0.30	291.8	291.8
0.43	290.7	290.62
0.50	290.8	290.88

Table XVII. gives the calculated values of the hypothetical critical temperature as compared with the real critical temperature. The former is found to be everywhere lower than the latter (except for mixture 0.43 $CO_2-C_2H_6$, where the calculated value is $0^{\circ}08$ above the observed T_c , a difference almost within the limits of experimental errors): for the maximum mixture the two naturally coincide. The differences are very small, especially with $N_2O-C_2H_6$ and $CO_2-C_2H_6$. For $C_2H_2-C_2H_6$ they are somewhat higher for mixtures containing more C_2H_6 : this is in agreement with the fact that the critical phenomena were more marked for those mixtures than for the others. In plotting the values of Table XVII. against the x 's, two curves are obtained, connecting the same points C_1 and C_2 and touching at a point corresponding to B. The curves are close together and the minimum of a/b

should belong to approximately the same value of x as the minimum T_c . The existence of a minimum for a/b depends on a_{12}/b_{12} being smaller than both a_1/b_1 and a_2/b_2 *. Table XVIII. gives the values of x for the minimum of a/b and for the observed minimum critical temperature (point A). The two sets agree very closely.

TABLE XVIII.

	Min. a/b .	Min. T_c : A.	Max. pressure.	
			Calculated.	Observed : B.
$N_2O-C_2H_6$...	$x=0.52$	0.50	0.01	0.20
$C_2H_2-C_2H_6$...	$x=0.50$	0.53	0.36	0.51
$CO_2-C_2H_6$...	$x=0.48$	0.45	0.10	0.30

39. In Table XVIII. are also given the mixtures of maximum pressure as observed and as calculated from the theory. The latter calculation is simple. In coexisting phases we have the three conditions that $\frac{\partial \psi}{\partial v}, \frac{\partial \psi}{\partial x}$, and $\psi - v \frac{\partial \psi}{\partial v} - x \frac{\partial \psi}{\partial x}$ are equal. Now ψ may be found from the equation $p = v T$, because $\frac{\partial \psi}{\partial v} = -p$. This leads to

$$\psi = -RT/(v - b_x) - \frac{a_x}{v} + \phi(x).$$

The two first conditions mentioned lead immediately to

$$\left(\frac{1}{v_1} + \frac{1}{v_2}\right) \frac{\partial b_x}{\partial x} = \frac{1}{a_x} \frac{\partial a_x}{\partial x} \dagger,$$

remembering that x is the same in the two phases; v_1 and v_2 are the volumes of the coexisting phases. At the critical point: $v_1 = v_2 = 3b_x$, and the equation becomes

$$\frac{2}{3b} \frac{\partial b_x}{\partial x} = \frac{1}{a_x} \frac{\partial a_x}{\partial x}.$$

There is not a very good agreement between the values

* Van der Waals, *Arch. Néerl.* xxiv. p. 23.

† Van der Waals, *l. c.* p. 23.

calculated from this formula and those observed. The points of agreement are that both sets of x 's are smaller than the x 's for the min. cr. temp., and that the smaller the x observed, the smaller also is the x calculated.

Considering how insufficient van der Waals's formula is, we should not expect a better agreement; still the equation seems to give correct indications as regards the relative position of the two points A and B. This connexion may be better shown in the following manner: the condition for the min. of a_x/b_x may be written thus:—

$$\frac{1}{a_x} \frac{\partial a_x}{\partial x} - \frac{1}{b_x} \frac{\partial b_x}{\partial x} = 0,$$

and the condition for point B

$$\frac{1}{a_x} \frac{\partial a_x}{\partial x} - \frac{2}{3b_x} \frac{\partial b_x}{\partial x} = 0.$$

When $\frac{\partial b_x}{\partial x} > 0$ (in which case also $\frac{\partial a_x}{\partial x} > 0$), as is the case in the three combinations investigated, the first equation gives a higher value of x than the second*.

40. But another question arises—What happens to the maximum at very low temperatures? This can be found from the same equation given above (in 39) by putting v_1 (volume of the liquid) equal to b_x and neglecting $1/v_2$. The result is the same equation as holds for the min. cr. temperature. Probably therefore, if the equation does not mislead us here too much, approximately the same mixture which has a min. cr. temp. will be the mixture of maximum vapour-pressure at low temperatures. We may therefore expect the mixtures of C_2H_6 with N_2O , C_2H_2 , and CO_2 which contain about 0.5 of each to have a max. vapour-pressure at low temperatures. By raising the temperature this maximum moves towards the substance of higher vapour-pressure (N_2O , C_2H_2 , and CO_2). As we saw for $C_2H_2-C_2H_6$, this motion has been very small, the points B and A almost coinciding. It is not impossible that for other mixtures which have a max. pressure at low temperature the maximum reaches one of the components at higher temperature and thus disappears. Those

* Van der Waals, *Kon. Ak.* 29 June, 1895, pp. 11-12.

mixtures would still have a minimum for a_x/b_x and, if the real critical temperatures are not too much above the hypothetical ones, they would also show a minimum critical temperature.

Instances of maximum vapour-pressures are:—ethyl alcohol and carbon disulphide (Berthelot *), chloroform and carbon bisulphide (Guthrie †), ether and carbon disulphide (Guthrie), propyl alcohol and water (Konowalow ‡). Some of these combinations would be worth investigating at higher temperatures up to the critical condition.

41. Similar considerations hold for maximum critical temperature and minimum vapour-pressure. A minimum vapour-pressure at low temperature means, according to the equation, a maximum for a/b : and a maximum of a/b is the same as a maximum of the hypothetical critical temperature, and therefore *a fortiori* of the real critical temperature. The minimum pressure will move towards the substance of lower vapour-pressure, with rise of temperature, and may either disappear or else exist up to the critical region. It is easy to understand what the p - t diagram would be like in this case. But the case has not been observed yet. A minimum vapour-pressure was found by Roscoe and Dittmar § for hydrochloric acid and water and for nitric acid and water, by Konowalow || for formic acid and water, by Friedel ¶ for hydrochloric acid and methyl ether. I hope to have an early opportunity of trying the latter combination, which will not require temperatures as high as the combinations with water.

42. One remark may be added. In the condensation of some substances it is assumed that an association takes place of molecules to more complicated systems. Substances with which that happens do not obey the laws of corresponding states, and van der Waals's formula would be even more inadequate in those cases than otherwise. We may expect similar phenomena to occur in the liquefaction of some

* Berthelot, *Comptes Rendus*, lxvii. p. 430.

† Guthrie, *Phil. Mag.* (5) xviii. p. 513.

‡ Konowalow, *Wied. Ann.* xiv. p. 34.

§ Roscoe & Dittmar, *Lieb. Ann.* cxii. p. 327, cxvi. p. 327.

|| Konowalow, *Wied. Ann.* xiv. p. 34.

¶ Friedel, *Comptes Rendus*, lxxxi. p. 152.

mixtures. An association of that kind one would feel inclined to identify with what is called the formation of molecular compounds. This would naturally have a great influence on the vapour-pressure, and it is not unlikely, that rules like those given above and deduced from van der Waals's formula would fail entirely in those cases. But these considerations can only be tested by experiment.

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DISCUSSION.

Dr. S. P. THOMPSON asked whether diagrams characteristic of cyanogen had been obtained. Its remarkable polymerism suggested an interesting case for critical phenomena.

Dr. KUENEN thought such a substance might be worth investigating.

XXII. *On the Steady Motion of an Electrified Ellipsoid.*

By G. F. C. SEARLE, M.A., Demonstrator in Experimental Physics, Cavendish Laboratory, Cambridge.*

AT the Meeting of the Royal Society on 19th March, 1896, I read a paper on "Problems in Electric Convection." The first part of the paper is printed in the 'Philosophical Transactions of the Royal Society'†, and contains the principles which are required in the solution of any problems about moving charges. The second part of the paper, which deals with the motion of a charged ellipsoid, was not published by the Royal Society. A few of the results are, however, stated in an abstract published in the 'Proceedings'‡. By the permission of the Royal Society I now publish my results for a moving ellipsoid. As frequent reference to my paper in the 'Philosophical Transactions' will be necessary, I shall use the notation §§ 5 and §§ 9 to indicate the paragraph or equation referred to in that paper.

* Read June 25, 1897.

† Phil. Trans. vol. 187 (1896) A. pp. 675-713.

‡ Proc. Roy. Soc. vol. lix. p. 343.

When any system of electric charges moves with uniform velocity through the æther, the electromagnetic field, when referred to axes moving forwards with the charges, can be completely defined by means of a quantity Ψ , as was first shown by Prof. J. J. Thomson*. The electric force $\mathbf{E}$ and the magnetic force $\mathbf{H}$ are simple functions of Ψ . But besides $\mathbf{E}$ and $\mathbf{H}$ there is another vector of great importance, viz. the mechanical force $\mathbf{F}$ experienced by a unit charge moving with the rest of the system. The value of $\mathbf{F}$ I have shown (§ 10) to be given by the vector equation

$$\mathbf{F} = \mathbf{E} + \mu \mathbf{V} \mathbf{u} \mathbf{H}. \quad (1)$$

The equations of the field are (§ 4)

$$\text{curl } \mathbf{F} = 0, \quad (2)$$

$$\mathbf{H} = K \mathbf{V} \mathbf{u} \mathbf{E}. \quad (3)$$

If $v = \frac{1}{\sqrt{K\mu}}$ is the velocity of light, and if α stand for $1 - \frac{u^2}{v^2}$, then when the motion takes place parallel to the axis of x , we have (§ 4)

$$F_1 = -\frac{d\Psi}{dx} \quad F_2 = -\frac{d\Psi}{dy} \quad F_3 = -\frac{d\Psi}{dz}, \quad (4)$$

$$E_1 = -\frac{d\Psi}{dx} \quad E_2 = -\frac{1}{\alpha} \frac{d\Psi}{dy} \quad E_3 = -\frac{1}{\alpha} \frac{d\Psi}{dz}, \quad (5)$$

$$H_1 = 0 \quad H_2 = \frac{Ku}{\alpha} \frac{d\Psi}{dz} \quad H_3 = -\frac{Ku}{\alpha} \frac{d\Psi}{dy}. \quad (6)$$

From these equations, since $\mathbf{E}$ has no divergence,

$$\alpha \frac{d^2\Psi}{dx^2} + \frac{d^2\Psi}{dy^2} + \frac{d^2\Psi}{dz^2} = 0. \quad (7)$$

Here, and throughout the paper, the axes are supposed to move forward with the same velocity as the electrical charges.

Prof. W. B. Morton has considered the motion of an ellipsoid in a paper read before the Physical Society on 27th March, 1896†. He obtains the two following results, viz. : (1) that the distribution of electricity is the same as if

* Phil. Mag. July 1889.

† Proc. Phys. Soc. No. 71, August 1896, p. 180; Phil. Mag. xli. p. 488.

the ellipsoid is at rest, and (2) the value of Ψ when the ellipsoid moves along one of its axes.

Prof. Morton obtains his result by the assumption first made by Mr. Oliver Heaviside, F.R.S.*, that a distribution of electricity on the surface of a charged body such as to give zero disturbance at all points inside the surface is an equilibrium distribution. Since $\mathbf{F}$ satisfies $\text{curl } \mathbf{F} = 0$ and $\mathbf{F}$ vanishes inside the surface, it follows that on the outside of the surface $\mathbf{F}$ is perpendicular to the surface. This implies that Ψ is constant over the surface. But as neither the electric force $\mathbf{E}$ nor the mechanical force experienced by each part of the charged surface (calculated from the Maxwell stress) is normal to the surface, I felt unable to accept the validity of Mr. Heaviside's assumption until I discovered §§ 15 that $\mathbf{F}$ is the mechanical force on an isolated moving unit charge, and that the term $-\mathbf{VGD}$, which appears in the expression for the force experienced by the surface, has no influence in causing convection of electricity from one part of the surface to another. Here $\mathbf{D}$ is the electric displacement, and $\mathbf{G}$ the "magnetic current" $\mu \frac{d\mathbf{H}}{dt}$.

Since Ψ is a true potential for the mechanical force $\mathbf{F}$, I have called Ψ the "electric convection potential."

When there has been established the boundary condition that Ψ is constant over the surface, with its consequence that there is zero disturbance within the surface, it is very easy to show that the distribution on an ellipsoid is the same for motion as for rest. Suppose the ellipsoid to have the same distribution as when it is at rest, so that $\sigma = qp/4\pi abc$, where q is the charge, a, b, c the axes of the ellipsoid, and p the perpendicular from the centre upon the tangent-plane at the point. Through any internal point M as vertex draw a slender double cone intercepting two areas N, N' on the surface. Now the electric force due to a moving point-charge is still radial and still varies inversely as the square of the distance, although it alters with change of direction of the radius vector. Thus it follows just as in electrostatics, since $\sigma \propto p$, that the effects at M of N and N' are exactly equal and

* 'Electrical Papers,' vol. ii. p. 514.

opposite. The whole surface can be treated in the same manner, and thus it follows that $\mathbf{E}=0$ at all internal points. Hence $\mathbf{H}=0$ also. Thus the assumed distribution is in equilibrium and is therefore the actual distribution. Thus the motion has no influence upon the distribution, and this result is true whatever the direction of motion with respect to the axes of the ellipsoid.

In order to find the state of the field near a charged ellipsoid moving with velocity u parallel to the axis of x , it is necessary to find a value of Ψ which shall be constant over the surface of the ellipsoid, shall vanish at infinity, and shall satisfy (7). We see at once that if $f(x, y, z)$ satisfies $\nabla^2 f = 0$, then $f(x/\sqrt{a^2 + \lambda}, y, z)$ satisfies (7). Now from electrostatics we know that

$$\Phi = \int_{\lambda}^{\infty} \frac{d\lambda}{\sqrt{(a'^2 + \lambda)(b^2 + \lambda)(c^2 + \lambda)}},$$

where λ is connected with x, y, z by the relation

$$\frac{x^2}{a'^2 + \lambda} + \frac{y^2}{b^2 + \lambda} + \frac{z^2}{c^2 + \lambda} = 1,$$

satisfies $\nabla^2 \Phi = 0$.

Hence

$$\Psi = \int_{\lambda}^{\infty} \frac{A d\lambda}{\sqrt{(a'^2 + \lambda)(b^2 + \lambda)(c^2 + \lambda)}}, \quad \dots \quad (8)$$

where λ is connected with x, y, z by the relation

$$\frac{x^2}{(a'^2 + \lambda)} + \frac{y^2}{b^2 + \lambda} + \frac{z^2}{c^2 + \lambda} = 1, \quad \dots \quad (9)$$

satisfies (7).

Writing a^2 for aa'^2 , (8) and (9) become

$$\Psi = \int_{\lambda}^{\infty} \frac{A d\lambda}{\sqrt{(a^2 + a\lambda)(b^2 + \lambda)(c^2 + \lambda)}}; \quad \dots \quad (10)$$

$$\frac{x^2}{a^2 + a\lambda} + \frac{y^2}{b^2 + \lambda} + \frac{z^2}{c^2 + \lambda} = 1. \quad \dots \quad (11)$$

This value of Ψ is constant over the surface of the ellipsoid a, b, c , for $\lambda=0$ at all points of this surface; it also vanishes at infinity, and it satisfies (7). It is therefore the value of Ψ required. To find the constant A we make σ have its proper value $q/4\pi bc$ at the end of axis a .

Now

$$\sigma = \frac{K}{4\pi} E_n = \frac{K}{4\pi} E_1$$

at the end of the axis.

But by (5)
$$E_1 = -\frac{d\Psi}{dx}.$$

Again, at $x=a$, $y=z=0$ we have $d\lambda/dx = 2a/\alpha$ and consequently

$$\frac{d\Psi}{dx} = \frac{d\Psi}{d\lambda} \frac{d\lambda}{dx} = -\frac{A}{abc} \cdot \frac{2a}{\alpha}.$$

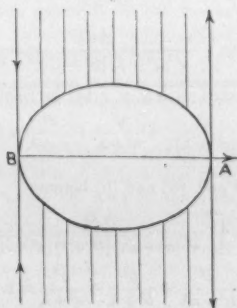
Hence
$$A = \frac{q\alpha}{2K}.$$

Thus, as Prof. Morton has also shown by the same method,

$$\Psi = \int_{\lambda}^{\infty} \frac{q\alpha d\lambda}{2K \sqrt{(a^2 + \alpha\lambda)(b^2 + \lambda)(c^2 + \lambda)}}. \quad \dots (12)$$

Now I have shown {§ 21} that if there is a surface A carrying a charge q , and any surface B is found for which Ψ is constant, then a charge q placed upon B and allowed to

Fig. 1.



acquire an equilibrium distribution will produce at all points not inside B the same effect as the charged surface A.

Hence the ellipsoid (11) when carrying a charge q produces at all points not inside itself exactly the same disturbance as the ellipsoid a, b, c with the same charge.

If we make $a=b=c=0$, the surfaces of equal "convection potential" are the ellipsoids given by

$$\frac{x^2}{a} + y^2 + z^2 = \lambda.$$

They are therefore all similar to each other. Thus the ellipsoid of this form produces exactly the same effect as a point-charge at its centre, and thus an ellipsoid of this form takes the place of the sphere in electrostatics. An ellipsoid with its axes in the ratios $\sqrt{a}:1:1$ I have called a Heaviside Ellipsoid, since Mr. Heaviside* was the first to draw attention to its importance in the theory of moving charges. Whatever be the ratios $a:b:c$, the equipotential surfaces approximate to Heaviside ellipsoids as λ is made very great.

The value of Ψ at the surface λ is $\frac{q\sqrt{a}}{\sqrt{\lambda}}$.

Putting $c=b$ so that we have an ellipsoid of revolution, the axis of revolution being the axis of x , we see by taking $\lambda = -b^2$ that a uniformly-charged line of length $2\sqrt{a^2 - b^2}$ lying along the axis of x produces exactly the same effect as the ellipsoid a, b, b . It may therefore be called its "image." When $b=a$ this length becomes $2au/v$. Thus, when a charged sphere is at rest it produces the same effect as a point-charge at its centre. When the sphere is in motion it produces the same effect as a uniformly-charged line whose length bears to the diameter of the sphere the same ratio as the velocity of the sphere bears to the velocity of light. When $u=v$, so that the sphere moves with the velocity of light, the line becomes the diameter of the sphere; and the same is true for an ellipsoid. Since when $u=v$ each element of the charged line produces a disturbance which is confined to the plane through the element perpendicular to the direction of motion {(46)}, it follows that the disturbance is entirely confined between the planes $x = \pm a$. Between them the electric force is radial to the axis of x and has exactly the same value, viz. $q/aK\rho$, as if the line had been of infinite length and had had the same

* 'Electrical Papers,' vol. ii. p. 514.

line-density $q/2a$. Here ρ stands for $\{y^2 + z^2\}^{\frac{1}{2}}$. The magnetic force is by (3) $qu/a\rho$. Hence the field between the planes $x = \pm a$ is independent of x . There are therefore no displacement-currents except in the two bounding-planes. There is an outward radial current in the front plane and an inward current in the back plane, the total amount of current in each case being qu , equal in amount to the convection-current carried by the ellipsoid.

It appears, however, that at the velocity of light *any* distribution on *any* surface is in equilibrium. For the value of Ψ at any point near a moving point-charge is $\{(43)\}$

$$\Psi = \frac{q\sqrt{\alpha}}{K\sqrt{x^2/\alpha + y^2 + z^2}},$$

and this vanishes when $u=v$ (so that $\alpha=0$), even when $x=0$. Thus the value of Ψ for a point-charge vanishes, and the value of Ψ for *any* distribution being derivable from that for a point-charge by integration, it follows that Ψ has the constant value zero everywhere. Hence the charge is in equilibrium however it may be distributed. The same result follows from the expression $\{\S 19\}$ for the force between two moving charges. When they move parallel to each other with the speed of light the force between them vanishes.

If the ellipsoid is more oblate than Heaviside's the limiting internal surface of ellipsoidal form, whose action is the same as that of the ellipsoid, is a disk of radius $\sqrt{b^2 - a^2/\alpha}$, the axis of the disk coinciding with the axis of x .

The form of the lines of the electric force $\mathbf{E}$ due to an ellipsoid of revolution is easily found. Putting ρ^2 for $y^2 + z^2$, the equilibrium surfaces are given by

$$\frac{x^2}{a^2 + \alpha\lambda} + \frac{\rho^2}{b^2 + \lambda} = 1. \quad \dots \dots (13)$$

Now the mechanical force $\mathbf{F}$ is normal to this surface, and therefore

$$\frac{F_\rho}{F_1} = \frac{\rho(a^2 + \alpha\lambda)}{x(b^2 + \lambda)},$$

where

$$F_\rho^2 = F_2^2 + F_3^2.$$

But by (5),

$$E_1 = F_1 \quad \text{and} \quad E_\rho = F_\rho/\alpha;$$

so that

$$\frac{E_0}{E_1} = \frac{1}{\alpha} \frac{\rho(a^2 + a\lambda)}{x(b^2 + \lambda)} \dots \dots \dots (14)$$

Now consider the conic

$$\frac{x^2}{a^2 + \alpha v} + \frac{\rho^2}{b^2 + v} = 1. \dots \dots \dots (15)$$

The tangent of the angle which the geometrical tangent makes with the axis of x is

$$-\frac{x(b^2 + v)}{\rho(a^2 + \alpha v)} \dots \dots \dots (16)$$

But if the point x, ρ lies on both (13) and (15), it follows that

$$-\frac{x(b^2 + v)}{\rho(a^2 + \alpha v)} = \frac{1}{\alpha} \frac{\rho(a^2 + a\lambda)}{x(b^2 + \lambda)}.$$

Hence by (14) and (16) the electric force is always tangential to the conic (15). But this conic has exactly the same equation as the equilibrium surfaces. Thus the single equation (13) represents both the equilibrium surfaces and the lines of electric force.

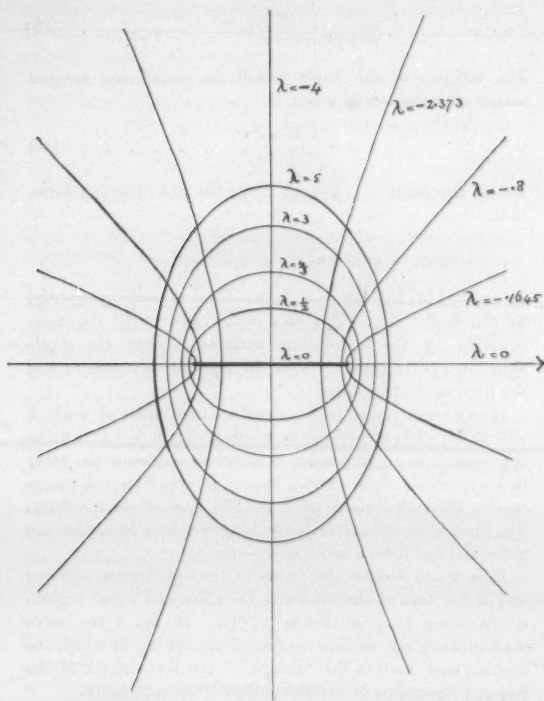
If any point x, ρ be taken, there are two values of λ which will satisfy (13) considered as a quadratic in λ . One value corresponds to an ellipsoidal equilibrium surface; the other to a hyperbolic surface whose lines of intersection with planes passing through the axis of x are the lines of electric force. The lines of electric force for a charged sphere in motion are not radial but form a series of hyperbolas.

Figs. 2 and 3 show the forms of the equilibrium surfaces and of the lines of electric force, for a line and a disk respectively, when $\alpha = \frac{1}{4}$ so that $u/v = .866$. In fig. 2 the curve marked $\lambda = \frac{2}{3}$ is a circle, a section of the sphere of which the line marked $\lambda = 0$ is the "image." The semi-length of the line and the radius of the disk are each taken as unity.

I have attempted to find the lines of the mechanical force **F** these being everywhere perpendicular to the equilibrium surfaces. But the process involved an impracticable integration, and thus led to no result.

I will now write down the values of E and H at any point near the ellipsoid of revolution with axes a, b, b . Instead of λ it will be convenient to take as the parameter of any one of

Fig. 2.



the equilibrium surfaces its x axis and to denote this by h . Thus

$$h^2 = a^2 + a\lambda;$$

and consequently if we put l^2 for $a^2 - ab^2$, so that l is the semi-

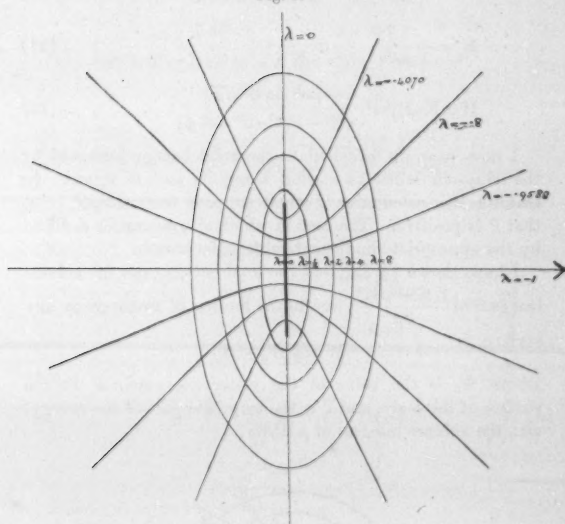
length of the line which is the "image" of the ellipsoid, we have

$$b^2 + \lambda = \frac{h^2 - l^2}{\alpha}.$$

The value of Ψ in terms of h thus becomes

$$\Psi = \frac{q\alpha}{K} \int_h^\infty \frac{dh}{h^2 - l^2} \quad \dots \quad (17)$$

Fig. 3.



Equation (11) now becomes

$$\frac{x^2}{h^2} + \frac{\rho^2 \alpha}{h^2 - l^2} = 1, \quad \dots \quad (18)$$

so that instead of the cylindrical coordinates x and $\rho (= \sqrt{y^2 + z^2})$

we can take h and ϕ where

$$x = h \cos \phi, \quad \rho = \frac{\sqrt{h^2 - l^2}}{\sqrt{\alpha}} \sin \phi. \quad (19)$$

From (18) we have in terms of h and ϕ

$$\frac{dh}{dx} = \frac{(h^2 - l^2) \cos \phi}{h^2 - l^2 \cos^2 \phi}, \quad \frac{dh}{d\rho} = \frac{h \sqrt{h^2 - l^2} \sin \phi \sqrt{\alpha}}{h^2 - l^2 \cos^2 \phi}.$$

Hence

$$E_1 = - \frac{d\Psi}{dh} \cdot \frac{dh}{dx} = \frac{\alpha q \cos \phi}{K(h^2 - l^2 \cos^2 \phi)}, \quad (20)$$

$$E_\rho = - \frac{1}{\alpha} \frac{d\Psi}{dh} \cdot \frac{dh}{d\rho} = \frac{qh \sin \phi \sqrt{\alpha}}{K \sqrt{h^2 - l^2} (h^2 - l^2 \cos^2 \phi)}, \quad (21)$$

$$H = K\mu E_\rho = \frac{q\mu h \sin \phi \sqrt{\alpha}}{\sqrt{h^2 - l^2} (h^2 - l^2 \cos^2 \phi)}. \quad (22)$$

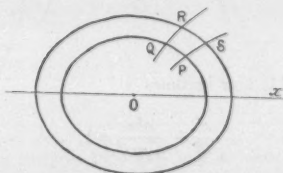
I now pass on to calculate the total energy possessed by the ellipsoid when in motion along its axis of figure. In making the calculation I shall suppose that $a^2 > ab^2$, *i. e.*, that l^2 is positive. The case in which $a^2 < ab^2$ can be deduced by the appropriate mathematical transformation.

I have shown (§ 22) that the total energy, *viz.* the volume integral of $\frac{KE^2 + \mu H^2}{8\pi}$, due to the motion of a charge on any surface, is

$$W = \frac{1}{2} q \Psi_0 + 2T,$$

where Ψ_0 is the value of the convection-potential at the surface of the body, and T is the magnetic part of the energy, *viz.*, the volume integral of $\mu H^2/8\pi$.

Fig. 4.



Now in fig. 4 let the ellipsoid PQ be determined by h , and

the ellipsoid RS by $h+dh$. Let the angular coordinate of P and S be ϕ , and let that of Q and R be $\phi+d\phi$. Then the area PQRS

$$\begin{aligned} &= \frac{d(x,\rho)}{d(h,\phi)} dh d\phi = \left(\frac{dx d\rho}{dh d\phi} - \frac{dx d\rho}{d\phi dh} \right) dh d\phi \\ &= \frac{h^2 - l^2 \cos^2 \phi}{\sqrt{a} \sqrt{h^2 - l^2}} dh d\phi. \end{aligned}$$

Now if the area PQRS revolve about the axis O*x* the volume of the ring traced out is

$$2\pi\rho \frac{d(x,\rho)}{d(h,\phi)} dh d\phi = \frac{2\pi(h^2 - l^2 \cos^2 \phi) \sin \phi}{a} dh d\phi.$$

Thus for the magnetic part of the energy we have

$$\begin{aligned} T &= \iint \frac{\mu H^2}{8\pi} 2\pi\rho \frac{d(x,\rho)}{d(h,\phi)} dh d\phi \\ &= \frac{\mu q^2 u^2}{4} \iint \frac{h^2 \sin^3 \phi dh d\phi}{(h^2 - l^2)(h^2 - l^2 \cos^2 \phi)}. \end{aligned}$$

Since λ goes from 0 to ∞ h goes from a to ∞ . The limits of ϕ are 0 and π .

Now

$$\begin{aligned} \int_0^\pi \frac{\sin^3 \phi d\phi}{h^2 - l^2 \cos^2 \phi} &= -\frac{1}{l^2} \left[\cos \phi - \frac{h^2 - l^2}{2hl} \log \frac{h + l \cos \phi}{h - l \cos \phi} \right]_0^\pi \\ &= \frac{1}{l^2} \left\{ 2 - \frac{h^2 - l^2}{hl} \log \frac{h + l}{h - l} \right\}. \end{aligned}$$

Hence

$$\begin{aligned} T &= \frac{\mu q^2 u^2}{4l^2} \int_a^\infty \left(\frac{2h^2}{h^2 - l^2} - \frac{h}{l} \log \frac{h + l}{h - l} \right) dh \\ &= \frac{\mu q^2 u^2}{4l^2} \left[h - \frac{h^2 + l^2}{2l} \log \frac{h + l}{h - l} \right]. \end{aligned}$$

When h is large the quantity in []

$$= h - l \left(\frac{h^2}{l^2} + 1 \right) \left(\frac{l}{h} + \frac{l^3}{3h^3} \dots \right)$$

vanishing when $h = \infty$.

Thus, making use of $\mu K v^2 = 1$ we have for the magnetic energy

$$T = \frac{q^2 u^2}{4l K v^2} \left\{ \frac{a^2 + l^2}{2l^2} \log \frac{a + l}{a - l} - \frac{a}{l} \right\}.$$

Now by (17) we have at the surface of the ellipsoid

$$\Psi_0 = \frac{q\alpha}{K} \int_a^\infty \frac{dh}{h^2 - l^2} = \frac{q\alpha}{2Kl} \log \frac{a+l}{a-l}.$$

Hence the total electromagnetic energy of the ellipsoid is

$$W = \frac{1}{2} q \Psi_0 + 2T = \frac{q^2}{4Kl} \left\{ \left(1 + \frac{u^2 a^2}{v^2 l^2} \right) \log \frac{a+l}{a-l} - 2 \frac{u^2 a}{v^2 l} \right\}. \quad (23)$$

Here we must remember that $l^2 = a^2 - ab^2$.

(A) *Energy of Heaviside Ellipsoid.* If we put $a/l = S$ and make S large we have

$$\begin{aligned} W &= \frac{q^2 S}{2Ka} \left\{ \left(1 + \frac{u^2 S^2}{v^2} \right) \left(\frac{1}{S} + \frac{1}{3S^3} + \dots \right) - \frac{u^2 S}{v^2} \right\} \\ &= \frac{q^2}{2Ka} \left(1 + \frac{1}{3} \frac{u^2}{v^2} \right) \text{ when } S = \infty. \quad \dots \dots \dots (24) \end{aligned}$$

This corresponds to the Heaviside ellipsoid, for when $S = \infty$ $a^2 = ab^2$. The energy of the same ellipsoid at rest is

$$\frac{q^2 \sqrt{a}}{2Ka} \cdot \frac{v}{u} \sin^{-1} \frac{u}{v}.$$

(B) *Energy of a Sphere.* Putting $b = a$ we have $l = au/v$, and thus

$$W = \frac{q^2}{2Ka} \left(\frac{v}{u} \log \frac{v+u}{v-u} - 1 \right). \quad \dots \dots (25)$$

If u is small compared with v we have

$$W = \frac{q^2}{2Ka} \left(1 + \frac{2}{3} \frac{u^2}{v^2} + \dots \right).$$

It will be found that as far as u^2/v^2 the magnetic energy is

$$\frac{q^2 u^2}{3Kav^2} = \frac{\mu q^2 u^2}{3a}$$

as has been found by Mr. Heaviside*. It follows from this that as far as terms in u^2/v^2 the electric part of the energy is unaltered by the motion.

* 'Electrical Papers,' vol. ii. p. 505.

(C) *Energy of a very slender Ellipsoid.* When the ellipsoid is so slender that b^2/a^2 may be neglected in comparison with unity we have

$$W = \frac{q^2}{2Ka} \left\{ \left(1 + \frac{u^2}{v^2} \right) \log \frac{2a}{b\sqrt{1 - \frac{u^2}{v^2}}} - \frac{u^2}{v^2} \right\}. \quad (26)$$

When u/v is small, this becomes

$$W = \frac{q^2}{2Ka} \left\{ \left(1 + \frac{u^2}{v^2} \right) \log \frac{2a}{b} + \frac{1}{2} \frac{u^2}{v^2} \right\}.$$

(D) *Energy of a Disk.*

When $a^2 < ab^2$ the ellipsoid is more oblate than Heaviside's, and l^2 becomes negative. In this case let us write

$$r^2 = b^2 - \frac{a^2}{\alpha},$$

so that r is the radius of the disk which is the "image" of the ellipsoid a, b . Then writing $\sqrt{-1} = i$ we have from (23)

$$W = \frac{q^2}{4Kir\sqrt{\alpha}} \left(1 - \frac{u^2 a^2}{v^2 r^2 \alpha} \right) \log \frac{1 + i\sqrt{\alpha}r/a}{1 - i\sqrt{\alpha}r/a} + \frac{q^2 u^2 a}{2Kv^2 r^2 \alpha}.$$

But

$$\frac{1}{i} \log \frac{1 + xi}{1 - xi} = 2 \left(x - \frac{x^3}{3} + \frac{x^5}{5} \dots \right) = 2 \tan^{-1} x,$$

so that (23) becomes

$$W = \frac{q^2}{2Kr\sqrt{\alpha}} \left\{ \left(1 - \frac{u^2 a^2}{v^2 r^2 \alpha} \right) \tan^{-1} \frac{r\sqrt{\alpha}}{a} + \frac{u^2 a}{v^2 r\sqrt{\alpha}} \right\}. \quad (27)$$

When $a=0$ we find for the energy of a disk of radius r moving along its axis

$$W = \frac{q^2 \pi}{4Kr\sqrt{\alpha}} \dots \dots \dots (28)$$

In all these cases it will be found that when $u=v$ the energy becomes infinite, so that it would seem to be impossible to make a charged body move at a greater speed than that of light.

DISCUSSION.

Prof. PERRY said the paper would help to solve many problems connected with the effect of the rotation of the earth upon electrical surface changes. An expression might be found for the mechanical and magnetic forces due to the motion of a charge at any point of the earth's surface. At the equator a point moves at different velocity at midday to its midnight velocity; it might now be possible to determine the magnetic and mechanical effects due to electric charges at equatorial points.

Mr. BLAKESLEY asked whether, in calculating the mutual action of two charged particles, proceeding at the velocity of light, it was assumed that the lines of motion were parallel.

Mr. SEARLE said he had always considered parallel lines of motion; he could not say whether the force would be zero in any other case. The results arrived at in the paper could be applied to problems connected with distributions of terrestrial charges.

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G. JOHNSTONE STONEY, M.A., F.R.S.

Secretaries.

T. H. BLAKESLEY, M.A., M.Inst.C.E.

3 Eliot Hill, Lewisham, S.E.

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50 City Road, E.C.

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PROF. S. P. THOMPSON, D.Sc., F.R.S.

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Physical Laboratory, South Kensington, London, S.W.

Other Members of Council.

WALTER BAILY, M.A.

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PROF. J. A. FLEMING, M.A., F.R.S.

R. T. GLAZEBROOK, M.A., F.R.S.

PROF. A. GRAY, M.A., F.R.S.

G. GRIFFITH, M.A.

PROF. G. M. MINCHIN, M.A., F.R.S.

PROF. W. RAMSAY, Ph.D., F.R.S.

J. WALKER, M.A.

Editor of Abstracts.

J. SWINBURNE.

66 Victoria Street, S.W.

Reporter.

R. APPELEYARD.

NAMES OF MEMBERS.

Honorary Members.

Date of Election.	
Feb. 10, 1883.	Boltzmann, Prof. Ludwig.
Feb. 9, 1889.	Bunsen, Prof. R. W. <i>Heidelberg.</i>
Feb. 3, 1877.	Cornu, Prof. M. A. <i>Ecole Polytechnique, Paris.</i>
Feb. 7, 1890.	Hittorf, Prof. W. <i>Münster, Westphalia.</i>
Feb. 3, 1877.	Janssen, P. J. C. Jules. <i>Mouzon, Seine-et-Oise, France.</i>
Feb. 14, 1885.	Mascart, Prof. E. <i>Collège de France, Paris.</i>
Feb. 11, 1882.	Quincke, Prof. G. <i>Heidelberg.</i>
Feb. 9, 1884.	Rowland, Prof. H. A. <i>Baltimore, U. S. A.</i>
Feb. 12, 1892.	Van der Waals, Professor J. D. <i>The University, Amsterdam.</i>
Feb. 12, 1881.	Wiedemann, Gustav.

Ex Officio Member.

Mar. 16, 1878. | The President of the Physical Society of Paris.

Ordinary Members.

The names of Original Members are marked thus **: the names of those who were admitted Members during the first Session of the Society are marked *; and Life Members are marked §.

	** Abel, Sir F. A., Bart., K.C.B., F.R.S. 2 <i>Whitehall Court, S.W.</i>
Jan. 15, 1876.	Abney, Capt. W. de W., R.E., C.B., F.R.S. <i>Rathmore Lodge, Bolton Gardens South, S.W.</i>
Mar. 25, 1876.	Ackroyd, William. <i>Borough Laboratory, Halifax, Yorkshire.</i>
	** Adams, Professor W. G., M.A., F.R.S. <i>King's College, Strand, London, W.C.</i>
May 12, 1888.	Addenbrooke, G. L. 21 <i>Lichfield Street, Wolverhampton.</i>
Dec. 4, 1891.	Anderson, Prof. Alex., M.A. <i>Queen's College, Galway.</i>
Jan. 19, 1878.	Angell, John. 6 <i>Beacon's Field, Derby Road, Fallowfield, Manchester.</i>
Nov. 9, 1894.	Appleyard, R. 139 <i>Church Lane, Charlton, S.E.</i>
Mar. 22, 1895.	Armstrong, Prof. H. E., D.Sc., F.C.S., F.R.S. 55 <i>Granville Road, Lewisham, London, S.E.</i>
Mar. 8, 1895.	Ashford, C. E., M.A., <i>Harrow, Middlesex.</i>
Jan. 22, 1897.	Ashworth, J. R., B.Sc. 105 <i>Frechold Street, Rochdale.</i>
	** Atkinson, Dr. <i>Portesbery Hill, Camberley, Surrey.</i>
Dec. 15, 1877.	Ayrton, Prof. W. E., F.R.S. <i>City and Guilds' of London Institute, Exhibition Road, London, S.W.; and 41 Kensington-Park Gardens, W.</i>

Date of Election.

- Nov. 12, 1887. Bacon, Lieut. R. H. S., R.N.
 Dec. 16, 1876. ||Baily, Walter, M.A. 4 Rosslyn Hill, Hampstead, N.W.
 Nov. 9, 1894. Barlow, William. Hillfield, Muswell Hill, London, N.
 Feb. 23, 1884. ||Barrell, F. R., M.A., B.Sc. University College, Bristol.
 **||Barrett, Prof. W. F., F.R.S.E., M.R.I.A. Royal College of Science, Stephen's Green, Dublin.
 Mar. 11, 1892. Barton, Edwin Henry. University College, Nottingham.
 *||Baynes, R. E., M.A. Christ Church, and 2 Norham Gardens, Oxford.
 June 14, 1895. Beale, S. Banbury Technical School, Banbury.
 Feb. 28, 1896. Bedell, Dr. F. Physical Laboratory, Cornell University, Ithaca, New York, U.S.A.
 May 22, 1886. Bell, Chichester A., M.B. 3 Mansfield Place, Richmond, Surrey.
 Dec. 13, 1895. Bell, H. G., B.A. Bancroft's School, Woodford, Essex.
 June 28, 1895. Berridge, Miss C. E. 17 Rotunda Terrace, Cheltenham.
 Nov. 25, 1892. Bessemer, A. G., Jun. 27 Killieser Avenue, Streatham Hill, London, S.W.
 Mar. 30, 1877. ||Bidwell, Shelford, M.A., LL.B., F.R.S. Riverstone Lodge, Wimbledon Park Road, Southfields, Wandsworth, S.W.
 Mar. 23, 1889. Biggs, C. H. W. 139 Salisbury Court, Fleet Street, E.C.
 May 28, 1881. ||Blaikley, D. J. 13 Elsworthly Terrace, Primrose Hill, London, N.W.
 Nov. 28, 1885. Blakesley, T. H., M.A., M.Inst.C.E. Royal Naval College, Greenwich, S.E.
 May 26, 1877. Blytheswood, Baron. 2 Seamore Place, Curzon Street, W.
 Mar. 8, 1895. Borns, H., Ph.D. 19 Alexandra Road, Wimbledon, Surrey.
 May 13, 1876. ||Bosanquet, R. H. M., M.A., F.R.S.
 *||Bottomley, J. T., F.R.S. University Physical Laboratory, Glasgow.
 Jan. 16, 1875. ||Bottomley, W. The University, Glasgow.
 *||Bourne, C. W., M.A. King's College, Strand, London, W.C.
 Feb. 12, 1892. ||Bower, William Richard. Technical College, Huddersfield.
 June 12, 1880. ||Boys, C. Vernon, F.R.S. 27 The Grove, Boltans, S.W.
 May 24, 1895. Bramwell, Sir F. J., Bart., D.C.L., F.R.S., M.Inst.C.E. 5 Great George Street, Westminster, S.W.
 Feb. 8, 1895. Briscoe, Albert Edward. Battersea Polytechnic, S.W.
 June 8, 1889. ||Brittle, J. R. Fural Villa, Vanbrugh Hill, Blackheath, London, S.E.
 Nov. 21, 1874. ||Brown, Prof. F. D., M.A. University College, Auckland, New Zealand.
 Feb. 12, 1887. ||Brown, J. Loughurst, Dunmurry, Belfast.
 Mar. 12, 1881. ||Browne, Colville. Plantation Cottages, Hextable, Kent.
 Dec. 14, 1894. ||Bryan, G. H., M.A., F.R.S. Thornlea, Trumpington Road, Cambridge.
 Feb. 25, 1882. ||Buchanan, J., B.Sc. Gordon's College, Aberdeen.
 May 10, 1895. ||Buckmaster, C. A., M.A. 16 Heathfield Road, Mill Hill Park, Acton, W.
 Mar. 8, 1895. Burbury, S. H., M.A., F.R.S. 1 New Square, Lincoln's Inn, W.C.
 Nov. 23, 1894. Burke, John, B.A. Owens College, Manchester.

Date of Election.

- Feb. 8, 1895. Burstall, Henry F. W., M.A., Assoc. M.Inst.C.E. *Mason College, Birmingham.*
- Jan. 24, 1896. Burstall, H. R. J. 14 *Old Queen Street, Westminster, S.W.*
- Mar. 10, 1888. Burton, Chas. V., D.Sc. 13 *Gray's Inn Residences, W.C.*
- Nov. 27, 1875. ||Busk, C.
- Apr. 26, 1895. Campbell, A., B.A. 15 *Rothwell Street, Primrose Hill, N.W.*
- Dec. 14, 1894. ||Capstick, J.W., D.Sc., B.A. *Trinity College, Cambridge.*
- May 8, 1896. ||Cardew, Major P., R.E. *Beechcroft, Woking.*
- June 22, 1896. Carter, E. T., F.R.A.S. *Broadclyst, 53 Cloudesdale Road, Upper Tooting, S.W.*
- Dec. 10, 1887. Carus-Wilson, C. A. *McGill University, Montreal; and 2 Holford Road, Hampstead, N.W.*
- Nov. 28, 1885. ||Casella, C. F. 17 *Campden-Hill Square, London, W.*
- Jan. 24, 1896. Cassie, W. *Lewington, Englefield Green, Staines.*
- Mar. 8, 1895. Catchpool, E., B.Sc.
- *||Cecil, Lord Sackville. *Hayes Common, Beckenham.*
- Feb. 28, 1885. Chadwick, O., A.Inst.C.E.
- Apr. 26, 1884. Chattock, A. P. *University College, Bristol.*
- Dec. 11, 1896. Chree, C., M.A., D.Sc. 25 *Mount Ararat Road, Richmond, Surrey.*
- May 12, 1877. ||Christie, W. H. M., M.A., F.R.S. *Royal Observatory, Greenwich, S.E.*
- Nov. 22, 1895. Clark, G. M., B.A. 39 *Eccleston Place, Belgravia, S.W.*
- *||Clark, Latimer, C.E., F.R.S. 11 *Victoria Street, London, S.W.*
- Dec. 10, 1881. ||Clayden, A. W., M.A. *St. John's, Polsloe Road, Exeter.*
- Nov. 27, 1875. ||Clifton, Prof. R. B., M.A., F.R.S. *The Museum, Oxford.*
- Mar. 23, 1889. ||Cockburn, A., F.C.S. 22 *Streantley Road, Brondesbury, N.W.*
- Mar. 11, 1876. ||Coffin, W. H. 94 *Cornwall Gardens, South Kensington, London, S.W.*
- Nov. 10, 1893. Cole, R. S., M.A. 26 *Victoria Place, Devonport.*
- May 16, 1890. ||Coleman, Professor W. M., A.B. *Sam Houston Normal Institute, Huntsville, Texas, U.S.A.*
- Jan. 29, 1876. ||Conroy, Sir John, Bart., M.A., F.R.S. *Balliol College, Oxford.*
- Feb. 22, 1895. Cooper, W. R., M.A. *Carishbrooke, Upper Tulse Hill, S.W.*
- *Core, Prof. T. H. *Owens College, Manchester.*
- *||Cotterill, Prof. J. H., M.A., F.R.S. *Royal Naval College, Greenwich, S.E.*
- June 23, 1893. Cox, John, M.A. *McGill University, Montreal, Canada.*
- June 12, 1875. Crawford and Balcarres, Earl of, F.R.S., F.R.A.S. 2 *Cavendish Square, London, W.*
- Mar. 8, 1895. ||Crawley, C. W. S. 44 *Castletown Road, West Kensington, London, W.*
- *||Crawshaw, Edward. 25 *Tollington Park, London, N.*
- *||Crisp, Frank, B.A., LL.B., J.P. 5 *Lansdowne Road, London, W.*
- Apr. 18, 1890. ||Croft, William Bleaden, M.A. *The College, Winchester.*
- Mar. 23, 1889. ||Crompton, R. E. *Thriplands, Kensington Court, London, W.*
- *||Crookes, W., F.R.S. 7 *Kensington Park Gardens, London, W.*

Date of Election.	
May 12, 1888.	Cunynghame, H. H. <i>Hurlingham Lodge, Fulham, London, S.W.</i>
Mar. 13, 1886.	Curtis-Hayward, A. C., M.A. <i>27 Chancery Lane, London, W.C.</i>
May 12, 1877.	Darwin, Major L., R.E. <i>12 Egerton Place, London, S.W.</i>
Dec. 4, 1891.	Davey, Henry, M.Inst.C.E. <i>3 Princes Street, Westminster, S.W.</i>
Feb. 13, 1875.	Day, R. Evan. <i>135 Fellows Road, London, N.W.</i>
May 26, 1877.	Debus, Dr. H., F.R.S. <i>1 Obere Sophienstrasse, Cassel, Germany.</i>
June 24, 1876.	Dewar, Prof. James, F.R.S. <i>Cambridge; and Royal Institution, Albemarle Street, W.</i>
June 12, 1891.	Dines, William H., B.A.
Feb. 25, 1882.	Dockrell, Staff-Commander H. J., R.N.
Oct. 30, 1896.	Donnan, F. G., M.A., Ph.D. <i>Knebeck Str., 3rd Charlottenberg, Berlin.</i>
	**Donnelly, Maj.-Gen. Sir J. F. D., R.E., K.C.B. <i>Science and Art Department, South Kensington, London, S.W.</i>
Mar. 23, 1889.	Douglass, Sir J. N., F.R.S. <i>Stella House, Bonchurch, I.W.</i>
Apr. 13, 1889.	Downson, J. E., M.Inst.C.E. <i>39 Old Queen Street, Westminster, London, S.W.</i>
	* Durham, Henry. <i>32 Fitzroy Street, London, W.</i>
May 26, 1877.	Dyer, W. T. Thiselton, C.M.G., M.A., B.Sc., F.R.S. <i>Kew Gardens, Surrey.</i>
Feb. 12, 1892.	Edser, Edwin. <i>62 Ringford Road, West Hill, Wandsworth.</i>
Apr. 26, 1895.	Edwards, Miss Eleanor M. Wynn. <i>Ladies' College, Cheltenham.</i>
Dec. 11, 1886.	Elder, H. M., M.A. <i>50 City Road, London, E.C.</i>
Jan. 25, 1895.	Emtage, W. T. A., M.A. <i>49 Schubert Road, Putney, S.W.</i>
June 26, 1891.	Enright, James, B.Sc. <i>202 Holland Road, London, W.</i>
Nov. 27, 1875.	Esson, W., M.A., F.R.S. <i>Merton College, Oxford.</i>
Mar. 8, 1895.	Eumorfopoulos, N., B.Sc. <i>1 Kensington Park Gardens, London, W.</i>
	*Eve, H. W., M.A. <i>University College School, Gower Street, London, W.C.</i>
Mar. 13, 1875.	Everett, Prof. J. D., D.C.L., F.R.S. <i>Queen's College and Derryfolgie Avenue, Belfast</i>
Nov. 22, 1895.	Everett, W. H., B.A. <i>University College, Nottingham.</i>
Feb. 21, 1890.	Evershed, Sydney. <i>1 Woodfield Road, Harrow Road, London, W.</i>
	** Festing, Maj.-Gen. E. R., R.E., F.R.S. <i>The Museum, South Kensington, London, S.W.</i>
Feb. 8, 1879.	Finch, G. B., M.A. <i>1 St. Peter's Terrace, Cambridge.</i>
May 23, 1885.	Fison, A. H., D.Sc. <i>25 Blenheim Gardens, Willesden Green, N.W.</i>
Mar. 23, 1889.	Fitzgerald, D. G. <i>94 Loughborough Road, Brixton, London, S.W.</i>
Feb. 25, 1882.	Fitzgerald, Prof. G. F., M.A., F.R.S. <i>Trinity College, Dublin.</i>
Dec. 14, 1894.	Fitzpatrick, Rev. T. C., M.A. <i>Christ's College, Cambridge.</i>

Date of Election.	
	*Fleming, Prof. J. A., M.A., D.Sc., F.R.S. <i>University College, Gower Street, W.C.</i>
Mar. 25, 1882.	Fletcher, L., M.A., F.R.S. 36 Woodville Road, Ealing, W.
Apr. 14, 1883.	Forbes, Prof. G., M.A., F.R.S. 34 Great George Street, London, S.W.
	** Foster, Prof. G. C., F.R.S. <i>University College, Gower Street, London, W.C.</i>
Nov. 25, 1892.	Fournier D'Albe, Edmund Edward, B.Sc. <i>Prisma, Dalkey, Co. Dublin, Ireland.</i>
June 14, 1879.	Fox, St. George Lane. <i>Travellers' Club, Pall Mall, London, S.W.</i>
Nov. 18, 1876.	Francis, Dr. W., F.R.A.S. <i>Red Lion Court, Fleet Street, London, E.C.</i>
May 12, 1877.	Frankland, Prof. E., D.C.L., F.R.S. <i>The Yews, Reigate Hill, Reigate.</i>
	* Freeman, Rev. A., M.A. <i>Murston Rectory, Sittingbourne, Kent.</i>
Dec. 13, 1895.	Fridlander, E. D. <i>Holyhead Road, Coventry.</i>
Nov. 22, 1895.	Frith, J. <i>The College, Marlborough, Wilts.</i>
Nov. 25, 1892.	Fry, Francis James. <i>Leigh Woods, Clifton, Bristol.</i>
Dec. 4, 1891.	Fulcher, L. W., B.Sc. <i>South Kensington Museum, S.W.; and 56 Burkleigh Road, Streatham Common, S.W.</i>
Apr. 29, 1876.	Fuller, Prof. F., M.A. 9 Palace Road, Surbiton, Kingston-on-Thames.
Feb. 14, 1885.	Fuller, G., M. Inst. C.E. 71 Lesham Gardens, Kensington, London, W.
Mar. 23, 1889.	Galton, Sir D., K.C.B., D.C.L., F.R.S. 12 Chester Street, Grosvenor Place, London, S.W.
Feb. 22, 1895.	Gardiner, J. H. 59 Wroughton Road, Balham, London, S.W.
	*Garnett, Prof. William, M.A., D.C.L.
Nov. 1, 1889.	Garrett, T. A., M.A. 30 The Avenue, Ealing, London, W.
May 13, 1892.	Gerrans, H. T. 20 St. John's Street, Oxford.
Nov. 13, 1896.	Gifford, J. W. <i>Oaklands, Chard, Somerset.</i>
	** Gladstone, Dr. J. H., F.R.S. 17 Pembroke Square, Bayswater, London, W.
Dec. 13, 1879.	Glazebrook, R. T., M.A., F.R.S. <i>Trinity College, Cambridge.</i>
Dec. 14, 1878.	Gleed, W., M.A. <i>Royal Naval College, Greenwich, S.E.</i>
	* Goolden, Walter T., M.A. 62 Warrington Crescent, Maida Vale, W.
Mar. 23, 1889.	Gorham, J. M. 2 Princes Mansion, Victoria Street, London, S.W.
Feb. 27, 1891.	Gray, Professor Andrew, M.A., F.R.S., F.R.S.E. <i>University College of North Wales, Bangor.</i>
Dec. 13, 1879.	Gray, J., B.Sc. <i>H.M. Patent Office, Chancery Lane, London, W.C.</i>
Mar. 13, 1880.	Gray, J. Macfarlane. 79 Mark Lane, London, E.C., and 4 Ladbrooke Crescent, Notting Hill, W.
	**Gray, M. 106 Cannon Street, London, E.C.
Dec. 4, 1891.	Gray, Philip Leman, B.Sc. 15 Francis Road, Edgbaston, Birmingham; and 106 Cannon Street, E.C.
Feb. 13, 1875.	Gray, R. K. <i>Telegraph Works Company, Silvertown, S.E.; and 106 Cannon Street, E.C.</i>

Date of Election.

- June 22, 1878. Grierson, F. W. *Holmwood Cottage, Stafford Road, Walsington, Surrey.*
- Nov. 21, 1874. Griffith, George, M.A. *Harrow.*
- Feb. 22, 1896. Griffiths, A., B.Sc. *Owens College, Manchester.*
- Feb. 8, 1895. Griffiths, E. H., M.A., F.R.S. *12 Parkside, Cambridge.*
- Feb. 10, 1893. Grove, C. E. *Thames Iron Works, Orchard Road, Blackwall.*
- Feb. 23, 1884. ||Grundy, J., Inspector of Mines in India. *27 Chowringhee Road, Calcutta.*
- Mar. 23, 1889. Gumpel, C. G. *22 Christchurch Road, Streatham Hill, London, S.W.*
- **||Gurney, Rev. H. P., M.A. *Rosworth, Gosforth, Northumberland.*
- **Haddon, A. *Royal Naval College, Greenwich, S.E.*
- Mar. 3, 1877. Hall, Samuel. *19 Aberdeen Park, Highbury, London, N.*
- Feb. 14, 1896. Halliday, George.
- *||Harding, Alfred B. *Belmont, Bromley Road, Catford, London, S.E.*
- Feb. 14, 1880. Hare, A. T. *Neston Lodge, East Twickenham.*
- Mar. 12, 1897. Harker, J. A., D.Sc. *22 Mount Ararat Road, Richmond.*
- Apr. 28, 1893. ||Harrison, Fred., M.A. *Dulwich College, London, S.E.*
- Dec. 9, 1882. ||Harrison, H. E., B.Sc. *2 Park Place, Regent's Park, N.W.*
- Nov. 27, 1896. Harrison, W. E. *70 Shardelwer Road, New Cross, S.E.*
- Jan. 24, 1896. Harvey, H. S. S. *157 Church Lane, Charlton, S.E.*
- Jan. 13, 1893. ||Haselfoot, C. E., M.A. *Hertford College, Oxford.*
- Apr. 26, 1896. Hay, A., B.Sc. *University College, Liverpool.*
- May 6, 1882. ||Heaton, W. H., M.A. *University College, Nottingham.*
- Mar. 13, 1896. Henkel, F. W. Jr., F.R.A.S. *73 Pembroke Road, Walthamstow.*
- **Henrici, Prof. O., Ph.D., F.R.S. *City and Guilds' of London Central Institute, South Kensington, London, S.W.; 34 Clarendon Road, Notting Hill, W.*
- May 9, 1885. ||Herroun, E. F. *Riverdale, Humber Road, Westcombe Park, Blackheath, S.E.*
- Mar. 9, 1888. Herschel, Prof. A. S., M.A., F.R.S. *Observatory House, Slough.*
- Mar. 16, 1878. Hicks, W. M., M.A., F.R.S. *Endcliffe Crescent, Sheffield.*
- Dec. 11, 1875. ||Higgins, Clement, M.A. *5 Paper Buildings, Temple, E.C.*
- June 12, 1880. Hilger, A. *204 Stanhope Street, Hampstead Road, London, N.W.*
- Feb. 12, 1876. Hodgkinson, W. R., Ph.D., F.G.S., F.R.S.E. *8 Park Villas, Blackheath, S.E.*
- Dec. 4, 1891. Hoffert, Hermann H., D.Sc. *4 Harewood Grove, Darlington.*
- Feb. 7, 1890. ||Holland, Charles E., B.A. *4 Redington Road, Hampstead, London, N.W.*
- Apr. 22, 1882. ||Hopkinson, Edward, M.A., D.Sc. *Oaksleigh, Timperley, Cheshire.*
- Mar. 16, 1878. ||Hopkinson, J., M.A., D.Sc., F.R.S., M.Inst.C.E. *Holmwood, Wimbledon.*
- **Hopps, J. *Coopers Hill, Staines.*
- Dec. 8, 1893. ||Hovenden, Fredk., F.L.S., F.G.S., F.R.Met.S. *Glenlea, Thurlow-Park Road, West Dulwich, S.E.*

Date of Election.

- Jan. 24, 1884. Howard, Alfred, A.R.S.M., A.N.S.S. 186 Stanstead Road, Forest Hill, S.E.
- May 13, 1876. ||Howard, David. Devon House, Buckhurst Hill, Essex.
- June 9, 1888. ||Howard, James Livsey, D.Sc. 86 St. John's Road, Waterloo, Liverpool.
- Nov. 21, 1874. Huggins, W., D.C.L., LL.D., F.R.S. Upper Tulse Hill, London, S.W.
- Mar. 13, 1880. ||Hughes, Prof. D. E., F.R.S. 40 Langham Street, Portland Place, W.
- **||Huntington, Prof. A. K. King's College, Strand, London, W.C.
- **||Huntington, J. B., M.Inst.C.E. 19 Bedford Row, London, W.C.
- Oct. 30, 1896. Hurst, G. H. J., B.A. Eton College, Windsor.
- *||Hurst, John Thomas. Raymond Villa, Geraldine Road, Wandsworth, London, S.W.
- May 26, 1877. ||Jack, Prof. W., M.A., LL.D. University, Glasgow.
- Mar. 25, 1882. ||Jackson, M. J., B.A., D.Sc. The Hawthorns, Ashurst Wood, nr. East Grinstead.
- Mar. 8, 1879. ||Jacob, F. Messrs. Siemens' Works, Woolwich, London, S.E.
- Feb. 23, 1884. Jervis-Smith, Rev. F. J., M.A. 28 Norham Gardens, Oxford.
- May 22, 1886. ||Johnson, W. Claude. Westcombe Park, Blackheath, London, S.E.
- Feb. 17, 1886. ||Jones, D. E., B.Sc. 4 Conway Road, Cardiff.
- Mar. 11, 1882. Jones, D. Rhys. 19 Paget Street, Cardiff.
- Nov. 27, 1880. ||Jones, H. C. Royal College of Science, South Kensington, London, S.W.
- June 24, 1882. ||Jones, Prof. J. V., M.A., F.R.S. University College, Cardiff.
- Feb. 13, 1891. Joyce, Samuel. Latchford House, Denmark Road, Manchester.
- Apr. 10, 1880. ||Judd, Prof. J. W., F.R.S. Royal College of Science, South Kensington, London, S.W.
- Dec. 15, 1877. ||Judson, J. E., B.A. Charterhouse, Godalming.
- Apr. 26, 1895. Kaye, S. R. Boys' High School, Allahabad, India.
- Feb. 23, 1894. Kearton, J. W. 6 & 8 Lime-Street Square, E.C.
- June 12, 1875. ||Kelvin, The Right Hon. Lord, LL.D., D.C.L., F.R.S. University, Glasgow.
- Mar. 13, 1875. Kennedy, Prof. A. B. W., C.E., F.R.S. 1 Queen-Anne Street, Cavendish Square, W.
- *||Kesselmeyer, C. A. Villa Mon Repos, Altrincham, Cheshire.
- *||Kesselmeyer, W. J. Rose Villa, Vale Road, Bowdon, Cheshire.
- Feb. 23, 1889. Kilgour, Martin H. 3 Clarence Parade, Cheltenham.
- May 28, 1881. Kilner, Dr. Walter, B.A., M.B. 104 Ladbroke Grove Road, London, W.
- Dec. 5, 1874. ||Kinch, Edward. Royal Agricultural College, Cirencester.
- Mar. 2, 1878. ||Kirkman, J. Parke, M.A. 4 St. Augustin's Road, Bedford.
- Apr. 8, 1876. Klaassen, H. M., F.G.S. Abersfeldy, Campden Road, Croydon.
- Mar. 13, 1896. Korda, Desiré. 64 Rue Cammartin, Paris.

Date of Election.

- * ||Lambert, Prof. C. J., M.A. *Royal Naval College, Greenwich, S.E.*
- Jan. 25, 1879. ||Lanckester, Prof. E. Ray, M.A., F.R.S. *2 Bradmore Road, Oxford.*
- Nov. 22, 1895. Larden, W. *37 Haddington Road, Devonport.*
- June 22, 1894. ||Larmor, Prof. Joseph, M.A., D.Sc., F.R.S. *St. John's College, Cambridge.*
- May 27, 1876. ||Law, Channell. *Ilsham Dene, Torquay.*
- May 25, 1889. ||Lawson, Dr. T. A. *114 Alexandra Road, St. John's Wood, London, N.W.*
- Nov. 6, 1891. Lee, Miss Alice, B.Sc., B.A. *Bedford College, Baker Street, W.*
- Nov. 22, 1895. ||Lehfeldt, R. A. *22 South Molton Street, W.*
- June 28, 1895. Leonard, Miss Agatha. *Ladies' College, Cheltenham.*
- May 11, 1889. Lewes, Prof. V. B. *Royal Naval College, Greenwich, S.E.*
- Feb. 28, 1896. Lewis, D. Morgan. *University College, Aberystwyth.*
- Feb. 14, 1880. ||Lewis, T. C., M.A. *Director of Public Instruction, N. W. P. and Oudh, Allahabad, India.*
- Jan. 17, 1882. ||Lewis, Prof. W. J., M.A. *Mineralogical Museum, Cambridge.*
- June 14, 1895. Littlewood, T. H. *Bancroft's School, Woodford Wells, Essex.*
- Feb. 22, 1879. ||Liveing, Prof. G. D., M.A., F.R.S. *Newnham, Cambridge.*
- Mar. 25, 1876. ||Liversidge, Prof. A., M.A., F.R.S. *University of Sydney, New South Wales.*
- Feb. 3, 1877. Lockyer, J. Norman, C.B., F.R.S. *South Kensington Museum, London, S.W.*
- Feb. 13, 1875. ||Lodge, Prof. O. J., D.Sc., F.R.S. *2 Grove Park, Liverpool.*
- Feb. 23, 1884. ||Love, E. F. J., M.A. *The University, Melbourne, Australia.*
- June 10, 1892. Lovibond, Joseph W. *26 St. Ann Street, Salisbury.*
- Nov. 27, 1896. Lowater, Miss Fanny. *Bryn Mawr College, Pennsylvania, U.S.A.*
- Mar. 6, 1891. Lucas, William, M.A. *53 Thornhill Square, Barnsbury, N.*
- * ||Lupton, Sydney, M.A. *A, Audley Mansions, 44 Mount Street, W.*
- Jan. 25, 1879. Macdonell, Prof. A., B.A. *O. E. College, Sibpur, Bengal.*
- Dec. 14, 1894. MacMahon, Major P., R.A., F.R.S. *52 Shaftesbury Mansions, W.C.*
- Nov. 6, 1891. ||McClean, Frank, M.A., F.R.S. *Rusthall House, Tunbridge Wells.*
- Dec. 14, 1878. McGregor, Prof. J. Gordon, M.A., D.Sc. *Dalhousie College, Halifax, Nova Scotia.*
- **M-Leod, Prof. Herbert, F.R.S. *Coopers Hill, Staines.*
- May 11, 1878. ||Magnus, Sir Philip, B.Sc.
- Oct. 30, 1896. Mair, D. B., M.A. *81 Onslow Road, Richmond, Surrey.*
- Nov. 24, 1893. Maitland, Col. E., C.B. *50 Norfolk Square, Hyde Park, W.*
- Nov. 18, 1876. ||Malcolm, Col. E. D., R.E., C.B.
- Mar. 7, 1890. ||Mance, Sir Henry. *32 Earl's Court Square, London, S.W.*
- June 10, 1882. Martin, Major-General C. N., R.E.
- Feb. 14, 1880. Martineau, Miss Caroline A. *1 Clifton Place, Sussex Square, London, W.*
- Mar. 23, 1889. ||Massey, W. H., M.Inst.C.E. *Twynford, R.S.O., Berks.*

Date of Election.

- May 14, 1887. ||Mather, T. 26 *Sumatra Road, West Hampstead, N.W.*
 ** ||Meldola, Prof. R., F.R.S. 6 *Brunswick Square, London, W.C.*
- Mar. 6, 1891. ||Miers, Prof. H. A., F.R.S., F.G.S. *Magdalen College, Oxford.*
 Feb. 13, 1886. ||Mills, A. E., M.A., M.Inst.E.E. 72 *Upper Tollington Park, London, N.*
 ** ||Mills, Dr. E. J., F.R.S. 60 *John Street, Glasgow.*
- Nov. 15, 1889. ||Milne, Prof. J., F.R.S. *Shide Hill House, Isle of Wight.*
 Mar. 13, 1880. ||Minchin, Prof. G. M., M.A., F.R.S. *Coopers Hill, Staines; and Malahide, Englefield Green, Surrey.*
- June 8, 1889. ||Mond, Ludwig, F.R.S. *The Poplars, 20 Avenue Road, London, N.W.*
 Dec. 4, 1891. ||Mond, Robert Ludwig, B.A. (Camb.), F.G.S. 20 *Avenue Road, London, N.W.*
- Apr. 13, 1889. ||Morley, W. M. *Redholm, Loughborough, Leicestershire.*
 May 12, 1877. ||Morley, H. F., M.A., B.Sc. 47 *Broadhurst Gardens, South Hampstead, London, N.W.*
- Mar. 22, 1895. ||Morris, D. K. 13 *Somers Place, Hyde Park Square, London, W.*
- Jan. 27, 1893. ||Morton, Thomas William. 18 *Hogarth Road, Earls Court, W.*
- Mar. 22, 1895. ||Morton, W. B., M.A. 9 *Chilworth Buildings, Stranmillis Road, Belfast.*
- Mar. 8, 1895. ||Mostyn, S. G., M.A. 49 *Grosvenor Road, Canonbury, N.*
 June 28, 1879. ||Moulton, J. F., M.A., Q.C., F.R.S. 57 *Onslow Square, London, S.W.*
- Mar. 23, 1889. ||Mudford, F. J. *Arrow Electrical Works, Jackson Road, Holloway, London, N.*
- ** ||Muirhead, Alexander, D.Sc. 3 *Elm Court, Temple, London, E.C.*
 * ||Müller, Dr. Hugo, F.R.S. 13 *Park Square East, London, N.W.*
- Nov. 27, 1896. ||Mundella, V. A., B.A., B.Sc. *Northern Polytechnic Institute, Holloway, N.*
- Dec. 11, 1896. ||Myers, J. E., Ph.D., M.Sc. 27 *Lovaine Crescent, Newcastle-on-Tyne.*
- Dec. 11, 1886. ||Natanson, Ladislav. *Professor, Cracow University; 32 Wolska str., Cracow (Austria).*
- Feb. 25, 1888. ||Neville, Francis Henry, M.A. *Sidney College, Cambridge.*
 Nov. 12, 1881. ||Newall, H. F., M.A. *Madingley Rise, Cambridge.*
- Oct. 25, 1895. ||Niven, W. D., M.A., F.R.S. *Royal Naval College, Greenwich, S.E.*
- Oct. 25, 1895. ||Noble, Sir Andrew, K.C.B., F.R.S. *Jesmond Dene House, Newcastle-on-Tyne.*
- June 9, 1877. ||Northcott, W. H. 7 *St. Mary's Road, Peckham, London, S.E.*
 Apr. 26, 1895. ||Notcutt, S. A., LL.M., B.A., B.Sc. 98 *Anglesea Road, Ipswich.*
- May 8, 1880. ||Obach, Eugene, Ph.D., F.I.C., F.C.S. *Messrs. Siemens Brothers & Co., Woolwich.*
- Apr. 10, 1875. ||Odling, Prof. W., M.A., F.R.S. *The Museum, Oxford.*

Date of Election.

- Apr. 13, 1889. O'Keefe, E. A., B.E. *City and Guilds' Technical Institute, Leonard Street, London, E.C.; and 6 Staple Inn, Holborn, London, E.C.*
- Feb. 24, 1883. ||Page, F. J. M. 54 *Sutherland Street, Pimlico, S.W.*
- Mar. 23, 1889. ||Paul, R. W. 44 *Hatton Garden, E.C.*
- Jan. 22, 1892. ||Peace, James Bennet, M.A. *Emmanuel College, Cambridge.*
- Feb. 26, 1897. ||Pearson, Prof. K., F.R.S. 7 *Well Road, Hampstead, N.W.*
- *Perigal, Henry, F.R.A.S. 9 *North Crescent, Bedford Square, London, W.C.*
- Jan. 16, 1875. ||Perkin, W. H., F.R.S. *The Chestnuts, Sudbury, Harrow.*
- **||Perry, Prof. J., F.R.S. 8 *Gilston Road, S. Kensington, S.W.*
- May 25, 1878. ||Phelps, W. R. *Montacute House, Ilminster.*
- Feb. 17, 1877. ||Philips, T. W.
- Mar. 13, 1886. ||Pickering, Prof. S. U., M.A., F.R.S. 48 *Bryanston Square, London, W.*
- Jan. 16, 1891. ||Pidgeon, Winter Randall, M.A. 42 *Porchester Square, W.*
- Jan. 16, 1891. ||Porter, Alfred William, B.Sc. *University College, Gower Street, W.C.*
- Nov. 11, 1892. ||Porter, Rev. T. C., M.A. *Eton College, Windsor.*
- Dec. 13, 1879. ||Poynting, Prof. J. H., M.A., F.R.S. *Foxhill, Alvechurch, Worcestershire.*
- Mar. 16, 1878. ||Prance, Miss Edith. *Frognall, Hampstead, London, N.W.*
- Nov. 4, 1876. ||Preece, W. H., C.B., C.E., F.R.S. *Gothic Lodge, Wimbledon.*
- Mar. 25, 1876. ||Preston, Dr. S. Tolver. *Stellinger Chaussee, Lokstedt, near Hamburg.*
- June 24, 1882. ||Price, Rev. Prof. Bartholomew, M.A., F.R.S. 11 *St. Giles', Oxford.*
- May 8, 1886. ||Price, W. A., M.A. 5 *Hardy Road, Westcombe Park, Blackheath, S.E.*
- Nov. 18, 1876. ||Purser, Prof. J., M.A. *Queen's College, Belfast.*
- Mar. 26, 1897. Quick, J. 8 *Upper Bedford Place, W.C.*
- June 27, 1885. ||Ramsay, Prof. W., Ph.D., F.R.S. *University College, Gower Street, London, W.C.; and 12 Arundel Gardens, W.*
- Dec. 9, 1892. Rawson, Major Herbert E., R.E.
- Dec. 15, 1877. ||Rayleigh, The Right Hon. Lord, M.A., D.C.L., F.R.S. *Terling Place, Witham, Essex; and 4 Carlton Gardens, London, S.W.*
- **||Reinold, Prof. A. W., M.A., F.R.S. *Royal Naval College, Greenwich, S.E.*
- Nov. 27, 1875. ||Reynolds, Prof. Osborne, M.A., F.R.S. *Owens College, Manchester.*
- Apr. 26, 1895. Rheam, W., B.Sc. *Freshfield, near Liverpool.*
- Apr. 13, 1894. ||Rhodes, W. G., M.Sc. *Royal Technical Institute, Peel Park, Salford, Manchester.*
- June 12, 1886. ||Rideal, S., D.Sc. 28 *Victoria Street, Westminster, S.W.*
- May 10, 1895. ||Rigby, Rev. E. *Lowe House, St. Helens, Lancashire.*
- June 12, 1886. ||Rimington, E. C. *Lynton House, The Park, Church Road, Norwood, S.E.*
- Nov. 9, 1894. Rintoul, David, M.A. 4 *Northcote Road, Clifton, Bristol.*

Date of Election.

- May 26, 1888. ||Roberts, Samuel Oliver, M.A. *Merchant Taylors' School, Charterhouse Square, London, E.C.*
 **Roberts-Austen, Prof. W. Chandler, C.B., F.R.S. *Royal Mint, London, E.*
- May 11, 1889. Robinson, W., M.E., A.M.Inst.C.E. *University College, Nottingham.*
- Feb. 22, 1895. ||Rodger, J. W. 80 *Anerley Park, London, S.E.*
- May 8, 1880. ||Roscoe, Prof. Sir Henry E., Ph.D., F.R.S. 10 *Bramham Gardens, Wetherby Road, London, S.W.*
- Jan. 24, 1885. ||Rose-Innes, J., B.Sc. 5 *Gayton Crescent, Hampstead, N.W.*
- June 26, 1875. ||Rosse, The Earl of, D.C.L., F.R.S. *Parsonstown, Ireland.*
 *||Rücker, Prof. A. W., M.A., F.R.S. 19 *Gledhow Gardens, London, S.W.*
- Jan. 19, 1878. ||Russell, J. W., M.A. 16 *Bardwell Road, Oxford.*
- June 24, 1876. Russell, The Hon. F. A. R., M.A. *Pembroke Lodge, Richmond Park, Surrey.*
- Mar. 2, 1878. ||Russell, W. J., Ph.D., F.R.S. 34 *Upper Hamilton Terrace, St. John's Wood, London, W.*
- Dec. 13, 1895. Rylands, L. G., B.A. 34 *Grosvenor Road, Newcastle-on-Tyne.*
- May 26, 1877. ||Sale, Capt. M. T., R.E., C.M.G. *Ugia Bank, Abbey Wood, Kent.*
- Mar. 25, 1876. ||Salisbury, The Marquis of, D.C.L., F.R.S. *Hatfield House, Herts.*
- Jan. 15, 1876. ||Salomons, Sir D. L., Bart. *Broom Hill, Tunbridge Wells.*
- Jan. 30, 1875. ||Salter, M. J. *Northcote, Beaconsfield Road, New Southgate, London, N.*
- June 27, 1885. ||Sanderson, F. W., B.A. *The School House, Oundle.*
- Dec. 9, 1882. ||Saunders, Rev. S. T. H., M.A. *Merchant Taylors' School, London, E.C.; and Kirkstall, 44 Christchurch Avenue, Brondesbury, N.W.*
 **||Schuster, Prof. A., Ph.D., F.R.S. *Owens College, Manchester.*
- Feb. 21, 1883. Scott, Prof. A. W. *St. David's College, Lampeter, South Wales.*
- Dec. 19, 1894. ||Searle, G. F. C., M.A. 11 *Fitz-William Street, Cambridge.*
- Mar. 17, 1877. ||Seaton, W. S. *Montague House, Richmond, Surrey.*
- May 9, 1885. ||Selby, A. L., M.A. *University College, Cardiff.*
- Mar. 8, 1895. Shaw, P. E., B.A., B.Sc. *University College, Nottingham.*
- Feb. 8, 1879. ||Shaw, W. N., M.A., F.R.S. *Emmanuel College, Cambridge.*
- Nov. 6, 1891. Shenstone, W. A. *Clifton College, Bristol.*
- Dec. 13, 1879. Shettle, R. C., M.D. *Carlton Lodge, Nailsworth, Gloucestershire.*
- Mar. 7, 1890. ||Shorter, Lewis R. 21 *Campden Hill Road, Kensington, W.*
- Oct. 25, 1895. ||Shurlock, F. W., B.A., B.Sc. *Leigh House, Devizes Road, Swindon, Wilts.*
- Apr. 13, 1889. ||Siemens, A. 12 *Queen Anne's Gate, London, S.W.*
- Jan. 22, 1881. ||Simpson, G. P., B.Sc. 12 *Queen Anne's Gate, London, S.W.*
- Dec. 10, 1881. ||Sircom, Prof. the Rev. Sebastian, M.A. *Presbytery, Holy Name, Oxford Road, Manchester.*
- Dec. 14, 1894. ||Skinner, S., M.A. *Cromwell House, Trumpington, near Cambridge.*
- Jan. 16, 1891. Skirrow, Benjamin Beck, B.A. *Yr Hen Dy, New Road, Llanelly.*
- May 10, 1884. Slade, Lieut. E. J. W., R.N.

Date of Election.

- Apr. 28, 1888. Slater, A. W.
 Dec. 13, 1895. ||Slater, R. C. *Charterhouse, Godalming, Surrey.*
 Nov. 23, 1878. ||Sleeman, Rev. P. R., F.R.A.S., 65 *Pembroke Road, Clifton, Bristol.*
 *||Smith, Basil Woodd, F.R.A.S. *Branch Hill Lodge, Hampstead Heath, London, N.W.*
 Feb. 7, 1890. ||Smith, Edward Wythe. 66 *College Street, Chelsea, London, S.W.*
 Apr. 10, 1880. ||Smith, W. O.
 June 26, 1896. ||Sodeau, W. H., B.Sc. 25 *Shore Road, South Hackney, N.E.*
 June 8, 1878. ||Solly, R. H., M.A., F.G.S. *Downing College, Cambridge.*
 Apr. 29, 1895. Soper, H. E., M.A. 1 *Christchurch Terrace, Cheltenham.*
 May 12, 1883. Soward, A. W. 28 *Therapia Road, Honor Oak, London, S.E.*
 Mar. 23, 1889. Spagnoletti, C., M.Inst.C.E. 2 *Craven Terrace, Ealing, W.*
 Mar. 8, 1895. ||Stansfield, H. S.-W. *London Polytechnic, Manresa Road, Chelsea.*
 Feb. 25, 1882. ||Stanley, W. Ford, F.G.S. *Cumberlow, South Norwood, London, S.E.*
 June 11, 1881. ||Stead, J. E. *Redcar.*
 June 28, 1895. ||Stewart, J. J., B.A., B.Sc. *University College, Nottingham.*
 *Stirling, Hon. Sir James, M.A. *Finchcocks, Goudhurst, Kent.*
 Dec. 15, 1877. ||Stocker, Prof. W. N., M.A. *Coopers Hill, Staines.*
 Nov. 18, 1876. ||Stoney, G. Johnstone, M.A., D.Sc., F.R.S. 8 *Upper Hornsey Rise, London, N.*
 Mar. 13, 1880. ||Stroh, Augustus. 98 *Haverstock Hill, London, N.W.*
 June 9, 1893. ||Stroud, H., D.Sc., M.A. *Durham College of Science, Newcastle-on-Tyne.*
 Feb. 26, 1887. ||Stroud, Prof. W., D.Sc. *Yorkshire College, Leeds.*
 Dec. 10, 1887. ||Sumpner, W. E., D.Sc. 19 *Greenfield Crescent, Edgbaston, Birmingham.*
 Dec. 6, 1889. ||Swan, J. W., F.C.S. 58 *Holland Park, W.*
 May 22, 1886. ||Swinburne, J. 66 *Victoria Street, S.W.*
 Feb. 8, 1895. Swinton, A. A. Campbell, A.M.Inst.C.E. 66 *Victoria Street, S.W.*
 May 27, 1876. Taylor, Herbert A. 25 *Collingham Road, London, W.*
 Apr. 26, 1895. Thomas, E., B.Sc. 36 *Sisters' Avenue, Clapham Common, S.W.*
 May 10, 1895. Thomas, G. L., B.Sc. 175 *East-Dulwich Grove, S.E.*
 Dec. 11, 1875. ||Thompson, Prof. S. P., D.Sc., F.R.S. *Finsbury Technical College, Leonard Street, London, E.C.*
 Nov. 27, 1875. Thomson, Prof. John Miller. 85 *Addison Road, Kensington, W.*
 Mar. 22, 1895. ||Thomson, Prof. J. J., M.A., F.R.S. 6 *Scroope Terrace, Cambridge.*
 May 8, 1896. Thornton, A., M.A. 67 *St. Mary's Terrace, Bradford, Yorks.*
 Feb. 13, 1891. Thorp, Walter, B.Sc. *Analytical Laboratory, Limerick.*
 **||Thorp, William, B.Sc. 24 *Crouch Hall Road, Crouch End, London, N.*
 Nov. 28, 1885. ||Thorpe, Prof. T. E., Ph.D., F.R.S. *Government Laboratory, Somerset House, W.C.*
 Mar. 9, 1889. ||Threlfall, Prof. R., M.A. *The University, Sydney.*

Date of Election.	
	* Tomlinson, Herbert, B.A., F.R.S. <i>South-West London Polytechnic, Manresa Road, Chelsea.</i>
Dec. 8, 1888.	Trotter, Alexander Pelham, B.A. <i>Treasury, Cape Town.</i>
Mar. 8, 1895.	Trouton, F. T., B.A., Sc.D. <i>Caerleon, Killiney, Co. Dublin.</i>
Dec. 2, 1876.	Tunzelmann, G. W. von. <i>65 Longridge Road, Earl's Court Road, London, S.W.</i>
Mar. 26, 1897.	Turner, Dawson, B.A., M.D., F.R.C.P. (Ed.). <i>37 George Square, Edinburgh.</i>
	** Unwin, Prof. W. C., B.Sc., F.R.S. <i>City and Guilds' of London Central Institute, Exhibition Road, London, S.W.</i>
Nov. 22, 1895.	Vincent, J. H., B.Sc. <i>Royal College of Science, South Kensington, S.W.</i>
June 9, 1893.	Wade, John, B.Sc., F.I.C. <i>Wyvenhoe, Purley, Surrey.</i>
Nov. 27, 1875.	Waghorn, J. W. W., R.N., D.Sc. <i>Royal Naval College, Greenwich, S.E.</i>
Dec. 1, 1883.	Walker, J., M.A. <i>30 Norham Gardens, Oxford.</i>
Dec. 5, 1874.	Walker, J. F., M.A. <i>45 Bootham, York.</i>
	** Walker, J. J., M.A., F.R.S. <i>12 Denning Road, Hampstead, London, N.W.</i>
Mar. 7, 1890.	Waller, Dr. Augustus D., F.R.S. <i>Weston Lodge, 16 Grove End Road, N.W.</i>
Nov. 24, 1888.	Walmsley, R. M., D.Sc. <i>Northampton Institute, London, E.C.</i>
Mar. 25, 1892.	Walter, Miss L. Edna, B.Sc. <i>38 Woodbury Grove, Finsbury Park, London, N.</i>
Dec. 11, 1891.	Watson, William, B.Sc. <i>7 Upper Cheyne Row, S.W.</i>
Mar. 8, 1895.	Watt, J. <i>Bradfield College, Berks.</i>
	**Watts, Dr. W. M. <i>Giggleswick Grammar School, and Goldielands, Settle.</i>
Dec. 11, 1886.	Wertheimer, Prof. Julius, B.A., B.Sc. <i>Merchant Venturers' Technical College, Bristol.</i>
June 9, 1877.	Whalley, L. J. <i>26 Park Place, Greenwich, S.E.</i>
Dec. 14, 1894.	Whetham, W. C. D., M.A. <i>Trinity College, Cambridge.</i>
May 10, 1895.	White, S. C. F., M.A. <i>King's College, W.C.</i>
Dec. 15, 1894.	Wilberforce, L. R., M.A. <i>6 Pemberton Terrace, Cambridge.</i>
Mar. 10, 1893.	Williams, W. <i>Physical Laboratory, Royal College of Science, South Kensington.</i>
Feb. 8, 1895.	Wilson, W. E., F.R.S., J.P. <i>Daramona, Streete, Westmeath, Ireland.</i>
Feb. 9, 1889.	Wimshurst, James. <i>7 Crescent Grove, Clapham Common, London, S.W.</i>
Mar. 13, 1880.	Wingfield, Lieut. G. T., R.N.
Jan. 22, 1887.	Womack, Frederick, B.Sc., M.B. <i>9 Dennington Park Road, West Hampstead, London, N.W.</i>
Dec. 15, 1877.	Wood, Sir H. T., B.A. <i>Society of Arts, Adelphi, London, W.C.</i>
Feb. 28, 1896.	Wood, S. <i>South Kensington Museum, S.W.</i>
May 8, 1880.	Woods, Edward, C.E. <i>45 Onslow Gardens, South Kensington, London, S.W.</i>
Dec. 10, 1881.	Woolcombe, W. G., M.A., B.Sc., F.R.A.S. <i>King Edward's High School, New Street, Birmingham.</i>

Date of Election.	
Jan. 24, 1884.	* Wormell, Dr. R. <i>Cowper-Street Schools, London, E.C.</i> Worthington, Prof. A. M., M.A., F.R.S. <i>R. N. E. College,</i> <i>and 6 Osborne Villas, Devonport.</i>
Mar. 27, 1886.	Wright, A. R. <i>46 Marmora Road, Honor Oak, London,</i> <i>S.E.</i>
Dec. 13, 1880.	Wrightson, T., C.E. <i>Norton Hall, Stockton-on-Tees.</i>
Oct. 28, 1892.	Wyatt, G. H., B.Sc. <i>Emmanuel School, Wandsworth Common,</i> <i>S.W.</i>
Feb. 27, 1886.	Young, Sydney, D.Sc., F.R.S. <i>University College, Bristol.</i>
Feb. 13, 1891.	Yule, G. U. <i>13 Addison-Court Gardens, Blythe Road, W.</i>

